Aid to Poor Resource Exporting Countries:
Which Role Should be Played by Resource Taxation?

Ruxanda Berlinschi† and Julien Daubanes‡

January 2009

Abstract

We study the revenue transfers between a rich oil importing country (North) and a two-class oil exporting country (South) where rich resource holders coexist with poor workers. On the one hand the North captures some of the South’s mining rents with distortional oil taxes. On the other hand the North transfers some of its revenue to the South with foreign aid. We use a dynamic general equilibrium model to illustrate the inefficiency of these simultaneous transfers and we propose a Pareto improving contractual solution.

JEL classification: Q3, O1, F1

Keywords: Foreign Aid, Oil Taxation

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*We thank Jean-Paul Azam and André Grimaud for their useful suggestions and encouragements. This paper has also benefited from rich comments by Sjak Smulders, Cees Withagen, Christian Gollier, Pepita Miquel-Florensa, Gilles Saint-Paul, Stéphane Straub, Yolande Hiriart, Christian Kiedaisch as well as by participants at the 1st Doctoral Meeting of Montpellier, the SURED 2008 conference, the Frontiers in Environmental Economics and Natural Resources Management TSE-AFSE conference and the 23rd Annual Congress of the EEA. The usual disclaimer applies.

† ARQADE at Toulouse School of Economics
‡ CER-ETH at Swiss Federal Institute of Technology Zürich
1. Introduction

In some oil exporting countries such as Nigeria, powerful rich resource holders coexist with large populations of poor people. The industrialized world is sending foreign aid to some of these countries while it is simultaneously capturing some of their oil rents with distortional oil taxes. Is there a more efficient way to help these populations? One solution suggested by the US Congressional Budget Office in 1986 is to lower the tariff on oil imported from low income countries. However, there is no guarantee that the resulting additional oil rents would be fairly shared with the poor populations. Recently, Sala-i-Martin and Subramanian (2003) proposed the drastic option of equally sharing the entire oil revenues with the whole population. Although very appealing, this solution is unlikely to be implemented by the elite controlling the oil revenues. This paper aims at designing an efficient and feasible redistribution scheme: a contract between the industrialized countries and the resource holders on the oil tax rate and on the share of oil rents to be redistributed to the poor.

Oil endowments are concentrated in some geologically specific grounds and their distribution is very heterogeneous across countries. The 19 countries with the largest per capita oil reserves hold more than 80% of the current world reserves. The top oil consumers are the rich industrialized countries. In these countries, oil products are taxed quite heavily. The part of taxes in the final price of a barrel was around 45% for the G7 countries in 2007\(^1\). Oil taxation allows the collection of important fiscal revenues. In the OECD countries for example, the taxes on oil products constitute as much as 6% of total fiscal revenues\(^2\). As oil is an exhaustible natural resource, its total supply is relatively inelastic in the long run, therefore these taxes fall almost entirely on the producers. By consequence, oil taxation leads to a revenue transfer from the resource holders to the fiscal authorities. According to the OPEC, during the period 2003-2007, the G7’s fiscal revenues from oil products taxation were equal to $2585 billion, exceeding the oil producers’ $2539 billion revenues\(^3\). These figures suggest that the G7 has captured almost $15 billion of oil revenues from Nigeria\(^4\), who represents 3% of the world production\(^5\).

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\(^1\) Source : OPEC (2008)

\(^2\) Source : IEA.

\(^3\) OPEC (2008).

\(^4\) This figure is based on the assumption that the share of the Nigerian oil in the G7’s consumption is the same as its share in the world consumption.

\(^5\) Source : IEA.
The top oil producers are mainly low or middle income countries\textsuperscript{6} with important inequality levels\textsuperscript{7}. The average per capita GNI of the 19 countries with the highest per capita oil reserves is less than $5800\textsuperscript{8}. In many of these countries, a small elite benefit from the oil revenues, while the rest of the population remains poor. In Nigeria for example, 70\% of the population lives on less than a dollar a day, despite the country’s $340 billion oil revenues since the 1970s\textsuperscript{9}. The industrialized world disburses considerable amounts of foreign aid to these populations. For instance, Nigeria received almost $10 billion of foreign aid from the G7 in 2006. The same year, the G7 disbursed $114 m to Algeria, $94 m to Ecuador and $48 m to Iran and $25 m to Venezuela\textsuperscript{10}. In these countries, oil taxes are very low or even negative. Bacon (2001) shows that low income oil producers charge significantly lower oil taxes than high income oil consumers.

The tax gap between the industrialized countries and the low income oil producing countries leads to significant differences in the oil price in these countries. This causes a distortion at the global level: oil is not used where it is the most productive. We think this distortion could be reduced with a better coordination of taxation policies and aid policies. This coordination should allow the industrialized countries to redistribute income to the poor populations living in oil rich countries in a more efficient way.

In this paper we formalize the interactions between a rich oil importing country and a poor oil exporting country. Our setting is a dynamic general equilibrium North-South model with three groups: the northern workers, who are relatively productive, the southern workers, who are relatively unproductive and the southern resource holders, who own the entire oil stock. The northern workers are altruistic towards the southern workers. The policy instruments used by the authorities are oil taxation and foreign aid. We identify the inefficiency that arises when the oil tax rates are set non cooperatively by the two countries and we propose a contractual solution that reduces this inefficiency and is Pareto improving. The contract consists of a commitment by the North to lower the oil tax, which reduces the global distortion in the oil prices and increases the resource holders’ rents, and a commitment by the South to redistribute the resulting additional rents to the poor population; which reduces the need for foreign aid.

\textsuperscript{6} The poor economic performance of resource rich countries, commonly called the resource curse, was the subject of many studies. See for example Sachs and Warner (1995).
\textsuperscript{7} On the inequality in oil rich countries, see Schubert (2002)
\textsuperscript{8} Source : The World Bank and Oil & Gas Journal
\textsuperscript{9} Lynn and Ian (2004)
\textsuperscript{10} Source : OECD.
This paper is at the crossing of two economic fields: development economics and resource economics. In the following we will briefly position this paper in the existing literature.

Many development economists have studied the motivations and the development impact of foreign aid. As far as the motivations are concerned, the literature seems to conclude that foreign aid is disbursed both for altruistic and for selfish reasons, the weight between the two depending on the donor (Berthelemy (2006)). The altruistic motivation is related to a moral demand for redistribution when income gaps are very important. In this context, foreign aid plays a similar role at the international level as do the progressive income taxes and the social security institutions at the national level (Mosley (1987)). Among the selfish motivations, we can cite political interests, such as influencing voting patterns in the United Nations (Alesina and Dollar (2000)), commercial interests such as helping national firms (Villanger (2003)) and security interests such as fighting terrorism (Azam and Thelen (2007)). In this paper foreign aid is motivated by pure altruism11. As in Azam and Laffont (2003), we suppose that the northern citizens put more weight on the welfare of the southern poor than do the southern authorities. Altruistic donors use foreign aid in order to reduce hunger in the short term, but also to promote sustainable growth and better policies and institutions in the long term. A number of authors have tested the effectiveness of foreign aid with respect to these longer term objectives. Their conclusions are quite mixed both on the growth impact of foreign aid (Burnside and Dollar (2000), Hansen and Tarp (2001), Easterly (2003)) and on its policy impact (Svensson (2003), Easterly (2005), Kilby (2005)). In this paper the only objective of foreign aid is to increase the consumption of the poor (reduce hunger). We model foreign aid as a lump sum transfer from the northern citizens to the southern workers.

