Pollution perception: A challenge for intergenerational equity

Ingmar Schumacher and Benteng Zou

December 2007
Pollution perception: A challenge for intergenerational equity

Ingmar Schumacher\textsuperscript{a,}*, Benteng Zou\textsuperscript{b}

\textsuperscript{a}Department of Economics, Universität Trier, C 407, FB IV, VWL, 54286 Trier, Germany
\textsuperscript{b}CREA, University of Luxembourg, 162A avenue de la Faïencerie, L-1511 Luxembourg, Luxembourg

Received 24 February 2006
Available online 23 December 2007

Abstract

In this article we extend the recent literature on overlapping generations and pollution by allowing generations to perceive the level of pollution differently than the actual level of pollution. We call this pollution perception. Pollution perception can visualize itself as either a concern for the flow of pollution only, or for the stock, or a combination of both. We derive this extension based on empirical evidence from recent advances in behavioural economics.

Pollution perception has not only significant consequences for the steady state levels of pollution and capital, but we also find a qualitative change in the dynamics from similar models without pollution perception [A. John, R. Pecchenino, An overlapping generations model of growth and the environment, Econ. J. 104 (1994) 1393–1410]. Specifically, we derive optimal non-linear dynamics through complex eigenvalues and Hopf or Flip bifurcations for a large set of parameters. This leads to violations of two standard criteria of sustainability, suggesting that pollution perception can be another source of intergenerational inequity.

\textcopyright 2007 Elsevier Inc. All rights reserved.

JEL classification: Q20; I31

Keywords: Pollution perception; Pollution; Overlapping generations; Inter-generational equity; Complex dynamics; Sustainability

1. Introduction

Economists have recently started to pay more and more attention to the inter-generational aspects of environmental degradation [23,22,35,41]. If generations are able to transfer the costs of their actions to the future, then this could deprive the latter of at least some of their welfare, creating a source of inter-generational inequity. Our focus in this article is to characterize a different source of inter-generational inequity, one where a policy maker need not look to the deep future to observe violations of equity criteria. This source of potential inter-generational inequity arises under a seemingly favourable condition when generations are able to adapt\textsuperscript{1} to existing levels of pollution.
In this article we investigate what happens if generations perceive the current level of pollution to differ from the actual level. We dub this pollution perception. We shall be more precise on the reasons for which we introduce this extension of preferences in the next section.

The idea that preferences can adapt to certain circumstances has already found application in the literature of habit formation in both continuous [37,43,8,32] and discrete time [11,46]. While our specification has much in common with the concept of habit formation, our focus is on inter-generational, as opposed to intra-generational, effects. We assume that people are not able to adapt to changing pollution levels during their lifetime (intra-generational), but instead we postulate that consecutive generations are not truly aware of, or cannot fully relate to, the environment as it was a generation ago (inter-generational).

One of our results is that pollution perception bears significant effects upon the steady state levels of pollution and capital. Our main result, however, is that incorrect perceptions of current pollution levels can have profound implications for inter-generational equity.

Overlapping generation models, even most continuous time growth models, augmented with an environmental constraint (e.g. [23]) possess clear dynamics. Utility either increases or decreases over time given an optimal choice of consumption and abatement. Only very few models actually create non-monotonic behaviour in form of cycles and bifurcations [6,41,49].

In the case of Bréchet and Lambrecht [6], these bifurcations or cyclical behaviour are a result of the choice of a specific resource function. They provide no attempt in trying to relate the effect of these cyclical dynamics to inter-generational equity. The articles which most closely correspond to our are Zhang [49] and Seegmüller and Verchère [41]. Utilizing the John and Pecchenino [23] model derives conditions for bifurcations. Seegmüller and Verchère derive the possibility of a Flip bifurcation in a model with pollution and a utility function linear in consumption. Both these papers therefore obtain, in models similar to our but without pollution perception, the possibility of bifurcations. However, this possibility corresponds to a degenerate set, a point, in the parameter space of the models. In contrast, we show how pollution perception results in an extensive set of parameters which allow for non-monotonic dynamics in the form of eigenvalues which are greater or smaller than one in absolute value or in the form of complex dynamics via Hopf or Flip bifurcations and complex eigenvalues.

The interest in non-monotonic dynamics derives from an inter-generational equity point of view. If some generations possess the capacity to reduce future generation’s utility relative to their own, then most theories of inter-generational equity demand policy makers to act upon this behaviour (e.g. egalitarianism). We are going to use an approach to inter-generational equity which is becoming standard in today’s literature, namely to judge the model’s implications upon its effects on the sustainability of welfare. One criterion of sustainability is Brundtland Sustainability, the other is Sustainable Development. We are able to show that pollution perception will, under extensive ranges of parameter choices, lead to violations of both criteria.

2. The preference formation

We argue that preference adaptation penetrates into every aspect of our lives. Prominent examples are pictures in the newspaper of smiling children living in garbage dumps in India, kind Inuits in the Arctic or helpful Sherpas in Nepal living in the severest of conditions. For the average person these examples provide extreme tales of people who adapted to tremendously unfavourable circumstances. In this section we discuss the intuition for adaptation, where adaptation is defined as a “process that attenuates the long-term emotional or hedonic impact of favourable and unfavourable circumstances” [16, p. 302].

As is well documented by now, the satisfaction with income does not depend on the level of income or consumption itself, but rather on the rate of improvement [15,17]. Similarly, evidence from psychology, sociology and economics suggests that people under various circumstances respond more to stimuli, i.e. changes, rather than levels [16,40]. This has also been called changing reference point, aspiration level or simply habit formation [8,16,25].

2 Other articles which deal with changing preferences are e.g. Bisin and Verdier [5] on cultural transmission, or Becker and Mulligan [4] on endogenous discounting.
Similarly, Amartya Sen notes that “in situations of long-standing deprivation, the victims...make great efforts to take pleasure in small mercies and to cut down personal desires...” [42, p. 55]. Therefore, adaptation also ought to be understood as a self-protection mechanism, protecting oneself from the perception of adverse situations or effects. Further prominent examples of adaptation to various adverse circumstances include bereavement [48], being in a small room in prison [14], or sensory deprivation like solitary confinement [18].

