Invitation à la soutenance publique de thèse

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Applications of the line bundle Laplacian to spanning forests and related lattice models

Over the last thirty years, conformal field theories (CFTs) have proved to be a powerful tool in physics to study two-dimensional lattice models at their critical point. In the scaling limit, discrete statistical correlators are expected to converge to those of a quantum field theory. In the simplest cases, these correlators are rational functions of the distances separating the insertion points. However, correlators with logarithms may also appear in so-called logarithmic conformal field theories (LCFTs), which possess a richer and more complicated structure than their rational counterparts.

One way to gain insight into LCFTs is by computing exact correlators in lattice models believed to be described by such theories. Among them is the Abelian sandpile model (ASM), which was introduced as a toy model for dynamical systems that naturally evolve toward their critical state. Its most natural observables are the height variables, which are controlled by a nonlocal probability measure. Computing height correlations in the ASM amounts to enumerating spanning trees with certain nonlocal properties, which is quite cumbersome via standard methods of graph theory. The first objective of this thesis consists therefore in using a recent and more efficient technique to produce explicit results, which is based on a connection encoded in the so-called line bundle Laplacian.

On the mathematical side, Schramm-Loewner evolutions (SLEs) were developed in the early 2000s as a more rigorous approach to study scaling limits of random paths and interfaces appearing in two-dimensional critical lattice models. Since then, SLEs have been proved to describe the continuum limit of a few well-known models, such as percolation, the Ising model, and the loop-erased random walk. One of the simplest explicit computations in the SLE framework is Schramm's formula, which gives the probability that a random curve between two boundary points of a continuous domain goes left of a marked interior point. The second objective of this thesis is to establish a discrete equivalent of this formula for a specific model, namely the loop-erased random walk (LERW), via its relation with spanning trees. The main tool is the line bundle Laplacian, which is also used to extend Schramm's formula to multiple LERWs. Explicit results are then compared to SLE and CFT predictions.