Some environmental and resource economists have studied the distributional aspects of non renewable resource taxation12. In particular, Bergstrom (1982) shows that resource taxation allows the resource importing country to capture some resource rents. Brander and Djajic (1983) study the optimal choices of two social planners representing a resource consuming country and a resource producing country. They show that the consuming country’s social planner is limited in the choice of the resource tax by the possibility of the

11 However, our analysis is not incompatible with a different assumption regarding the motivations of foreign aid disbursements.
producing country to use the resource itself. Despite this possibility, the resource tax remains higher in the resource consuming country. Daubanes and Grimaud (2006) use a dynamic, decentralized, general equilibrium model with two countries consuming a non renewable polluting resource entirely held by one country. They show that the resource importing country is limited in the possibility of rent extraction by the loss of competitiveness resulting from a high resource tax. They also show that even in the presence of pollution, the resource importing country taxes the resource too heavily with respect to what would be globally optimal. Besides the heterogeneity in resource endowments considered in these papers, we consider intra national heterogeneity of endowments (resource holders versus poor workers) and a mechanism of international redistribution (foreign aid).

The rest of the paper is organized as follows. The model is presented in Section 2. The general equilibrium for given policy instruments is determined in Section 3. The strategic choices of taxation and foreign aid are examined in Section 4. The contractual solution is presented in Section 5. Finally, Section 6 provides a discussion of the implementation issues of our theoretical solution.

2. The Model

At each date $t^{13}$, one final good is produced in both countries using labor and oil. The aggregate production functions are:

\[ Y_i(t) = (A_i(t)L_i(t))^{1-\alpha} R_i(t)^{\alpha}, \quad i = N, S, \]

where $A_i(t)$ is an index of labor productivity, $L_i(t)$ is the quantity of labor employed and $R_i(t)$ is the quantity of oil used by the final sector firms of country $i$.

The labor productivity in the South is a constant fraction of the labor productivity in the North. The growth rate of labor productivities is constant and exogenously given:

\[ A_N(t) = A(t), \quad A_S(t) = \varphi A(t), \quad \text{with} \quad 1 > \varphi > 0, \quad g_A = x^{14}, \quad \text{and} \quad A(0) > 0. \]

Oil is extracted without any cost from an initial stock $Q_0$:

\[ \dot{Q} = -(R_N + R_S), \quad Q(0) = Q_0. \]

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13 When no confusion is possible, we will drop the time index in order to simplify the notations
14 We denote by $g_X$ the growth rate of a variable $X$. 
The northern population is composed of \( L_N \) identical agents who are endowed with one unit of labor. The southern population is composed of two groups: one group of \( L_S \) workers (southern poor) who are endowed with one unit of labor and one group of resource holders (southern rich) who own the oil stock \( Q_0 \). The resource holders’ group size is normalized to one. All the agents are infinitely lived.

There is no international labor mobility\(^{15}\).

The consumption levels of the northern workers, the southern poor and the southern rich are respectively denoted by \( C_N \), \( C_{SP} \) and \( C_{SR} \). The preferences of the representative agent of each group are given by the utility functions:

\[
(4) \quad U_N = \int_0^{\infty} [L_N \ln(C_N/L_N) + \delta L_S \ln(C_{SP}/L_S)]e^{-\rho t} \, dt,
\]
\[
(5) \quad U_{SP} = \int_0^{\infty} [L_S \ln(C_{SP}/L_S)]e^{-\rho t} \, dt,
\]
\[
(6) \quad U_{SR} = \int_0^{\infty} \ln(C_{SR})e^{-\rho t} \, dt,
\]

where \( 0<\rho<1 \) is the future discount rate common to all groups and \( 0\leq\delta<1 \) is the altruism rate of the northern citizens towards the southern poor.

The world budget constraint is:

\[
(7) \quad Y_N + Y_S = C_N + C_{SP} + C_{SR}.
\]

There are world competitive markets for the final good, oil and financial assets, and local competitive labor markets.\(^{16}\). The final good price is normalized to one and we denote by \( p \), \( r \), \( w_N \) and \( w_S \) the world oil price, the interest rate and the wages in the North and in the South respectively.

We suppose that the northern government represents the northern citizens whereas the southern government only represents the resource holders, who constitute a small elite close to the power. The governments maximize the utility function of the group that they represent.

\(^{15}\) This implies that labor is a fixed input.

\(^{16}\) Our results are robust to the introduction of market power in the extraction sector. The market power is considerably limited by the fact that the total oil stock is finite (Stiglitz (1976)).
The authorities can impose ad valorem tax rates on the local use of oil. We denote the oil taxes by $\theta_i, i = N, S$. For simplicity, we suppose that the taxes are constant over time. With our notations, the consumer oil price at date $t$ is equal to $(\theta_N + 1)p(t)$ in the North and $(\theta_S + 1)p(t)$ in the South. The fiscal revenues $\theta_N p(t) R_N(t)$ and $\theta_S p(t) R_S(t)$ are respectively redistributed to the northern citizens and the resource holders.

Finally, we suppose that the northern government can make a lump sum transfer $F(t) \geq 0$ of foreign aid to the southern poor. We suppose that foreign aid grows at the same rate as the northern citizens’ revenue.

The timing of the game is the following. At date zero, the two governments set their policies non cooperatively. These policies determine the production of the final good in the two countries and the net revenues of the three groups. We solve the problem by backward induction. First, we characterize the competitive general equilibrium for given oil tax rates and foreign aid. Second, we determine the Nash equilibrium policies.

### 3. Competitive General Equilibrium for Given Policies

The final sector firms maximize their profits with respect to the quantity of labor $L_i$ and the quantity of oil $R_i$:

$$\max_{L_i, R_i} (A_i L_i)^{1-\alpha} R_i^\alpha - w_i L_i - (\theta_i + 1)p R_i, \text{ for } i = N, S,$$

In equilibrium, the firms equalize the marginal productivity of each input with its price:

$$\alpha (Y_i / R_i) = p (\theta_i + 1), i = N, S,$$

$$\left(1 - \alpha\right) (Y_i / L_i) = w_i, i = N, S.$$

It is important to note that a necessary condition for a global optimum is $\theta_N = \theta_S$. When the tax rates differ, the marginal oil productivities are not equalized and a higher global output could be obtained with a different oil allocation between the North and the South.

The extraction sector firms maximize their profits with respect to the flow of oil $R(t)$, under the exhaustibility constraint :

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17 This assumption is made for simplicity and it is not restrictive. In our model the governments have no reason to set dynamic tax rates. In the presence of an environmental distortion such as pollution, the optimal taxes rates may not be constant. See Daubanes and Grimaud (2006) for reference to such a case.

18 This assumption is made for computational simplicity and is not a real restriction.
\[
\max_{R(t)} \int_t^\infty p(t)R(t)e^{-\int_t^u r(u)du}ds \text{ s.t. (3).}
\]

In equilibrium, oil is managed as a financial asset and its extraction follows the usual Hotelling rule:

\[\frac{\dot{p}}{p} = r.\]  

The instantaneous budget constraints of the representative northern citizen, southern poor and southern rich are respectively:

\[\frac{C_N}{L_N} + \frac{\dot{B}_N}{L_N} = w_N + \frac{rB_N}{L_N} + \frac{\theta_N p R_N}{L_N} - \frac{F}{L_N},\]

\[\frac{C_{SP}}{L_S} + \frac{\dot{B}_{SP}}{L_S} = w_S + \frac{rB_{SP}}{L_S} + \frac{F}{L_S},\]

\[C_{SR} + \dot{B}_{SR} = p(R_N + R_S) + rB_N + \theta_S p R_S.\]

where \(B_i\) is the group \(i\)'s stock of financial assets.