More closely related to our interpretation, studies also found near complete adaptation to disability like loss of limb or burn injuries [45,48,38].\(^4\) Using survey methods, these authors found that the reported happiness of people who lost a body part was only marginally lower than the reported happiness of population means. Therefore, people are simply able to learn to live with certain health problems.

This adaptation process has also been observed for pollution or the environment in general. For example, a 1963 St. Louis metropolitan area study which suggests that age is the most decisive factor influencing a person's attitude towards air pollution, one likely interpretation being that people get used to air pollution over a longer period of time [9].

What comes as a surprise is that people very often underestimate their own ability to adapt to changing environments. Using a survey approach, Loewenstein and Frederick [28] find that people in general overestimate the effect on them from an environmental change (like air pollution, loss of endangered species or rain forest destruction). Although we do not explicitly model expectation formations, this implicitly suggests the strength of human adaptation to changing environments.

Based on this recent evidence on adaptation to adverse circumstances, we attempt to model and analyze the notion that “we are stressed out by unprecedented levels of environmental (...) destabilization and somehow we are getting used to it” [29]. A simple example should help to illustrate the idea.

Let us imagine that a generation will be born at a certain point in time with an existing stock of pollution. Being just born implies that this generation will not know the world any different. Hence, it will grow up in this environment and simply view this as a part of life. So, the only effect of pollution that this generation might notice is the change in the pollution stock during the time of its existence. We generalize this idea by allowing the generations to be concerned with either the stock of pollution, or the change in pollution during their lifetime, or anything in between. If one interprets pollution perception thus as a change in the reference point level, or aspiration level, then several additional concrete examples come to the mind. For example, pollution is well known to reduce the amenity or recreational value of ecosystems or landscape. But being born in an already polluted world, one might not feel disturbed by this pollution and can therefore enjoy the landscape in a similar way as the ancestors did, even though it might be more polluted.\(^5\)

Wohlwill and Kohn [47] come closest to testing—and supporting—our idea of pollution perception. They conclude that migrants from rural areas perceive a city as more noisy than migrants from metropolitan areas. Thus, people born in a noisy environment like a metropolitan area are better adapted to deal with pollution like noise than people born in rural areas. A final remark seems appropriate: all these studies only deal with intra-generational adaptation—the adaptation to a situation during a person’s lifetime. It seems clear that the inter-generational adaptation which we are considering here must be much stronger than the intra-generational one. After all, the new generation has no connection with past selves, in contrast to the intra-generational case.

In terms of modelling approaches one can then generally distinguish between two adaptation processes. One approach, introduced by Abel [1], is called desensitization, which changes the intensity of any given stimulus. He defines an adaptation level \((X_t)\) as the average of past stimuli \((P_t, \forall t)\), thus giving: \(X_t = 1/\gamma \sum_{t=0}^\gamma P_t\). Then the way the stimuli are perceived is according to \(H_t = P_t X_t\), where \(\gamma \in [-\infty, 0]\).

\(^3\)The current authors interviewed poor Chinese living in slums in Beijing. They answered upon the question whether they were happy: “Yes, because the less you demand, the happier you are.”

\(^4\)Other studies found that people did not adapt to progressive diseases, which most likely implies that the progression of the disease was faster than the adaptation.

\(^5\)Other possible interpretations, which we would not follow here, could be through health effects. As pollution increases, the amount of skin diseases, asthma cases or various kinds of non-fatal diseases like respiratory diseases rise as well. If you are then born in a generation where your parents already had to increasingly cope with these illnesses, you yourself might not view this situation as a negative effect on utility but simply as a part of your life.
The other approach, due to Helson [20, 21], is the shifting adaptation levels, which changes the stimulus level which is perceived as neutral. Using slight simplification, he proposes that a person adapts to a stimulus according to \( H_t = P_t - X_t \). A slight generalization, which we utilize in the article, is by Deaton [13], who suggests that \( H_t = P_t - hX_t \), with \( h \in [0, 1] \).

As is evident from the previous discussion, the adaptation process which we have in mind here is the shifting adaptation levels process. In the extreme case of \( h = 1 \) we will have that each new generation perceives the existing level of the environment it is born into as the neutral stimulus level. Finally, as we are dealing with an inter-generational process here, this implies there exists only one past stimulus, namely a stimulus when the generation is born. Therefore, our specification reduces to \( H_t = P_t - hP_{t-1} \), with \( h \in [0, 1] \). This is also the final specification which we are using in the subsequent model.

3. The model

We consider a perfectly competitive overlapping generations economy. We allow for perfect foresight and discrete time with an infinite horizon, \( t = 0, 1, 2, \ldots \). For simplicity we assume that population is constant and each generation consists of a single representative individual. At each date a generation lives for two periods, young and old. Furthermore, the young generations supply their labor inelastically and decide whether to save or invest (in abatement), and the old generations obtain utility from consuming their savings. In addition, we assume that the old generations feel the effects of pollution as a disutility, but perceive pollution differently for the various reasons as laid out in the previous section.

3.1. The pollution accumulation

Pollution is assumed to accumulate as described by the following equation:

\[
P_{t+1} = (1 - b)P_t + \beta c_t - \gamma A_t, \tag{1}
\]

where \( b \in [0, 1] \) is the rate of pollution absorption, \( \beta (>0) \) is a parameter of consumption externality, representing the rate of pollution emissions from a unit of consumption, and \( \gamma (>0) \) represents the effectiveness of the abatement effort, \( A_t \), on pollution. Hence, the stock of tomorrow’s pollution is partially depending on today’s pollution stock and is being increased by consumption and reduced by abatement. What is important is the fact that the costs of today’s consumption are transferred to tomorrow, which thus directly addresses the issue of inter-generational cost transferal—the costs of today’s actions only affect tomorrow’s generation. Notice also that we do not assume irreversibilities here.

Our interpretation of the assumption that the young generation chooses the level of abatement is as follows: As the old generation will not feel the effect of decreases in pollution if they pay for it when old, and without any altruistic link to the young generation, they will leave it up to the young generation to decide on the level of pollution which they are willing to accept. This, clearly, is a rather pessimistic view, which is, however, the predominant one in the literature. Nevertheless, if one believes that altruism plays a minor or even no role in inter-generational decision taking, then this is the correct approach. We are here only concerned with the completely decentralized problem, thus excluding a governmental taxation system.

