Optimization and control techniques for cyber-physical systems: a new paradigm

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***Abstract** – Our research tackles fundamental control problems on complex systems, known as cyber-physical systems, with a particular focus on the rich class known as switching systems. We show that using state-of the art optimization techniques, such as sum-of-squares programming, and taking into account the logical structure of the system, we may obtain powerful control tools for problems which, when asked on such complex systems, are known to be extremely hard in general.*

Dynamical systems are models that describe the evolution in time of physical quantities. Typical modern systems involve many non-idealities, like disruptions in the feedback loop, nonlinearities, safety regulations (these can be listed as a set of requirements, sometimes with no clear mathematical translation), quantization due to communication channels, etc. Classical control techniques have not been designed for such situations, and making use of these classical techniques can lead to severely sub-optimal solutions. Typically, in order to address at best all these non-idealities, one resorts to the *bisimulation technique*, a fundamental tool to model and control Cyber-Physical Systems [6]. In a nutshell, this tool allows to represent a (complicated, continuous-time, nonlinear,...) system by a *finite abstraction, say, an automaton*.

Let us consider a practical and simple example, where a dynamical system is subject to failures of its controller. We represent failures by a state that evolves in time. When $\sigma(t) = 1$, it means that the controller behaves as expected, and when $\sigma(t) = 2$, it means a failure occurred. In many applications, we may assume that the failure mode cannot occur more than a few times in a row, and one may enquire about the behavior and stability properties of the system, under this assumption (see [4]). A typical way of approaching this example is by modeling it as a *constrained switching system*.

A switching system is a multi-modal system, that is, a system whose law of evolution can change from time to time. These models encapsulate the complexity of many different applications, ranging from the optimal design of drug treatment for viral diseases like HIV [1], to the analysis of consensus systems of autonomous agents [2], and other applied and theoretical topics [3].

They are described by the following difference equation, $x_{t+1} = A_{\sigma(t)}x_t$ where $x_t \in \mathbb{R}^n$ is the state of the system at time t, $\sigma(t)$ is the mode at time t, and where we may consider that the sequence of modes must satisfy some rules. For our example, the rule could be "no more than 2 failures in a row". These rules can be encoded using an automaton, as in Fig. 1.

Applying classical control techniques to modern cyber-physical systems may lead to severely suboptimal solutions.

We developed several tools for the analysis of these systems, where the dynamics applied follows rules that can be modeled by an automaton. Our tools leverage techniques from automata theory, optimization, and Lyapunov control. They generalize and unify previously known techniques into a powerful analysis framework. The philosophy of our technology is not to apply blindly off-the-shelf optimization techniques (like so-called LMIs criteria available in the literature), but rather to leverage our knowledge of both the physical and logical structure of the system under study in order to apply a 'smart control' approach.

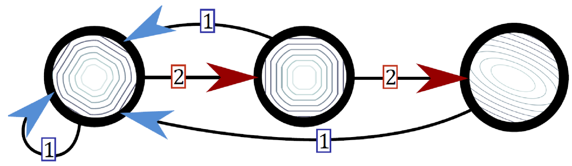

Figure 1: The automaton does not accept mode "2" for more than 2 times in a row. In our work, we leverage this combinatorial a priori knowledge of the system, combining it with semidefinite optimization [4] or sum-of-square techniques [5].

A well-known problem of the bisimulation technique is that, contrary to the simple example we have just seen, the size of the obtained automaton, which models the actual system, is typically huge. This often hampers our ability to use the bisimulation technique on practical systems.

Our approach alleviates this pitfall: instead of computing an abstraction of a particular system, and then looking for a control solution on it, *our technique allows to co-design the abstraction and the control solution*, thereby circumventing the curse of dimensionality by leveraging our physical knowledge of the system, without ending up with a huge representation of the system. We believe that such techniques will unlatch our ability to apply Cyber-Physical Control techniques to practical systems, which is currently strongly limited by the curse of dimensionality.

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