The households maximize their utility \(U_i, i = N, SP, SR\) under their respective budget constraints and the no Ponzi game condition:

\[\lim_{t \to \infty} B_i(t)e^{-\int_0^t r(s)ds} = 0, \ i = N, SP, SR.\]

The first order conditions of the households’ maximization problems lead to the usual Ramsey-Keynes conditions:

\[g(C_N) = g(C_{SP}) = g(C_{SR}) = r - \rho.\]

The financial market is in equilibrium at date zero if \(B_N(0) + B_{SP}(0) + B_{SR}(0) = 0\). The initial debt of each group being arbitrary, we assume that it is nil for all groups\(^{19}\):

\[B_N(0) = B_{SP}(0) = B_{SR}(0) = 0.\]

**Proposition 1** The following properties are satisfied at the decentralized general equilibrium:

1. The national production levels \(Y_N\) and \(Y_S\), as well as the consumption levels of the three groups \(C_N\), \(C_{SP}\) and \(C_{SR}\), grow at the same rate \(g = (1 - \alpha)g_A - \alpha \rho\).
2. The ratio of the production levels \(Y_N/Y_S\) is a decreasing function of the ratio of oil taxes: \((\theta_N + 1)/(\theta_S + 1)\).

\(^{19}\) Our results are robust to a different assumption on the level of initial debt.
3. For given production levels $Y_N$ and $Y_S$ and a given level of foreign aid $F$, $\theta_N$ has a positive effect on $C_N$, a negative effect on $C_{SR}$ no effect on $C_{SP}$ whereas $\theta_S$ has no effect on the consumption levels.

**Proof**: See the Appendix.

The first property is due to the fact that the resource extraction path is not biased. This property simplifies the analysis a good deal as it reduces the intertemporal utility maximization problems to the maximization of date zero utilities.

The second property is due to the effect of the relative oil tax rates on the split of the final good production between the North and the South. When the oil tax rate is higher in the North, the northern firms are less competitive as one of their inputs is more expensive. Then the southern firms will use a higher share of the extracted oil. As oil is the only mobile input in our model, less oil use implies a lower production level. Therefore, a unilateral increase in $\theta_N$ decreases $Y_N$ and increases $Y_S$ and the opposite is true for $\theta_S$.

The third property is due to the effects of the absolute oil tax rates on the split of the oil rents between the governments and the resource holders. As oil is exhaustible, its total supply is inelastic. Therefore, when a tax on oil use is imposed, the producer price of oil falls by an amount equal to the tax. Thus, oil taxes shift some of the oil rents from the resource holders to the fiscal authorities. One can see from the first order conditions (8) that the total oil remuneration by the final sector firms, $p(\theta_N + 1)R_N + p(\theta_S + 1)R_S$, is equal to $\alpha Y_N + \alpha Y_S$. This remuneration is composed of three parts: $p(R_N + R_S)$ are the resource holders’ oil revenues, $\theta_N pR_N$ are northern fiscal revenues, redistributed to the northern citizens, and $\theta_S pR_S$ are southern fiscal revenues, redistributed to the resource holders. Thus, the resource holders’ net revenues are equal to $\alpha Y_N/(\theta_N + 1) + \alpha Y_S$ and they are not affected by $\theta_S$. For given $Y_N$ and $Y_S$, a higher $\theta_N$ leads to lower revenues for the resource holders and higher revenues for the northern citizens. The southern poor’ consumption depends on their labor revenues and on the level of foreign aid, therefore it is determined for given $Y_S$ and $F$.

Thus, northern tax rate $\theta_N$ has a negative effect on the northern citizens’ wage revenues $(1 - \alpha)Y_N$ because of its negative effect on $Y_N$ and a positive effect on their fiscal revenues $\theta_N pR_N$. The northern government will take these two effects into account when setting the optimal tax rate.
We denote by $C_N(\theta_N, \theta_S, F)(t)$, $C_{SP}(\theta_N, \theta_S, F)(t)$ and $C_{SR}(\theta_N, \theta_S)(t)$ the equilibrium consumption levels of each group at date $t$ as functions of the tax rates and the level of foreign aid. Their expressions are computed in the Appendix. The following section examines the choice of the instruments by the governments at the Nash equilibrium.

4. Foreign Aid and Oil Taxation

The northern government chooses the oil tax rate $\theta_N$ and the level of international aid $F(t) \geq 0$ that maximize northern agents’ intertemporal utility, taking as given the equilibrium consumption functions $C_N(\theta_N, \theta_S, F)(t)$ and $C_{SP}(\theta_N, \theta_S, F)(t)$ and the southern government’s strategy $\theta_S$:

$$\max_{\theta_N, F(t) \geq 0} \int_0^\infty L_N \ln \left( \frac{C_N(\theta_N, \theta_S, F)(t)}{L_N} \right) + \delta L_S \ln \left( \frac{C_{SP}(\theta_N, \theta_S, F)(t)}{L_S} \right) e^{-\rho t} dt$$

s.t. $F(t) \geq 0 \forall t \geq 0$

The southern government chooses the tax rate $\theta_S$ that maximizes the resource holders’ intertemporal utility, taking as given the equilibrium consumption function $C_{SR}(\theta_N, \theta_S)(t)$ and the northern government’s strategy $\theta_N$:

$$\max_{\theta_S} \int_0^\infty \ln(C_{SR}(\theta_N, \theta_S)(t)) e^{-\rho t} dt$$

We have shown in Proposition 1 that $C_N(\theta_N, \theta_S, F)(t), C_{SP}(\theta_N, \theta_S, F)(t)$ and $C_{SR}(\theta_N, \theta_S)(t)$ grow at the same rate. Therefore, the governments’ optimization problems reduce to the maximization of utilities at date $0$.20

We denote by $\theta_N^e$, $\theta_S^e$ and $F^e(t)$ the Nash equilibrium strategies.

Proposition 2 The Nash equilibrium policies verify the following properties:

1. The oil tax rate is strictly positive in the North and nil in the South: $\theta_N^e > 0$, $\theta_S^e = 0$.
2. Foreign aid is positive if the North is sufficiently altruistic: $F^e(t) > 0$ if $\delta > \delta^e$.

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20 A formal proof of this property is given in the Appendix
**Proof:** See the Appendix.

Proposition 2 shows that at the non cooperative Nash equilibrium, the two governments set different oil tax rates. This is due to the difference in oil endowments of the groups that they represent.

The southern government has no incentives to tax oil. We have shown in proposition 1 that for given production levels $Y_N$ and $Y_S$, $\theta_S$ is neutral to $C_{SR}(\theta_N, \theta_S)(t)$ because the fiscal revenues that would be collected with a positive tax rate are exactly equal to the loss in oil revenues that this tax would induce. Moreover, we have seen that an increase in $\theta_S$ would shift some of the final good production to the North, increasing $Y_N$ with respect to $Y_S$. This would increase the share of oil consumed by the northern firms, on which the South perceives no fiscal revenues. So a positive oil tax would induce a net loss in the resource holders’ revenues. Therefore, $\theta_S^e = 0$ is a dominant strategy for the southern government.

The northern government has some incentives to tax oil. We have seen in proposition 1 that $\theta_N$ allows the capture of oil rents by the northern government from the resource holders. For given production levels $Y_N$ and $Y_S$, $\theta_N$ increases $C_N(\theta_N, \theta_S, F)(t)$ and decreases $C_{SR}(\theta_N, \theta_S)(t)$. As the northern citizens do not internalize the negative effect of $\theta_N$ on the resource holders’ revenues, they would like to set the highest possible tax rate. However, we have also seen that an increase in $\theta_N$ shifts some of the final good production from the North to the South, decreasing $Y_N$ with respect to $Y_S$ and therefore decreasing the northern citizens’ wages. The northern government considers this tradeoff between fiscal revenues and labor revenues. Therefore, $\theta_N^e$ is positive but not infinite.