3.2. The generations

Generations derive utility over consumption and pollution only when old. This is a widely used assumption in the literature and based on John and Pecchenino [23]. The generation’s utility function is of the form

\[
U(c_{t+1}, H_{t+1}), \tag{2}
\]

\[
H_{t+1} = P_{t+1} - hP_t, \tag{3}
\]

\footnote{For models that use this assumption, see [24].}

\footnote{This could, however, easily be incorporated: One could imagine that young and old vote together on an abatement level which is paid through taxing the income of the young.}
where \( c_{t+1} \) refers to (per capita) consumption in period \( t+1 \), \( H_{t+1} \) reflects the amount of pollution which the generation essentially observes or is affected by, whereas \( P_t \) and \( P_r \) refer to the actual stock of pollution in periods \( t+1 \) and \( t \), respectively. The parameter \( h \in [0,1] \) then stands for the strength of pollution perception. If \( h = 0 \) generations perceive only the flow of pollution yielding a similar model to \([23,49]\); if \( h = 1 \), generations only perceive the stock; if \( 0 < h < 1 \), generations perceive both the stock and the flow. We make use of the following assumptions for the utility function.

**Assumption 1.** (i) \( U(c, H) \) is twice continuous differentiable on \( \mathbb{R}^2_+ \); (ii) \( U_c > 0, U_{cc} < 0, U_H < 0, U_{HH} < 0, \forall c, H \in \mathbb{R}^+; \) (iii) \( \lim_{c \to 0} U_c = \infty, \lim_{H \to 0} U_H = -\infty. \)

The utility function is thus assumed to be twice continuously differentiable in both its arguments, increasing and concave in \( c_{t+1} \), but decreasing and concave in \( H_{t+1} \).

Generations then maximize their utility with respect to savings and subject to their budget constraints which are given by

\[
w_t - A_t = s_t, \tag{4}\]

\[
(1 + r_{t+1}) s_t = c_{t+1}, \tag{5}\]

and the pollution accumulation equation (1). Here, \( w, A, s \) and \( r \) refer to the wages obtained, the abatement effort, the savings carried forward to the next period and the interest obtained on the savings, respectively.

The first order condition from the generation’s maximization problem implies

\[
(1 + r_{t+1}) U_c = -\gamma U_H, \tag{6}\]

which equalizes the marginal benefit of consuming an additional unit of consumption with the marginal cost of feeling a higher disutility of adapted pollution. Assuming a constant elasticity \( -(HU_H)/(cU_c) = \alpha > 0 \) \([33,49]\), we can rewrite the first order conditions as

\[
\alpha \gamma s_t = P_{t+1} - hP_t. \tag{7}\]

As is easily observable, the first-order condition links the non-negativity of savings to the non-negativity of \( H_{t+1} \). The left-hand side of Eq. (7) then gives the marginal benefit to utility of an additional unit of savings now, whereas the right-hand side gives the marginal costs to utility of a unit reduction in abatement. The lower the relative preference of pollution with respect to consumption, as given by \( \alpha \), the more will each generation save in order to obtain a higher level of consumption when old. Also, the less each generation cares about the actual stock of pollution, i.e. a high \( h \), the lower the level of savings. Finally, as generations are not altruistic, they do not take the effect of their consumption on next generation’s utility into account. Therefore, they are only concerned with cleaning up some of the pollution their ancestors did. However, if they notice that their abatement efforts are not very effective, thus \( \gamma \) is low, then they will prefer to save more to obtain a higher level of consumption when old.

Optimality is assured from the second order conditions, requiring

\[
(1 + r_{t+1})^{2} U_{cc} + \gamma^{2} U_{HH} + 2(1 + r_{t+1})\gamma U_{cH} < 0. \tag{8}\]

Due to our previous assumptions, \( U_{cc} < 0 \) and \( U_{HH} < 0 \). We did not find an explicit functional form allowing for \( U_{cH} \leq 0 \) and keeping \( \alpha \) constant. However, a functional form satisfying our assumptions and satisfying the second order conditions is e.g. \( U(c, H) = -e^{-\mu H} \), where \( 1 < \mu < \infty \) implying \( \alpha \in (1, \infty) \). We shall therefore concentrate on \( \alpha > 1 \) in the remainder of the article. We wish to remind that this condition is sufficient, but not necessary for optimality.8

3.3. The representative firm

The representative firm produces with a constant returns to scale technology, \( y = f(k)L \), where we normalize the labour supply to \( L = 1 \). We furthermore assume the standard conditions \( f''(k) > 0 \) and \( f''(k) < 0.\)

---

8 The case of \( u_{HH} > 0 \) and \( \alpha < 1 \) is treated in [39].
The firm then maximizes profits in a competitive market that clears, where
\[
\max_{\{L_t\}} P_t = f(k_t) - w_t
\]  
implies that wages are given by \(f(k_t) - f'(k_t)k_t = w_t\). The profits are then distributed to the owners of the capital, which is the old generation. Thus, a firm that receives savings in period \(t - 1\) will produce in period \(t\) the profit \(\Pi_t = f'(k_t)k_t\). Therefore, the return on savings in \(t - 1\) is equal to the marginal product of capital at time \(t\) minus capital depreciation. Summarizing, the following equations hold:
\[
\begin{align*}
    f'(k_{t+1}) - \delta &= r_{t+1}, \\
    f(k_t) - f'(k_t)k_t &= w_t, \\
    s_t &= k_{t+1}.
\end{align*}
\]
We use the Cobb–Douglas output function to specify the production technology, with \(f(k) = k^m\), where \(m \in (0, 1)\) is the capital share. Furthermore, we assume full depreciation, \(\delta = 1\), during the course of one generation.

### 3.4. The intertemporal equilibrium

We first define the intertemporal equilibrium of this economy.

**Definition 1.** The intertemporal equilibrium of the above depicted economy is a sequence of \(\{k_t, P_t\}_{t=0}^{\infty}\) with given initial conditions \(\{k_0, P_0\}\) which satisfies the two equations that rule the dynamics (13) and (14).

