Frames and locales: topology without points

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Point-free topology is the study of the category Loc of locales and localic maps and its dual category Frm of frames and frame homomorphisms (see [1]). In some respects the category Loc is nicer than its classical counterpart Top (of topological spaces and continuous maps): it contains many objects that are not space-like, it can be analysed by algebraic methods, and it provides tools that are not so obvious from a Top perspective.

In this mini-course, an overview of the basic ideas and motivation for point-free topology will be presented. We plan to cover the following topics, explaining the similarities and dissimilarities with the classical setting and emphasizing the new features.

1. Frames: the algebraic facet of spaces.
From topological spaces to frames: the category Frm of frames and frame homomorphisms; the (contravariant) open functor $\mathcal{O}\colon \mathsf{Top} \to \mathsf{Frm}$. Other relatives of frames: semilattices and distributive lattices.

2. Categorical aspects of Frm.
Galois adjunctions. Frames as complete Heyting algebras. Categorical behaviour of Frm.

3. Locales: the geometric facet of frames.
Making the picture covariant: the category Loc of locales and localic maps; the covariant functor $\Omega\colon \mathsf{Top} \to \mathsf{Loc}$. Points of a locale. The spectrum functor $\Sigma\colon \mathsf{Loc} \to \mathsf{Top}$. The adjunction $\Omega \dashv \Sigma$. The lattice of sublocales of a locale.

4. Doing topology in Loc.
Locales as generalised spaces. Some illustrative examples, results and constructions.

References

[1] J. Picado and A. Pultr, *Frames and locales: Topology without points*, Frontiers in Mathematics, vol. 28, Springer, Basel (2012).