There is a third channel through which $\theta_N$ influences the northern citizens’ utility. By shifting some productive activities to the South, $\theta_N$ increases the southern workers’ wages and consumption. This is an additional incentive for the altruistic North to tax oil. Of course, another way to help the southern poor is to give them foreign aid. These two redistribution instruments do not have the same costs. A marginal increase in foreign aid decreases the consumption of the northern citizens and increases that of the southern poor by the same amount. A marginal increase in $\theta_N$ below the level that maximizes the northern citizen’s revenue has a positive effect on the southern poor’s wages and a negative effect on the northern citizen’s total revenue, but these two effects are not equal in absolute value. In equilibrium, the North will choose the least expensive redistribution instrument.
The Graph 1 represents the equilibrium policies as a function of the northern altruism rate $\delta$. The northern tax rate $\theta_0$ maximizes the northern citizens’ revenue. It is implemented when the North is not altruistic, i.e. when $\delta = 0$. If the North increases $\theta_N$ above this level, the loss in labor revenues exceeds the gain in fiscal revenues for the northern citizens, while the southern poor’s wages increase. For small increases in $\theta_N$ above $\theta_0$, the resulting increase of the southern poor’s wages exceeds the loss of the northern citizens’ revenues. Thus, the tax instrument allows the Northern authorities to increase the poor’s consumption by one unit while decreasing the northern citizens’ consumption by less than one unit. When $\theta_N$ exceeds $\theta$, the opposite is true and foreign aid becomes a cheaper redistribution instrument. In equilibrium the Northern tax rate increases with the altruism rate up to $\theta$, which is implemented when $\delta = \overline{\delta}$. If $\delta > \overline{\delta}$, $\theta_N^e$ remains equal to $\overline{\theta}$ and $F^e$ becomes positive, increasing with $\delta$.

Graph 1: Nash Equilibrium Policies

![Graph 1](image)

We show in the Appendix that $\overline{\theta}$ is defined by

$$\overline{\theta} = \arg\max_{\theta_N} [C_N(\theta_N, 0, 0) + C_{SP}(\theta_N, 0, 0)]$$

and $\overline{\delta}$ is defined by

$$\overline{\delta} = \frac{C_{SP}(\overline{\theta}, 0, 0)/L_S}{C_N(\overline{\theta}, 0, 0)/L_N}$$

When $\delta > \overline{\delta}$, foreign aid splits the total revenue $C_N(\theta, 0, 0) + C_{SP}(\theta, 0, 0)$ according to the weight of each group in the northern citizen’s utility function:

$$C_N(\overline{\theta}, 0, F^e) = \frac{L_N}{L_N + \delta L_S} [C_N(\overline{\theta}, 0, 0) + C_{SP}(\overline{\theta}, 0, 0)]$$

$$C_{SP}(\overline{\theta}, 0, F^e) = \frac{\delta L_S}{L_N + \delta L_S} [C_N(\overline{\theta}, 0, 0) + C_{SP}(\overline{\theta}, 0, 0)]$$
The Graph 1 also illustrates that there is a gap between $\theta^c_N$ and $\theta^e_S$. This gap in taxation leads to a gap in the consumer oil price and therefore to an inefficient oil allocation. Global output could be increased if a larger share of oil were used in the North, where its marginal productivity is higher. The following section presents a contractual solution that reduces this inefficiency.

5. The Contract

The global distortions in the oil allocation would be reduced with a lower oil tax in the North. A lower $\theta_N$ would increase global production and total revenue. However, this increase would be unevenly split between the three groups. The resource holders’ oil rents would increase while the sum of the northern citizens’ and the southern poor’s revenues would decrease. The northern government has no incentive to lower $\theta_N$ as it does not internalize the resource holder’s revenues. A contractual solution could solve this problem by redistributing the additional income resulting from a lower $\theta_N$ in order to obtain a Pareto superior allocation.

Consider the following contract between the northern and the southern authorities. With this contract, the North lowers $\theta_N$ and the South redistributes some of the additional oil rents to the poor population, whose consumption is internalized by the altruistic North. The contract would specify the northern oil tax $\theta_N$ and some level of internal redistribution from the resource holders to the southern poor, denoted $\{I(t)\}_{t \geq 0}$.

The southern authorities will accept the contract if and only if it increases the resource holders’ utility. Their participation constraint is:

\[
\int_0^\infty \ln(C_{SR}(\theta^c_N,0)(t) - I(t))e^{-\rho t} dt \geq \int_0^\infty \ln(C_{SR}(\theta^e_N,0)(t))e^{-\rho t} dt
\]

Then the North solves:

\[
\max_{\theta_N,\{I(t)\}_{t \geq 0},\{F(t)\}_{t \geq 0}} \int_0^\infty L_N \ln \left( \frac{C_N(\theta_N,0,F(t))}{L_N} \right) + \delta L_S \ln \left( \frac{C_{SP}(\theta_N,0,F(t) + I(t))}{L_S} \right) e^{-\rho t} dt
\]

s.t. (17) and $\forall t, F(t) \geq 0, I(t) \geq 0$

As $I(t)$ enters the North’s utility function with a positive sign, the South’s participation constraint will be binding. All the additional oil rents will be redistributed to the poor: $\forall t, I(t) = C_{SR}(\theta^c_N,0)(t) - C_{SR}(\theta^e_N,0)(t)$. Therefore, $I(t)$ grows at the same rate as
all the other variables and the North’s optimization problem is reduced again to the date zero utility maximization.

We denote the optimal contract by \((\theta_N^c, I^c(t))\) and the optimal level of foreign aid with the contract by \(F^c(t)\).

**Proposition 3** The equilibrium policies with the contract verify the following properties:

1. The contract improves global efficiency: \(0 < \theta_N^c < \theta_N^e\) and \(\theta_N^c\) decreases with \(\delta\).
2. Internal redistribution is always positive. Foreign aid is positive if the North is sufficiently altruistic: \(I^c > 0\) and \(F^c > 0\) if \(\delta > \underline{\delta}\).

**Proof:** See the Appendix.

Proposition 3 shows that the constraint \(I(t) \geq 0 \iff \theta_N^c \leq \theta_N^e\) is never binding, which means that the North has always an incentive to use the contract and decrease \(\theta_N\). The reason for this is the following. Without the contract, the oil tax rate set by the North is above the one that maximizes the northern citizens’ revenue. When this tax is reduced, the northern citizen’s revenue increases. The southern poor’s revenue, equal to their wages plus the transfer from the resource holders, also increases. Therefore, as long as \(\delta > 0\) the contracting opportunity is used by the North.

However, lowering \(\theta_N\) below \(\theta_0\) decreases the northern citizens’ net revenues as their fiscal losses start to exceed their wage gains, while it continues to increase the southern poor’s revenues. As long as \(\theta_N > 0\), the increase of the southern poor’s net revenues following a marginal decrease in \(\theta_N\) exceeds the loss of the northern citizens’ revenue. Therefore the contract is a more efficient redistribution instrument than foreign aid. As long as \(\delta < \underline{\delta}, \theta_N^c\) decreases with \(\delta\) (and therefore \(I^c\) increases with \(\delta\)), as a more altruistic North is ready to sacrifice more of its own revenues for the southern poor. When \(\delta = \underline{\delta}\), the northern tax rate is minimum: \(\theta_N^c = 0\). Above this altruism level, the north no longer decreases the tax but starts using foreign aid as a redistribution instrument; when \(\delta > \underline{\delta}, \theta_N^c = 0\) and \(F^c\) increases with \(\delta\).

World efficiency thus is restored when \(\delta \geq \underline{\delta}\). This result is surprising, given that the North is not a world social planner. The reason for this result is the following. The inefficiency of the Nash equilibrium policies was due to the fact that the North did not internalize the effects of the oil tax rate on the resource holders’ revenues. The contract
transforms the additional oil rents into lump sum transfers to the poor population, whose consumption enters the North’s utility function. Therefore, it makes the North internalize the effects of $\theta_N$ on the oil revenues. When the North is very altruistic, (i.e. when $\delta > \overline{\delta}$), the contract transforms the North’s objective function into maximizing the global production minus a constant, which is the resource holders’ rent in the absence of the contract. For this reason, the optimal tax rate for a very altruistic North is the one that maximizes global output.

We have seen that global output is maximized when $\theta_N = \theta_S = 0$. Therefore, $0 = \arg\max_{\theta_N} [C_N(\theta_N, 0, 0) + C_{SP}(\theta_N, 0, 0) + C_{SR}(\theta_N, 0)]$.

