Ring epimorphisms, Gabriel topologies and contramodules

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During the 1960s considerable work was done by Mac Lane in order to understand the meaning of "epimorphism". The subject got particular attention in categories of rings where the abstract category-theoretic meaning is now of common use.

The notion of ring epimorphism has relations with torsion theory and localization theory. In particular, perfect right Gabriel topologies ([3, Ch.IX]) correspond bijectively to left flat ring epimorphisms ([3, Ch. XI]).

In this mini-course, we will introduced two classes of modules defined in term of a ring epimorphism: the comodules and the contramodules ([6]).

Adding some conditions on the ring epimorphism we will extend classical results from commutative algebra ([5]) showing an equivalence between suitable subcategories of the two classes of modules.

Prerequisites

1. Basics on Categories: functors, natural transformations ([3, Ch.1] or [4, Ch.1]).
2. Basics on Module Categories: (bi)modules, isomorphism theorems, Hom and Tensor product functors, exact sequences, projective, injective and flat modules ([3, Ch.1] or [1, Ch.1 and 5] or [2, Ch. 2 and 3]).

References

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