By combining the first order condition with the market clearing condition, the output function, as well as the budget constraints and the pollution equation, we obtain
\[
k_{t+1} = -\frac{1 - b - h}{\gamma(1 - z)} P_t - \frac{m\beta + m\gamma - \gamma}{\gamma(1 - z)} k_t^m
\]  
and
\[
P_{t+1} = \frac{h + b\gamma - z}{1 - z} P_t - \frac{(m\beta + m\gamma - \gamma)z}{(1 - z)} k_t^m.
\]

By taking \(k_t = \bar{k}\) and \(P_t = \bar{P}\), we derive the steady states of this economy. There exist two steady states, one is trivial with \(\{\bar{k}, \bar{P}\} = (0, 0)\), the other is
\[
\bar{k} = \left(\frac{1}{\gamma(b\gamma + h - 1)}\right)^{1/(1-m)},
\]
for \(m\beta + m\gamma - \gamma \neq 0\) and \(b\gamma + h - 1 \neq 0\), as well as
\[
\bar{P} = \frac{\gamma}{1 - h} \left(\frac{(1 - h)(m\beta + m\gamma - \gamma)}{\gamma(b\gamma + h - 1)}\right)^{1/(1-m)}.
\]

In order to insure positive steady states, both \(m\beta + m\gamma - \gamma\) and \(b\gamma + h - 1\) must have the same sign. Intuitively, \(m\beta + m\gamma - \gamma\) represents the direction of the effect from an additional unit of capital on pollution. It therefore describes the technological side of accumulation. Slightly rewriting gives \(m\beta + m\gamma - \gamma > 0\) which is equivalent to \(\beta > 0\). In general, the capital share is around \(m = \frac{1}{3}\), which leads to \(\beta\) being more (less) than twice the value of \(\gamma\). Hence, this condition also corresponds to the idea that it takes significantly less (slightly less or more) effort to pollute than to clean up. In comparison, \(b\gamma + h - 1\) represents the direction of the valuation of an additional unit of capital. This term therefore describes the preference side of accumulation. The existence of a positive steady state requires then that, if marginal abatement is very

---

For empirical evidence, see [3,30,31].
effective, the agent must not feel the impact of pollution perception (or changes in pollution) on utility too strongly, as otherwise she would spend everything on abatement (and vice versa).

We shall now discuss the implications of pollution perception on the steady state values of capital and pollution under the different parameter configurations. The results we obtain hinge crucially on the sign of $xb + h - 1$.

Parameter subset 1: $m\beta + m\gamma - \gamma < 0$ and $xb + h - 1 < 0$. This case implies that for relatively efficient abatement the agent either does not value changes in pollution too highly (a low $z$), or she perceives pollution close to its real level ($h$ low), or pollution has a long lifetime ($b$ low).\(^9\) Comparative statics for this subset of parameters suggest that

$$\frac{\partial k}{\partial h} = -\frac{b\beta k}{(1-h)(1-m)(b\beta + h - 1)} > 0, \tag{17}$$

$$\frac{\partial P}{\partial h} = \frac{P}{(1-h)(1-m)} \left[ (1-m) + \frac{xb}{1-b\beta - h} \right] > 0. \tag{18}$$

Therefore, increases in $h$ lead to larger steady state values of $k$ and $P$. If generations perceive the stock of pollution to be lower than it actually is, then they will feel less concerned with it and thus produce more and abate less. But, of course, this depends crucially on the condition that agents view the overall contribution of pollution to be lower than it actually is, then they will feel less concerned with it and thus produce more and abate less. One would presume that this result would be dominant in the case of developed countries.

Parameter subset 2: $m\beta + m\gamma - \gamma > 0$ and $xb + h - 1 > 0$. Given this subset of parameters, the effect of pollution perception on the steady state can again be discovered by taking the derivative of (15) and (16) with respect to $h$. We obtain

$$\frac{\partial k}{\partial h} = -\frac{b\beta k}{(1-h)(1-m)(b\beta + h - 1)} < 0, \tag{19}$$

$$\frac{\partial P}{\partial h} = \frac{P}{(1-h)(1-m)} \left[ (1-m) + \frac{xb}{1-b\beta - h} \right] < 0. \tag{20}$$

As $h \in [0, 1]$ and $m \in (0, 1)$, it is clear that the steady state capital stock as well as the steady state pollution stock decrease with increases in $h$.\(^1\) This is because under $xb + h > 1$, the agent feels a strong impact of changes in pollution on her utility and therefore will abate significantly more and save less. One would presume that this result would be dominant in the case of developed countries.

Our intention now is to investigate the effect of pollution perception on the dynamics in order to show that it can lead to significant violations of inter-generational equity. We shall concentrate on the possibility of oscillatory dynamics as we wish to focus on the effect of pollution perception on inter-generational equity.

4. The dynamics

By linearizing Eqs. (13) and (14) around the non-trivial steady state we obtain the dynamics around the steady state. The Jacobian matrix at steady state is

$$\begin{bmatrix}
\frac{m(b\beta + h - 1)}{(1-z)(1-h)} & \frac{1-b-h}{(1-z)(1-h)} \\
\frac{m\gamma(b\beta + h - 1)}{(1-z)(1-h)} & \frac{\gamma(1-z)}{h+b\beta - z} \\
\frac{xbm(b\beta + h - 1)}{(1-z)(1-h)} & \frac{1-b-h}{(1-z)(1-h)}
\end{bmatrix}, \tag{21}
$$

and the Characteristic equation is given by

$$\lambda^2 + \frac{z(h-1)(h-1) + zbm - (1-h)(h+m)}{(z-1)(h-1)}\lambda - \frac{(zb + h - 1)hm}{(z-1)(h-1)} = 0, \tag{22}$$

\(^9\)These last two results come from the equality of marginal utilities on the optimal path.

\(^1\)Since $\frac{dP}{dh} < 0$ as $h > 1 + (bzm)/(1 - m)$ is impossible.
where $\lambda$ gives the eigenvalue. Now, in discrete time, oscillatory dynamics can occur (in the short run) if the absolute value of one eigenvalue is larger than one, whereas the other is less than one. This, generally, only requires a two-dimensional system like ours.\textsuperscript{12} Thus the claim which we would like to put forward, namely that pollution perception leads to oscillatory dynamics, looks then more like a mere result of an increase in dimension.