We show in the Appendix that $\delta$ is given by $\delta = \frac{[C_{SP}(0,0,0) + C_{SR}(0,0) - C_{SR}(\theta_N^e, 0)]/L_S}{C_N(0,0,0)/L_N}$. When $\delta > \overline{\delta}$, the level of foreign aid $F^c$ splits the total available revenue $C_N(0,0,0) + C_{SP}(0,0,0) + C_{SR}(0,0) - C_{SR}(\theta_N^e, 0)$ according to the weight of each group in the northern citizen’s utility function:

$$C_N(0,0,F^c) = \frac{L_N}{L_N + \delta L_S} [C_N(0,0,0) + C_{SP}(0,0,0) + C_{SR}(0,0) - C_{SR}(\theta_N^e, 0)]$$

$$C_{SP}(0,0,F^c) = \frac{\delta L_S}{L_N + \delta L_S} [C_N(0,0,0) + C_{SP}(0,0,0) + C_{SR}(0,0) - C_{SR}(\theta_N^e, 0)]$$

The Graph 2 illustrates the difference in policies with and without the contract.

**Graph 2: Policies with the Contract**

![Graph 2](image)

We can see from Graph 2 that the contract decreases the northern oil tax, decreases the level of foreign aid and increases the level of internal redistribution in the South. Thus, additionally to increasing the world output, the contract alters its split between the different groups. The following proposition assesses the welfare effects of the contract.
**Proposition 4** The equilibrium allocation with the contract Pareto dominates the Nash equilibrium allocation.

**Proof**: See the Appendix.

The welfare of the North is obviously improved with an additional instrument. The resource holders’ welfare is unchanged as their participation constraint is binding. The contract has three effects on the consumption of the southern poor. First, it decreases their wage revenue because some productive activities move to the North as the latter becomes more competitive. Second, it decreases the amount of foreign aid from the North. Third, it introduces a positive amount of internal redistribution. Proposition 4 shows that the overall effect on the southern poor’s revenue is positive.

Thus a simple contract between the authorities of the oil importing countries and the resource holders from poor oil exporting countries is sufficient in order to correct the current distortions and improve welfare. Obviously, our model abstracts from some difficulties that could arise when implementing such a contract the real world. We discuss some of the implementation issues in the following section.

### 6. Discussion

In this paper we have shown that the industrialized countries could transfer revenues to the poor populations living in oil rich countries in a more efficient way by coordinating their taxation policies and aid policies. The contractual solution presented in this paper improves global efficiency and welfare. However, policy makers have to take into account several implementation issues that are absent from our theoretical model.

First, reducing the tax rates on oil products may not be a very popular measure at the times where global warming and pollution have become important concerns for the industrialized countries’ electorate. However, as the supply of this exhaustible natural resource is relatively inelastic in the long run, oil taxes have little influence on the global oil production. The recent papers of Grimaud and Rougé (2005), Groth and Schou (2007), Grimaud and Rougé (2008) and Sinn (2008) show that the world pollution accumulation is determined by the anticipated dynamics of the oil tax rates and not by the absolute levels of the tax rates. The northern authorities could lower the oil tax levels in order to reduce the
global productive distortions and modify the tax growth rates in order deal with the global pollution externality. Moreover, the industrialized countries seem to orient themselves to a regulation of carbon gas emissions using international markets for polluting rights instead of oil taxes.

Second, as the northern and southern authorities put different weights on the welfare of the poor, how can the North make sure that the additional oil revenues resulting from the oil tax cut will really be transferred to the poor? This is a relevant question. However, it also applies to foreign aid. Once the millions of dollars of aid are in the South governments’ accounts, the donors do not have a perfect control of how this money is spent. In fact, for the southern authorities, oil revenues are equivalent to budget aid and there is no obvious reason why they should spend them differently. If the industrialized countries trust them enough to give them budget aid, then they should trust them enough to sign contracts as the one proposed in this paper.

Third, foreign aid has the advantage of being more visible than contracts. The revenue transfers resulting from foreign aid are easier to quantify than the transfers resulting from an oil tax cut. If the industrialized countries are willing not only to help the southern poor, but also to let everyone know by how much they are helping, then the current situation, although inefficient, could be their preferred option.

Finally, maybe the most important implementation issue is targeting. While foreign aid can be directed to the recipient countries according to their needs and to the donors’ interests, lowering the oil taxes would enrich not only Nigeria but also Saudi Arabia. In order to discriminate between the different oil exporting countries, oil tariffs instead of oil consumption taxes should be used. Admittedly, targeting could be more complicated than with direct foreign aid.

These additional costs for the policymakers should be weighed against the benefits of helping the poor living in oil rich countries in a more efficient way. Quantifying these costs and benefits would be a useful approach for future research on this topic.
Appendix

Proof of Proposition 1

1. Let’s prove that all the variables grow at the rate $g = (1 - \alpha)g_A - \alpha \rho$

The production functions (1) $=> Y_N/Y_S = (L_N/\varphi L_S)^{1-\alpha} (R_N/R_S)^\alpha$.

The first order conditions (8) $=> R_N/R_S = (Y_N/(1 + \theta_N))/(Y_S/(1 + \theta_S))$.

Then $Y_N/Y_S = (L_N/\varphi L_S)((\theta_S + 1)/(\theta_N + 1))^{\alpha/(1-\alpha)}$.

As $L_N, L_S, \theta_N, \theta_S$ are fixed, $g_{\gamma_N} = g_{\gamma_S} = g_{\gamma}$.

The world budget constraint (7) $=> g_{\gamma} = g_{C}$.

The Ramsey Keynes conditions (15) $=> g_{\gamma} = r - \rho$.

The first order conditions (8) and $g_{\gamma_N} = g_{\gamma_S} = g = g_{\gamma_N} = g = g_{C}$.

The Hotelling rule (10) $=> g_{\gamma} = g_{\gamma} - r$. Then $g_{\gamma} = r - \rho$ $=> g_{\gamma} = -\rho$.

The production functions (1) $=>$ $g_{\gamma} = (1 - \alpha)g_A + \alpha g_{\gamma}$. Then $g_{\gamma} = -\rho$ $=> g_{\gamma} = (1 - \alpha)g_A - \alpha \rho$.

Thus $g_{\gamma} = (1 - \alpha)g_A - \alpha \rho = g_{\gamma_N} = g_{\gamma_S} = g = g_{C}$.

2. Follows directly from $Y_N/Y_S = (L_N/\varphi L_S)((\theta_S + 1)/(\theta_N + 1))^{\alpha/(1-\alpha)}$ proven in 1.

3. Let’s show that

$C_N(t) = (1 - \alpha/(\theta_N + 1))Y_N(t) - F(t)$

$C_{SP}(t) = (1 - \alpha)Y_S(t) + F(t)$

$C_{SR}(t) = \alpha/(\theta_N + 1) Y_N(t) + \alpha Y_S(t)$

Let’s explicit the instantaneous budget constraints of the three groups.

From the first order conditions (9), $w_N = (1 - \alpha)Y_N/L_N$ and $w_S = (1 - \alpha)Y_S/L_S$.

From the first order conditions (8), $p\theta_N R_N = \alpha Y_N \theta_N/(\theta_N + 1)$, $p\theta_S R_S = \alpha Y_S \theta_S/(\theta_S + 1)$ and $p (R_N + R_S) = \alpha Y_N/(\theta_N + 1) + \alpha Y_S/(\theta_S + 1)$.

By substituting in these revenues in (11), (12) and (13) and rearranging, the instantaneous budget constraints become:

$C_N/L_N + \dot{B}_N/L_N = (1 - \alpha/(\theta_N + 1))Y_N/L_N + r B_N/L_N - F/L_N$

$C_{SP}/L_S + \dot{B}_{SP}/L_S = (1 - \alpha)Y_S/L_S + r B_{SP}/L_S + F/L_S$

$C_{SR} + \dot{B}_{SR} = \alpha Y_N/(\theta_N + 1) + \alpha Y_S + r B_{SR}$.

