In this thesis, we study a variation of the two-dimensional Euler's equations where the topography of the basin is a non constant function. This variation of the model is referred to as the lake equations. The non homogeneity of the topography makes the fluid behaves as if it was a compressive one, modifying the behaviour of singular vortices by making them follow lines of constant depth. This behaviour highly contrasts with what is known for the 2D Euler's equations, where the vortex core is known to move according to the global geometry of the domain. Although formal sketches were given in the study of vortices in lakes, no mathematical proof of the existence of singular vortices was ever given for the lake equations. We show in our work that the lake model is robust enough to yield singular-vortex solutions, and we give a mathematical justification of the vortex core evolution law. We give other interpretations of folklore results in this topic, based on relevant physical conserved quantities. A particular attention is drawn on the steady case, where the vortex core remains unchanged in time. In this case, we propose to describe the behaviour of the vortex core through the theory of rearrangements of sets and functions. This theory enters the larger field of measure theory and functional analysis. The measure-theoretic approach allows us to relax the required regularity to study the problem, making reachable cases of irregular lakes with vanishing topography. We prove that one can construct steady vortex pairs by energy maximisation, in a lake where some Coriolis force has been prescribed. The first order behaviour of such pair was unknown from the literature, and we prove that at second order, the system follows the same rules available for the 2D Euler's equations. These results fill a conceptual gap between the lake and 2D Euler's equations. The techniques of proof to attack the singular vortex problem require adaptation of symmetrisation techniques and tools from calculus of variation. In these topics, we also solve an open problem concerning the almost-sure convergence of shape symmetrisation along Markov processes, and we investigate an interesting question about the strict convergence of vector valued measures, whose study turned out to give rise to new interplays between measure-theory and convex analysis.