In order to show that oscillatory dynamics are more than a mere artifact of the increase in dimension in our model, we shall concentrate on the case of complex eigenvalues, which we refer to as “complex dynamics”. For the case of two roots, thus when we deal with $\lambda^2 + a_1\lambda + a_2 = 0$, so that necessary and sufficient conditions for converging complex dynamics to occur are $a_1^2 < 4a_2$ and $a_2 < 1$. The following proposition summarizes the conditions necessary for $a_2 < 1$ to hold, as well as for $a_2 = 1$, which we show implies a Hopf bifurcation for a range of parameter configurations. In addition, the conditions for a Flip bifurcation are given for $zb+h<1$.

**Proposition 1.** In the case of complex dynamics, the system asymptotically converges to its non-trivial steady state if and only if $h$ is such that $1 - zb < h < h_2$, where $h_2$ is given by

$$h_2 = \sqrt{(m(bz - 1) + z - 1)^2 + 4m(z - 1) - (m(bz - 1) + z - 1))/(2m)} < 1.$$ 

Moreover, at the critical value of the parameter $h_2$, which solves $|\lambda_{1,2}(h_2)| = 1$, an Andronov–Hopf bifurcation occurs if $h_2$ satisfies, for given parameters $z, b$ and $m$, the condition

$$1 - \frac{zb(1 - m)}{2 - z - m + (z - 1)/m} < h_2 < 1 - \frac{ab(1 - m)}{3z - 2 - m + (z - 1)/m}.$$ 

(23)

Finally, given $zb + h < 1$, if the parameters $z, m$ and $b$ satisfy $(zb(1 + m) - 2z + 1)^2 > 4(1 + m)(zb(m - 1) + 2z - m - 1)(> 0)$ and $z > 1 + m/2$, then there exist eigenvalues $\lambda_{1,2}(h)$ of the characteristic equation, such that $\lambda_1(h_{1,2}) = -1, \lambda_2(h_{1,2}) > 0$.\textsuperscript{13} Furthermore, if $2m(z - 1)(3 - zb) - (z - 1)^2 + m^2(zb - 1)^2$, then $\lambda_2(h_{1,2}) < 1$, there is a Flip bifurcation at $h_{1,2}$; if $2m(z - 1)(3 - zb) - (z - 1)^2 + m^2(zb - 1)^2$, $z_2(h_{1,2}) < 1$, the system diverges.

The proof, which is relegated to the appendix, gives the full mathematical conditions for the existence of an Andronov–Hopf and a Flip bifurcation. However, unfortunately, to prove the existence of converging, complex dynamics, the condition of $a_1^2 < 4a_2$ requires the solution of a fourth-order polynomial for all the ranges and combinations of parameters such that the inequality is satisfied. We shall therefore follow the subsequent path: We fix the capital share at an empirically reasonable value $m = \frac{1}{2}$. We then graph the joint conditions on $b$, $h$ and $z$ leading to converging, complex dynamics, taking the parameter restrictions into account.

Fig. 1 represents the same graph from different angles\textsuperscript{14} allowing the following observations: for $b = 0$ or $h = 0$ no complex dynamics can occur. If $b$ increases then the possibility for complex dynamics increases. Only intermediate values of $h < h_2$ lead to converging complex dynamics, unless $b$ is very small. Large $h$ leads to instability, a small one to stability. The larger is $z$ the larger is the possibility for complex dynamics. Two further parameter restrictions need to be imposed for positive abatement.\textsuperscript{15} One sufficient condition is $1 - b - h < 0$, thus not allowing for any parameter combinations in the area made by ABCDEF of the figure on the right. The other sufficient condition is $1 < \beta m/\gamma (1 - m) < z$, leading to a minimum level of $z$. Both restrictions are only sufficient conditions, and not necessary. The Hopf bifurcation will occur at the largest value of $h$ which still leads to converging, complex dynamics.

The main result we wish to draw from this analysis is that adaptation to pollution, given that it is large enough relative to the self-regeneration rate of the environment, ultimately leads to non-monotonic, oscillatory dynamics. This result not only holds for a degenerate group of parameters, but in our case for a whole set of parameter combinations, as is visible in the two graphs.

\textsuperscript{12}Using inequality plots like the subsequent one we are easily able to confirm that this model can have $|\lambda_1| < 1$ and $|\lambda_2| > 1$.

\textsuperscript{13}In fact, $\lambda_2(h_{1,2}) = -m(zb + h_{1,2} - 1)h_{1,2} / (z - 1)(1 - h_{1,2})$, where $h_{1,2} = ((zb(1 + m) - 2z + 1) \pm \sqrt{A}) / 2(1 + m)$ and $A = (zb(1 + m) - 2z + 1)^2 - 4(1 + m)(zb(m - 1) + 2z - m - 1)$.

\textsuperscript{14}Mathematica does not draw the top, front and back so the area depicted is a volume.

\textsuperscript{15}See the Appendix.
4.1. Interpretation of the complex dynamics

We notice that, at the intertemporal equilibrium, changes in capital and pollution are non-monotonic for a range of parameter combinations due to the interplay of two elements: firstly, the savings of the old \((s_t)\) are utilized to produce the wages of the young and the consumption of the old \((c_{t+1})\). This transformation is subject to decreasing returns \((f''(k) < 0)\). The second element is a direct result of the pollution perception. For \(0 < h < 1\), generations are partly able to adapt to existing stocks of pollution \((P_{t+1} - hP_t)\), wherefore they are spending less money on abatement and more on consumption. From a certain level of capital stock onwards, the additions to savings are so small that the increases in pollution outweigh the advantages from higher savings \((s_t)\). Therefore, the generation spends more money on abatement \((A_t)\), which reduces savings \((s_t)\). This reduction has a twofold impact. Firstly, the reduction in savings \((s_t)\) reduces next periods capital stock \((k_{t+1})\) and thus consumption \((c_{t+1})\) and the pollution stock of the consecutive period \((P_{t+1})\); secondly, the increase in abatement \((A_t)\) reduces the stock of pollution \((P_{t+1})\). Assuming the parameter combination that leads to complex dynamics, the next generation is now in a position where they view the effect of pollution on their utility \((P_{t+1} - hP_t)\) as sufficiently small, which leads them to reduce abatement and increase savings. Depending on the parameter combinations, this adjustment process can be convergent or divergent.