We have shown in 1 that $g_{\gamma} = r - \rho$ and $g_{\gamma} = (1 - \alpha)g_A - \alpha \rho$ $=> r = (1 - \alpha)(g_A + \rho)$.

Solving the instantaneous budget constraints as first order differential equations in $B_N$, $B_{SP}$ and $B_{SR}$, one obtains the following inter temporal budget constraints satisfied $\forall T \geq 0$:

$B_N(T)e^{-\alpha T} + \int_0^T C_N(t)e^{-\alpha t} dt = (1 - \alpha/(\theta_N + 1)) \int_0^T Y_N(t)e^{-\alpha t} dt - \int_0^T F(t)e^{-\alpha t} dt + B_N(0)$,
\[ B_{SP}(T)e^{-rT} + \int_{0}^{T} C_{SP}(t)e^{-rxt}dt = (1 - \alpha) \int_{0}^{T} Y_{S}(t)e^{-rxt}dt + \int_{0}^{T} F(t)e^{-rxt}dt + B_{SP}(0) \]

\[ B_{SR}(T)e^{-rT} + \int_{0}^{T} C_{SR}(t)e^{-rxt}dt = \frac{\alpha}{\theta_{N} + 1} \int_{0}^{T} Y_{N}(t)e^{-rxt}dt + + \alpha \int_{0}^{T} Y_{S}(t)e^{-rxt}dt + B_{SR}(0) \]

The no Ponzi game conditions, (14) \( \Rightarrow \lim_{T \rightarrow \infty} B_{i}(T)e^{-rT} = 0, i = N, SP, SR. \)

Using the assumption \( B_{N}(0) = B_{SP}(0) = B_{SR}(0) = 0 \) we obtain

\[ \int_{0}^{\infty} C_{N}(t)e^{-rxt}dt = (1 - \alpha/\theta_{N} + 1) \int_{0}^{\infty} Y_{N}(t)e^{-rxt}dt - \int_{0}^{\infty} F(t)e^{-rxt}dt, \]

\[ \int_{0}^{\infty} C_{SR}(t)e^{-rxt}dt = \frac{\alpha}{\theta_{N} + 1} \int_{0}^{\infty} Y_{N}(t)e^{-rxt}dt + \alpha \int_{0}^{\infty} Y_{S}(t)e^{-rxt}dt, \]

\[ \int_{0}^{\infty} C_{SP}(t)e^{-rxt}dt = (1 - \alpha) \int_{0}^{\infty} Y_{S}(t)e^{-rxt}dt + \int_{0}^{\infty} F(t)e^{-rxt}dt. \]

We have proven in 1 that \( C_{N}, C_{SP}, C_{SR}, Y_{N}, Y_{S} \) and \( F \) all grow at the same rate \( r - \rho \).

\[ \forall X: g_{X} = r - \rho, \int_{0}^{\infty} X(t)e^{-rxt}dt = \int_{0}^{\infty} X(0)e^{(r-p)}e^{-rxt}dt = X(0) \int_{0}^{\infty} e^{-pt}dt = X(0)/\rho. \]

Then the inter temporal budget constraints imply

\[ C_{N}(0) = (1 - \alpha/\theta_{N} + 1)Y_{N}(0) - F(0) \]

\[ C_{SP}(0) = (1 - \alpha)Y_{S}(0) + F(0) \]

\[ C_{SR}(0) = \alpha/\theta_{N} + 1 Y_{N}(0) + \alpha Y_{S}(0) \]

and

\[ C_{N}(t) = (1 - \alpha/\theta_{N} + 1)Y_{N}(t) - F(t) \equiv C_{N}(\theta_{N}, \theta_{S}, F)(t), \]

\[ C_{SR}(t) = \alpha/\theta_{N} + 1 Y_{N}(t) + \alpha Y_{S}(t) \equiv C_{SR}(\theta_{N}, \theta_{S})(t). \]

\[ C_{SP}(t) = (1 - \alpha)Y_{S}(t) + F(t) \equiv C_{SP}(\theta_{N}, \theta_{S}, F)(t). \]

These expressions imply that for given \( Y_{N}(t) \) and \( Y_{S}(t) \), \( \theta_{N} \) has a positive effect on \( C_{N}(t) \), a negative effect on \( C_{SR}(t) \) and no effect on \( C_{SP}(t) \) while \( \theta_{N} \) has no effect on the consumption levels.

**Proof of Proposition 2**

From the above expressions of the consumption functions at date \( t \) note that

\[ C_{N}(\theta_{N}, \theta_{S}, F)(t) = C_{N}(\theta_{N}, \theta_{S}, 0) - F(t) \]

\[ C_{SP}(\theta_{N}, \theta_{S}, F)(t) = C_{SP}(\theta_{N}, \theta_{S}, 0)(t) + F(t) \]

Let’s show that the governments’ optimization problems reduce to the constrained maximization of date 0 utilities.

From Proposition 1, \( C_{N}, C_{SP}, C_{SR} \) and \( F \) grow at the same rate \( g = (1 - \alpha)g_{A} + \alpha g_{R} \)
\[ C_N(\theta_N, \theta_S, 0)(t) = C_N(\theta_N, \theta_S, 0)(0)e^{-gt}, \]
\[ C_{SP}(\theta_N, \theta_S, 0)(t) = C_{SP}(\theta_N, \theta_S, 0)(0)e^{-gt}, \]
\[ C_{SR}(\theta_N, \theta_S)(t) = C_{SR}(\theta_N, \theta_S)(0)e^{-gt}. \]

Then the governments’ optimization problems become:
\[
\max_{\theta_N, \theta_S, 0} \left[ L_N \ln \left( \frac{C_N(\theta_N, \theta_S, 0)(0)-F(0)}{L_N} \right) + \delta L_S \ln \left( \frac{C_{SP}(\theta_N, \theta_S, 0)(0)+F(0)}{L_S} \right) \right] \int_0^\infty e^{-(g+\rho)t} dt, \text{ s.t. } F(0) \geq 0
\]
\[
\max_{\theta_N, \theta_S, 0} \ln \left( C_{SR}(\theta_N, \theta_S)(0) \right) \int_0^\infty e^{-(g+\rho)t} dt,
\]
and are reduced to:
\[
\max_{\theta_N, \theta_S, 0} \left[ L_N \ln \left( \frac{C_N(\theta_N, \theta_S, 0)(0)-F(0)}{L_N} \right) + \delta L_S \ln \left( \frac{C_{SP}(\theta_N, \theta_S, 0)(0)+F(0)}{L_S} \right) \right], \text{ s.t. } F(0) \geq 0
\]
\[
\max_{\theta_N, \theta_S, 0} \ln \left( C_{SR}(\theta_N, \theta_S)(0) \right)
\]

ii. Let’s explicit the expressions of \( C_N(\theta_N, \theta_S, 0)(0) \), \( C_{SP}(\theta_N, \theta_S, 0)(0) \) and \( C_{SR}(\theta_N, \theta_S)(0) \)

We have shown in proposition 1 that
\[ C_N(\theta_N, \theta_S, 0)(0) = (1 - \alpha/(\theta_N + 1))Y_N(0) \]
\[ C_{SP}(\theta_N, \theta_S, 0)(0) = (1 - \alpha)Y_S(0) \]
\[ C_{SR}(\theta_N, \theta_S)(0) = \alpha/(\theta_N + 1) Y_N(0) + \alpha Y_S(0) \]

Let’s compute \( Y_N(0) \) and \( Y_S(0) \) as functions of \( \theta_N \) and \( \theta_S \).

