

Categories for universal algebra

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In categorical algebra some structural properties of varieties of universal algebras are investigated by replacing the arguments involving elements and algebraic operations with other ones using relations and commutative diagrams. A typical example is provided by the study of *Mal'tsev categories* [1]: instead of requiring the existence, in the algebraic theory, of a ternary term $p(x, y, z)$ verifying the identities $p(x, y, y) = x$ and $p(x, x, y) = y$, one asks that any (internal) reflexive relation is an equivalence relation. This categorical property is strong enough to establish, in the regular context, many of the well known properties of Mal'tsev varieties (in the sense of Smith [2]).

In this mini-course, some fundamental ideas of this approach to Mal'tsev categories will be first presented (see the survey [3], and the references therein). Then some recent results concerning *Goursat categories* [4, 5, 6, 7] will be explained, which are useful to illustrate many of the links between exactness properties in categorical algebra, the so-called Mal'tsev conditions in universal algebra, and the validity of suitable homological lemmas.

References

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