4.2. Comparison to \(h = 0\) case

Of course, one will now be inclined to ask: What is the effect of pollution perception? Is there any qualitative change to the current literature? The answer is yes, and for this we shall compare the case of \(h > 0\) to the one of \(h = 0\).

In the case of \(h = 0\), agents do not adapt to pollution; in this case our model conforms in spirit to [41,49]. Both articles find that convergence towards equilibrium is monotonic unless a degenerate combination of parameters happens to be satisfied. In that case the possibility of bifurcations exists, which implies non-monotonic dynamics. Clearly, the possibility of a degenerate combination of parameters, given these are fixed and do not change (as it is the case in these models and ours), is rather unlikely. The theoretical possibility of non-monotonic dynamics therefore exists, but obtaining the specific parameter combination required for complex dynamics in practice is difficult.

In comparison to this let us now take the case of \(h > 0\). We find that for \(h > 0\) there exists a whole set of parameter combinations that leads to non-monotonic dynamics. Therefore one can speak of a qualitative change in the dynamics of the model when \(h\) increases from zero to a positive number. If one takes the standard estimate for the capital share of \(m = \frac{1}{3}\) [3] then, as Fig. 1 shows, a larger combinations of \(h, b\) and \(\alpha\) lead to complex dynamics. Furthermore, the larger is the pollution perception parameter \(h\), that means the more important is pollution perception, the more likely will be the occurrence of oscillatory dynamics.

In the following section we are going to concentrate on this qualitative change in the model and therefore focus on the possibility and implication of non-monotonic dynamics. We shall utilize this case to highlight the consequences of pollution perception for inter-generational equity.

5. Welfare analysis and intergenerational equity

As suggested in the previous section, pollution perception can cause oscillatory dynamics for a large set of parameter values. Our focus in this section will then be to emphasize the implications of oscillatory dynamics on two widely used criteria of inter-generational equity. One criterion is Brundtland Sustainability, the other is Sustainable Development.\(^{16}\)

The first notion of sustainability, *Brundtland Sustainability*, was shaped in 1987 in the United Nation’s report *Our Common Future*, more commonly referred to as the Brundtland Report [7]. The most widely quoted sentence of this report is that sustainability should be thought of as “meeting the needs of the present without compromising the ability of the future generations to meet their own needs”. However, this sentence,

\(^{16}\)We are aware that the Brundtland report originally called its criterion Sustainable Development, but choose to follow recent expositions [19,34].
on its own, is not a complete account of what the Brundtland Report has in mind by sustainability. The report furthermore suggests that sustainability “(...) requires meeting the basic needs of all and extending to all the opportunity to fulfill their aspirations for a better life” [7, p. 24]. The second part of this interpretation of sustainability seems to have been neglected in today’s literature. It is more closely connected to the new egalitarian thinking on capability and responsibility [36]. It is nevertheless not clear though, how we are to interpret the request to “extend to all the opportunity to fulfill their aspirations” in terms of an economic approach to inter-generational equity.

One interpretation could be that meeting the basic needs suffices and does not require further redistributions [34]. This is clearly a sufficientarian notion of justice. This theory of justice suggests that a distribution is just if all basic needs are covered. This then can be rewritten in utility terms, where it comes to denote that a minimum level of utility, \( u_t \geq u_0 \), is to be obtained for all subsequent, indefinite number of generations. We shall have this interpretation in mind when we refer to Brundtland Sustainability in the subsequent paragraphs.

**Definition 2.** A path of utility \( \{u(t)\}_{t=0}^{\infty} \) conforms with the Brundtland Sustainability criterion if \( u_t \geq u_0 \), \( \forall t \), where \( u_0 > 0 \) is a minimum level of utility.

Another interpretation could be as follows: if we were to stay within the boundaries of this model, then each generation will have the same aspirations—maximizing their utility. Given rationality and perfect foresight\(^\text{17}\) this implies that every generation should obtain at least the same level of utility as their ancestors did\(^\text{18}\). This is a much stronger demand than Brundtland Sustainability and—at least in our model—is closely connected to our second notion of sustainability. However, as this second interpretation of Brundtland Sustainability adds nothing more to our analysis, we shall leave it aside.

The second notion, Sustainable Development, is by now the predominant notion used in economic analysis [44,10,34], as well as egalitarian thinking. In economic terms it has been interpreted to mean that a certain level (or development) of utility is to be achieved. This has been taken to imply that \( u_{t+1} \geq u_t \), for all consecutive time periods. Hence a world in which this criteria is utilized is one in which utility is either kept constant or increases over time, but is not reduced.

\(^{17}\)Plus abstracting from various issues like population changes, changes in bundles of goods transferred between generations, etc.

\(^{18}\)The part “at least the same level of utility” comes from the fact that capital is productive, \( r(t) > 0 \).
Definition 3. A path of utility \( \{u(t)\}_{t=0}^{\infty} \) conforms with the Sustainable Development criterion if \( du_t/dt \geq 0, \forall t \).

The Sustainable Development criterion has also been derived by use of axiomatic theory by [2]. If a policy maker believes the axioms of equity and efficiency\(^{19} \) (the combination of which implies the Suppes–Sen grading principle) to be relevant for policy taking, then this directly implies that she has to discard any path which does not conform to the Sustainable Development criterion.

Another way to arrive at the Sustainable Development criterion is via the use of an argument similar to the no-envy criterion.\(^{20} \) We are going to use a slightly weaker version of no-envy, namely no-envy with respect to past generations. An allocation conforms with no-envy with respect to past generations if the current generation, given its own utility, does not prefer the preceding generation’s bundle. Mathematically, we write \( u_{t+1}(c_{t+1}, H_{t+1}) \geq u_{t+1}(c_t, H_t) \). The advantage of this approach is that no interpersonal comparison of utility is required, which substantially eases the policy making. We can then write recursively \( \cdots \geq u(c_{t+1}, H_{t+1}) \geq u(c_{t-1}, H_{t-1}) \geq \cdots \) to arrive at the non-declining utility requirement imposed by the Sustainable Development criterion.