The production functions (1) \( \Rightarrow Y_N(0) = (A_0L_N)^{1-\alpha} R_N(0)^{\alpha}, Y_S(0) = (A_0\varphi L_N)^{1-\alpha} R_S(0)^{\alpha} \).

Let’s explicit \( R_N(0) \) and \( R_S(0) \).

From the proof of proposition 1, \( g_R = -\rho \).

Then (3) \( \Rightarrow Q_0 = \int_0^\infty R(t) dt = \int_0^\infty R(0)e^{-\rho t} dt = R(0)/\rho. \) Thus \( R(0) = Q_0/\rho. \)

The first order conditions (8) \( \Rightarrow R_N/R_S = (Y_N/(1 + \theta_N))/(Y_S/(1 + \theta_S)) \)

Then (1) \( \Rightarrow Y_N/Y_S = (L_N/\varphi L_S)((\theta_S + 1)/(\theta_N + 1))^{\alpha/(1-\alpha)} \)

and \( R_N(0)/R_S(0) = ((\theta_N + 1)/(\theta_S + 1))^{1/(1-\alpha)} L_N/(\varphi L_S). \)

Using \( R_N(0) + R_S(0) = R(0) \), we obtain
\[ R_N(0) = \rho Q_0/[1 + (\varphi L_S/L_N)((\theta_N + 1)/(\theta_S + 1))^{1/(1-\alpha)}], \]
\[ R_S(0) = \rho Q_0/[1 + (L_N/\varphi L_S)((\theta_S + 1)/(\theta_N + 1))^{1/(1-\alpha)}]. \]

Thus
\[ Y_N(0) = (A_0L_N)^{1-\alpha}\left[ \rho Q_0/[1 + (\varphi L_S/L_N)((\theta_N + 1)/(\theta_S + 1))^{1/(1-\alpha)}] \right]^\alpha, \]
\[ Y_S(0) = (A_0\varphi L_S)^{1-\alpha}\left[ \rho Q_0/[1 + (L_N/\varphi L_S)((\theta_S + 1)/(\theta_N + 1))^{1/(1-\alpha)}] \right]^\alpha, \]
and
\[
C_N(\theta_N, 0, 0)(0) = \left(1 - \frac{\alpha}{(\theta_N + 1)}\right)(A_0 L_N)^{1-\alpha} \left[\frac{\rho Q_0}{1+(\frac{L_N}{\phi L_S})(\frac{\theta_N+1}{\theta_S+1})^{1/(1-\alpha)}}\right]^\alpha
\]

\[
C_{SP}(\theta_N, 0, 0)(0)= (1 - \alpha)(A_0 \phi L_S)^{1-\alpha} \left[\frac{\rho Q_0}{1+(\frac{L_N}{\phi L_S})(\frac{\theta_N+1}{\theta_S+1})^{1/(1-\alpha)}}\right]^\alpha
\]

\[
C_{SR}(\theta_N, \theta_S)(0) = \frac{\alpha}{(\theta_N + 1)}(A_0 L_N)^{1-\alpha} \left[\frac{\rho Q_0}{1+(\frac{L_N}{\phi L_S})(\frac{\theta_N+1}{\theta_S+1})^{1/(1-\alpha)}}\right]^\alpha + \alpha(A_0 \phi L_S)^{1-\alpha} \left[\frac{\rho Q_0}{1+(\frac{L_N}{\phi L_S})(\frac{\theta_N+1}{\theta_S+1})^{1/(1-\alpha)}}\right]^\alpha
\]

iii. Let’s solve the South’s maximization problem.
One can check that \(\partial C_{SR}(\theta_N, \theta_S)(0)/\partial \theta_S < 0 \Leftrightarrow \theta_S > 0\).
After some simplifications, \(\partial C_{SR}(\theta_N, \theta_S)(0)/\partial \theta_S < 0 \Leftrightarrow\)

\[
\left(\frac{L_N}{\phi L_S}\right)^{1-\alpha} \left[\frac{1+(\phi L_S/L_N)((\theta_N+1)/(\theta_S+1))^{1/(1-\alpha)}}{1+(L_N/\phi L_S)((\theta_N+1)/(\theta_S+1))^{1/(1-\alpha)}}\right]^{-\alpha-1} \phi L_S \left(\frac{\theta_N+1}{(\theta_S+1)}\right)^{-1/(1-\alpha)}
\]

\[\times \left(\frac{L_N}{\phi L_S}\right)^{1/(1-\alpha)} (\theta_N + 1)\]

After some other simplifications this condition becomes

\((\theta_N + 1)^{-1}((\theta_N + 1)/(\theta_S + 1))^{\alpha/(1-\alpha)} < ((\theta_S + 1)/(\theta_N + 1))^{1/(1-\alpha)} \Leftrightarrow (\theta_S + 1) > 1\).
Thus \(\theta_S^e = 0\) maximizes \(C_{SR}(\theta_N, \theta_S)(0)\) for all \(\theta_N\).

iv. Let’s solve the North’s optimization problem:
Anticipating \(\theta_S^e = 0\), the North solves

\[
\max_{\theta_N, F(0) \geq 0} \left[L_N \ln \left(\frac{C_N(\theta_N, 0, 0)(0) - F(0)}{L_N}\right) + \delta L_S \ln \left(\frac{C_{SP}(\theta_N, 0, 0)(0) + F(0)}{L_S}\right)\right]
\]

\[L = L_N \ln \left(\frac{C_N(\theta_N, 0, 0)(0) - F(0)}{L_N}\right) + \delta L_S \ln \left(\frac{C_{SP}(\theta_N, 0, 0)(0) + F(0)}{L_S}\right) + \pi F\]

The first order conditions are

(C1) \(\partial L/\partial \theta_N = 0\)

(C2) \(\partial L/\partial F(0) = 0\)

(C3) \(\pi F(0)\)

(C4) \(F(0) \geq 0\)

(C5) \(\pi \geq 0\)

Case 1 : \(F(0) = 0\)
(C1) becomes:

$$\frac{L_N}{C_N(\theta_N,0,0)(0)} \frac{\partial C_N(\theta_N,0,0)(0)}{\partial \theta_N} + \frac{\delta L_S}{C_{SP}(\theta_N,0,0)(0)} \frac{\partial C_{SP}(\theta_N,0,0)(0)}{\partial \theta_N} = 0$$

Let’s show that this has a unique solution $\theta_N^e(\delta) > 0$.

Using the expressions of $C_N(\theta_N,0,0)(0)$ and $C_{SP}(\theta_N,0,0)(0)$ computed in ii) and after some simplifications, (C1) is equivalent to $Z1(\theta_N) = 0$, where

$$Z1(\theta_N) \equiv \left(\frac{\theta_N + 1}{\theta_N + 1 - \alpha}\right)^{1/(1-\alpha)} + \left(\frac{\phi L_S/L_S}{\theta_N + 1 - \alpha}\right) - \left(\frac{\phi L_S/L_S}{1 - \alpha}\right) + \left(\frac{\delta L_S}{L_N}\right) \frac{1}{1 - \alpha} \left(\theta_N + 1\right)^{-1/(1-\alpha)}$$

One can check that $Z1'(\theta_N) < 0$, $Z1(0) > 0$ and $\lim_{\delta \to \infty} Z1(\theta_N) < 0$. These properties imply that $\forall \delta, \exists! \theta_N^e(\delta)$, such that $Z1(\theta_N^e(\delta)) = 0$ and that $\theta_N^e(\delta) > 0$.

(C2) and (C5) require $\delta \leq (C_{SP}(\theta_N^e(\delta),0,0) L_S)/(C_N(\theta_N^e(\delta),0,0)(0)/L_N)$.

As $\forall \theta_N, \frac{\partial C_{SP}(\theta_N,0,0)(0)}{\partial \theta_N} > 0$, (C1) implies that $\theta_N^e$ is such that $\frac{\partial C_N(\theta_N^e(\delta),0,0)(0)}{\partial \theta_N^e(\delta)} < 0$ and the higher $\delta$, the lower $\frac{\partial C_N(\theta_N^e(\delta),0,0)(0)}{\theta_N^e(\delta)}$. Therefore, $\theta_N^e(\delta)$ is increasing with $\delta$.

