Invitation à la soutenance publique de thèse

Pour l’obtention du grade de Docteur en Sciences

Monsieur Alban JAGO
Master en sciences physiques à finalité approfondie

Traces, fixed points and quantization of symmetric spaces

Since the advent of quantum mechanics, we know that the microscopic world exhibits some strange quantum effects that do not occur in our usual classical world. Whereas the measurable quantities of a classical system (the energy, the position, ...) are described by functions – which form a commutative algebra for the pointwise product -, those quantum effects are well described using noncommutative algebras of operators instead. For instance, the potentially discrete spectrum of an operator allows to reproduce the discrete energy levels of an atom.

That being said, it is natural to ask how those effects disappear as we go from the quantum world to the classical one, that is, as the scale of the system changes. From a mathematical point of view, this has lead to the subject of deformation quantization which tackles the problem by constructing a family of noncommutative associative products on an algebra of functions on the classical space, that deform the usual commutative pointwise product. That deformation encodes the scaling process that allows to pass from the quantum space (the noncommutative algebra) to the classical one (the commutative algebra).

In the case of the so-called symmetric spaces, Weinstein’s conjecture gives a beautiful geometric interpretation to such a deformed product, in terms of the fixed points of their symmetries. One of the goal of this thesis is to get a better grasp on this conjecture. Since the computation of traces of operators turns out to be a central tool toward that goal, another part is devoted to that subject.

In the first part of the thesis, we prove a fixed point formula for the distributional trace of a family of geometric operators. The result was known in the case of transformations of compact manifolds, and is here extended to the noncompact setting.

In the second part, we build a quantization map on symmetric spaces, which, under some hypotheses, leads to an associative noncommutative product on those spaces. A key point of the construction is that, since the quantization is realized by a family of geometric operators, the results of the first part allow to prove a fixed point formula for the deformed product, as in Weinstein’s conjecture.

Membres du jury :

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