We are now going to study the evolution of utility in order to understand which of the two criteria of sustainability are satisfied within our model. The following proposition summarizes the motion of utility at the intertemporal equilibrium.

**Proposition 2.** The level of utility at the intertemporal equilibrium can be expressed as a function of the optimal capital stock only.

**Proof.** At the intertemporal equilibrium, substituting (5), (7), (10) and (12) into the utility function, we have

\[
U(c_{t+1}, H_{t+1}) = U(mk_{t+1}^{P_0}, x_{t+1}k_{t+1}), \quad \text{due to the choice of Cobb–Douglas output function.} \]

Proposition 2 thus allows us to see that utility, at the intertemporal equilibrium, can be written as a function of capital only \( u(k, P(k)) \).

It is possible to observe that generations will face different levels of utility depending on when they are born.\(^{21} \) Hence, any policy maker who believes that policy decisions should be based—at least partly—on either the axioms of equity and efficiency or the axiom of no-envy towards past generations (or on the Sustainable Development criterion directly), will observe that the Sustainable Development requirements can be violated. If we then assume that a policy maker assesses inter-generational equity by comparing the motion of utility at the intertemporal equilibrium with the requirements of the Sustainable Development criterion, then the oscillatory motion of utility at the intertemporal equilibrium prevents achieving this equity target without adequate policy interventions. These interventions could for example take the form of inter-generational transfers.\(^{22} \)

If we assume that a policy maker assesses inter-generational equity by comparing the motion of utility at the intertemporal equilibrium with the requirements of the Brundtland Sustainability criterion, then the result is far less clear. What we can say, however, is that besides the level of minimum utility, the initial conditions as well as the level of the pollution perception parameter \( h \) and \( z \) play the predominant roles.

What this model is therefore able to point out is that preferences, which seem initially favourable towards sustainable welfare through the possibility to adapt to pollution levels, can trivially result in the violation of the now commonly used Sustainable Development criterion, and might also violate the Brundtland Sustainability, basic needs criterion. This therefore suggests that sustainability criteria will (potentially) not only be violated in the future due to a run-down of resources or strong increases in pollution, but they can also be violated in the near future through preferences which initially seem favourable towards sustainability of welfare.

\(^{19}\)The axiom of equity states that a policy maker should not discriminate between generations simply because they happen to exist at different points in time. Efficiency implies no waste of resources and sensitivity to small changes.

\(^{20}\)We are not aware of any references linking no-envy and Sustainable Development, the link of which, however, seems trivial. No-envy stems from Kolm [26].

\(^{21}\)Furthermore, using simulations we notice that increases in the pollution perception parameter increase the oscillations, and parameter combinations which are closer to the bifurcation parameters will further amplify the cycles.

\(^{22}\)For arguments questioning the possibility of inter-generational transfers, see [27].
6. Conclusion

The main contribution of this article is that pollution perception provides a qualitative change in the way steady states are reached. Whereas overlapping generation models without pollution perception usually have monotonic dynamics [23], we find that for a large set of parameter combinations the generation’s behaviour at the intertemporal equilibrium can lead to oscillations in utility of subsequent generations. This is in contrast to the case of no pollution perception, where there then exists only one specific combination of parameters which allows for bifurcations [49,41].

We analyze these oscillatory dynamics for their effect on two standard criteria of inter-generational equity, Brundtland Sustainability and Sustainable Development. In case these oscillations are to occur, then Sustainable Development will be impossible to achieve without adequate policy interventions. Furthermore, whether the Brundtland Sustainability criterion will be satisfied depends on the level of the minimum utility, the initial conditions as well as the level of the pollution perception parameter and the relative importance of pollution in generating utility.

Our results can be slightly generalized. When a model generates endogenous cycles then both predominantly used criteria of inter-generational equity, Brundtland Sustainability as well as Sustainable Development, can be easily violated in case there are no policy interventions. This thus requires a certain trade-off between the value that generations place on efficiency, and the value that the generations or a policy maker place on inter-generational equity.

Pollution perception is evidently a challenging extension for standard OLG models of the environment and deserves greater attention in consecutive research. Especially interesting would be to see how a policy maker could affect the behaviour of the consecutive generations and how forward-looking the policy maker must be in order to avoid the inter-generational inequities.

Acknowledgments

The authors would like to thank two anonymous referees as well as Claude d’Aspremont, Raouf Boucekkine, Thierry Brechet, David de la Croix, Luca Marchiori, Katheline Schubert, Thomas Seegmüller, Cees Withagen and the participants of SURED 2006 in Ascona and those at the Rencontre de l’environnement in Paris. All remaining errors are ours. The first author acknowledges financial support from the Chair Lhoist Berghmans in Environmental Economics and Management and the Belgian Scientific Policy under the CLIMNEG project (contract CP/10/243). The second author acknowledges financial support from DFG (EBIM, under GRK1134/1).

Appendix

Proof (Convergence). \( a_2 < 1 \) is equivalent to \( mh(bz + h - 1) < (1 - h)(z - 1) \), which can be rewritten as \( G(h) \equiv mh^2 + [m(bz - 1) + (z - 1)]h - (z - 1) < 0 \). The roots of \( G(h) = 0 \) are

\[
h_{1,2} = \frac{-(m(bz - 1) + z - 1) \pm \sqrt{A_h}}{2m},
\]

where we take \( h_1 < h_2 \) and \( A_h = [m(bz - 1) + (z - 1)]^2 + 4m(z - 1) \). It is easy to see from the definition of \( A_h \), that \( A_h > [m(bz - 1) + (z - 1)]^2 \) for any \( z > 1 \). It follows that the smaller root of \( G(h) \), denoted wlog by \( h_1 \), is negative, and the larger one is positive. As \( h \in [0, 1] \), then \( h_2 < 1 \) if and only if \( \sqrt{A_h} < 2m + [m(bz - 1) + z - 1] > 0 \). Taking squares on both sides, and rearranging the terms, we obtain that \( h_2 < 1 \) if and only if \( 0 < bmz \), which is true always.

Proof (Hopf bifurcation). The complex eigenvalue checks \( |\lambda(h_2)|^2 = a_2(h_2) = 1 \), and \( |\lambda(h)|^2 = a_2(h) < 1 \) for \( 0 \leq h < h_2 \). Obviously, the crossing condition at \( h_2 \) holds as well. Hence, the system admits a Hopf bifurcation at \( h = h_2 \), provided \( h_2 \) is inside the interval of \( h \) which allows for complex eigenvalues. Therefore, we now need to show that, taking the other parameters \( m, z, b \) as given, condition (23) holds.
In other words, we need to show that \( a_2'(h_2) < 4a_2(h_2) = 4 \), which is equivalent to the condition \([z(b-1)+bmz/(h_2-1)+h_2+m]^2 < 4(z-1)^2\), or \(-2(z-1)<z(b-1)+bmz/(h_2-1)+h_2+m<2(z-1)\). Rearranging the terms, we have

\[
-2(z-1)-z(b-1)-m < \frac{bmz}{h_2-1} + h_2 < 2(z-1)-z(b-1)-m. \tag{*}
\]

From \( a_2(h_2) = 1 \), it follows \( h_2^2 + ((m(bz-1)+z-1)/m)h_2 - (z-1)/m = 0 \), and rewriting as \( bmz/(h_2-1)+h_2 \), we obtain

\[
\frac{bmz}{h_2-1} + h_2 = \frac{b(1-m)z}{h_2-1} - \frac{(bz+\frac{z-1}{m})}{h_2-1}.
\]

Therefore, Eq. (*) is equivalent to

\[
2 - z - m + \frac{z-1}{m} < \frac{b(1-m)z}{h_2-1} < 3z - 2 - m + \frac{z-1}{m}. \tag{+}
\]

Given \( z > 1 \), \( 3z - 2 - m + (z-1)/m \) is always positive. We claim that \( 2 - z - m + (z-1)/m \) is also positive. Suppose that \( 2 - z - m + (z-1)/m \leq 0 \). Rearranging the terms, we have \( z(1-m)/m \leq (1+m^2-2m)/m = (1-m)^2/m \), which is \( z \leq 1-m < 1 \), due to \( 0 < m < 1 \). However, that contradicts \( z > 1 \). Therefore, arranging terms in Eq. (+), condition (23) follows.

The proof of the Flip bifurcation is analogous. \( \square \)

**Proof (Positive abatement).** The condition on positive abatement can be derived as follows: As \( A_t = w_t - s_t \) and \( s_t = k_{t+1} \), we can then substitute the solutions \( w_t = f(k) = f'(k)k \) as well as the dynamical equation for \( k_{t+1} \), as given by Eq. (13). Hence \( A_t = (1-m)k_t^n + ((1-b-h)/\gamma(1-z))P_t + ((m\beta + m\gamma - \gamma)/(1-z))k_t^n \).

The coefficient on pollution is always positive if \( 1-b-h < 0 \), and a sufficient condition for positive abatement is then \( \beta < \gamma(1/m - 1) \), or \( 1 < \beta(m)/\gamma(1-m) < \alpha \). \( \square \)

### References

Environmental Economics & Management Memoranda

75. Giorgia OGGIONI and Yves SMEERS. Evaluating the impact of average cost based contracts on the industrial sector in the European emission trading scheme. January 2008 (also CORE DP 2008/1).
69. Thierry BRECHET en Pierre PICARD. Economische instrumenten voor de regulering van de geluidshinder in de omgeving van luchthavens? Brussels Studies, nummer 12, 3 december 2007
64. Parkash CHANDER and Subhashini MUTHUKRISHNAN. Green consumerism and collective action. June 2007 (also available as CORE DP 2007/58).
62. María Eugenia SANIN and Skerdilajda ZANAJ. Environmental innovation under Cournot competition. June 2007. (also available as CORE DP 2007/50)
61. Thierry BRECHET et Stéphane LAMBRECHT. Family altruism with a renewable resource and population growth. October 2006 (also available as CORE DP 2006/35).
60. Thierry BRECHET, François GERARD and Henry TULKENS. Climate Coalitions: a theoretical and computational appraisal. February 2007 (also available as CORE DP 2007/3).
54. Ingmar SCHUMACHER. Endogenous discounting via wealth, twin-peaks and the role of technology. November 2006 (also available as CORE DP 2006/104).

53. Ingmar SCHUMACHER. On optimality, endogenous discounting and wealth accumulation. October 2006 (also available as CORE DP 2006/103).


51. Maria Eugenia SANIN. Market Design in Wholesale Electricity Markets. October 2006 (also available as CORE DP 2006/100).

50. Luisito BERTINELLI, Eric STROBL and Benteng ZOU. Polluting technologies and sustainable economic development. June 2006 (also available as CORE DP 2006/52).


47. Philippe TULKENS, Henry TULKENS. The White House and the Kyoto Protocol: Double standards on uncertainties and their consequences. May 2006 (also TERI School of Advanced Studies WP Series #1).

46. Thierry BRECHET, Pierre-André JOUVET. Environmental innovation and the cost of pollution abatement. January 2006 (also available as CORE DP 2006/40).


43. Ingmar SCHUMACHER and Benteng ZOU. Habit in Pollution, A Challenge for Intergenerational Equity. March 2006 (also available as CORE DP 2006/6).


41. Urs LUTERBACHER. Climate Change, the Kyoto Protocol, and Transatlantic Relations. November 2005.


27. Thierry BRECHET, Stéphane LAMBRECHT, Fabien PRIEUR. Intergenerational transfers of pollution rights and growth. May 2005 (also available as CORE DP 2005/42).

26. Maryse LABRIET, Richard LOULOU. From non-cooperative CO₂ abatement strategies to the optimal world cooperation: Results from the integrated MARKAL model. April 2005.


3. Thierry BRECHET, Marc GERMAIN et Philippe MONTFORT. Spécialisation internationale et partage de la charge en matière de réduction de la pollution. (also available as IRES discussion paper n°2003-19).


Environmental Economics & Management Memorandum

Chair Lhoist Berghmans in Environmental Economics and Management
Center for Operations Research & Econometrics (CORE)
Université catholique de Louvain (UCL)
Voie du Roman Pays 34
B-1348 Louvain-la-Neuve, Belgium

Hard copies are available upon request : env@core.ucl.ac.be
Papers are available in pdf format on line : http://www.uclouvain.be/en-16845.html