**Case 2**: $F(0) > 0$.

(C3) $\Rightarrow \pi = 0$ and (C2) $\Rightarrow F(0) = \frac{\delta L_S C_N(\theta_N,0,0)(0) - L_N C_{SP}(\theta_N,0,0)(0)}{L_N + \delta L_S}$.

Replacing $F(0)$ in (C1) and simplifying, (C1) becomes

$$\frac{\partial C_N(\theta_N,0,0)(0)}{\partial \theta_N} + \frac{\partial C_{SP}(\theta_N,0,0)(0)}{\partial \theta_N} = 0$$

Let’s show that this equation has a unique solution $\theta > 0$.

Using the expressions of $C_N(\theta_N,0,0)(0)$ and $C_{SP}(\theta_N,0,0)(0)$ computed in ii) and simplifying, (C1) becomes $Z2(\theta_N) = 0$, where

$$Z2(\theta_N) \equiv (L_N/\phi L_S)(\theta_N + 1)^{-(1/(1-\alpha))} + (2 - \alpha)/(1 - \alpha) - (\theta_N + 1)/(1 - \alpha)$$

One can check that $Z2'(\theta_N) < 0$, $\lim_{\delta \to \infty} Z2(\theta_N) < 0$ and $Z2(0) > 0$. These properties imply that $\exists! \theta$, such that $Z1(\theta) = 0$ and that $\theta > 0$.

(C4) requires

$$\delta \geq (C_{SP}(\theta,0,0)(0)/L_S)/(C_N(\theta,0,0)(0)/L_N)$$

Let’s check that the North’s problem has a unique solution for all $\delta$.

First note that when $\delta = (C_{SP}(\theta_N^e(\delta),0,0)(0)/L_S)/(C_N(\theta_N^e(\delta),0,0)(0)/L_N)$, $\theta_N^e(\delta) = \theta$.

As $\theta_N^e(\delta)$ is increasing with $\delta$, $\delta \geq \frac{C_{SP}(\theta,0,0)}{L_S} \frac{\partial C_{SP}(\theta,0,0)(0)}{\partial \theta_N} \Rightarrow \theta_N^e(\delta) > \theta$.

Second note that as $\frac{\partial C_{SP}(\theta,0,0)(0)}{\partial \theta_N}$ and $\frac{\partial C_N(\theta,0,0)(0)}{\partial \theta_N}$ are decreasing with $\theta_N$, and $\theta$ is defined by
\[
\frac{\partial C_{\text{SP}}(\theta, 0, 0)(0)}{\partial \theta_N} = - \frac{\partial C_N(\theta, 0, 0)(0)}{\partial \theta_N},
\]
\[
\theta_N > \theta \Leftrightarrow \frac{\partial C_{\text{SP}}(\theta_N, 0, 0)(0)}{\partial \theta_N} \leq - \frac{\partial C_N(\theta_N, 0, 0)(0)}{\partial \theta_N}
\]
Suppose that \( \delta \geq (C_{\text{SP}}(\theta, 0, 0)/L_S)/(C_N(\theta, 0, 0)/L_N) \). Then \( \theta_N^e = \theta \) and \( F^e(0) = \frac{\delta L_S C_N(\theta, 0, 0)(0) - L_N C_{\text{SP}}(\theta, 0, 0)(0)}{L_N + \delta L_S} \) is a solution. Let’s show that \( \theta_N^e = \theta_N^e(\delta) \) and \( F^e(0) = 0 \) is not a solution.

From the definition of \( \theta_N^e(\delta) \),
\[
\left( \frac{C_{\text{SP}}(\theta_N^e(\delta), 0, 0)}{L_S} \right) / \left( \frac{C_N(\theta_N^e(\delta), 0, 0)}{L_N} \right) = \delta \frac{\partial C_{\text{SP}}(\theta_N^e(\delta), 0, 0)(0)}{\partial \theta_N} / \left( - \frac{\partial C_N(\theta_N^e(\delta), 0, 0)(0)}{\partial \theta_N} \right)
\]
Thus \( \delta > \frac{C_{\text{SP}}(\theta_N^e(\delta), 0, 0)/L_S)}{(C_N(\theta_N^e(\delta), 0, 0)/L_N)} \).

Suppose that \( \delta < (C_{\text{SP}}(\theta, 0, 0)/L_S)/(C_N(\theta, 0, 0)/L_N) \). Then \( \theta_N^e = \theta \) and \( F^e(0) = \frac{\delta L_S C_N(\theta, 0, 0)(0) - L_N C_{\text{SP}}(\theta, 0, 0)(0)}{L_N + \delta L_S} \) is a solution. Let’s show that \( \theta_N^e = \theta_N^e(\delta) \) and \( F^e(0) = 0 \) is not a solution.

From the definition of \( \theta_N^e(\delta) \),
\[
\left( \frac{C_{\text{SP}}(\theta_N^e(\delta), 0, 0)}{L_S} \right) / \left( \frac{C_N(\theta_N^e(\delta), 0, 0)}{L_N} \right) = \delta \frac{\partial C_{\text{SP}}(\theta_N^e(\delta), 0, 0)(0)}{\partial \theta_N} / \left( - \frac{\partial C_N(\theta_N^e(\delta), 0, 0)(0)}{\partial \theta_N} \right)
\]
Thus \( \delta < \frac{C_{\text{SP}}(\theta_N^e(\delta), 0, 0)/L_S)}{(C_N(\theta_N^e(\delta), 0, 0)/L_N)} \).

So the solution to the North’s problem is \( (\theta_N^e, F^e(0)) \):
\[
\theta_N^e = \theta_N^e(\delta) \text{ and } F^e(0) = 0 \text{ if } \delta \leq \left( \frac{C_{\text{SP}}(\theta, 0, 0)}{L_N} \right) / \left( \frac{C_N(\theta, 0, 0)}{L_N} \right)
\]
\[
\theta_N^e = \theta \text{ and } F^e(0) = \frac{\delta L_S C_N(\theta, 0, 0)(0) - L_N C_{\text{SP}}(\theta, 0, 0)(0)}{L_N + \delta L_S} \geq 0 \text{ if } \delta \geq \left( \frac{C_{\text{SP}}(\theta, 0, 0)}{L_N} \right) / \left( \frac{C_N(\theta, 0, 0)}{L_N} \right)
\]
\[
(C_N(\theta, 0, 0)/L_N).
\]

**Proof of Proposition 3**

As in proposition 2, we can check that the North’s problem reduces to the utility maximization at date 0:
\[
\max_{\theta_N, J(0) \geq 0, F(0) \geq 0} \left[ L_N \ln \left( \frac{C_N(\theta_N, \theta_S, 0)(0) - F(0)}{L_N} \right) + \delta L_S \ln \left( \frac{C_{\text{SP}}(\theta_N, \theta_S, 0)(0) + F(0) + I(0)}{L_S} \right) \right]
\]
s.t. \( C_{\text{SR}}(\theta_N^e, 0) \leq C_{\text{SR}}(\theta_N, 0) - I(0) \)

As the North’s objective function is increasing with \( I(0) \), the South’s participation constraint is binding: \( I(0) = C_{\text{SR}}(\theta_N, 0) - C_{\text{SR}}(\theta_N^e, 0) \).

As \( C_{\text{SR}}(\theta_N, 0) \) decreases with \( \theta_N \), \( I(0) \geq 0 \Leftrightarrow \theta_N \leq \theta_N^e \).
So the North’s problem becomes

\[
\max_{\theta_N, F(0)} \left[ L_N \ln \left( \frac{C_N(\theta_N, \theta_S, 0)(0) - F(0)}{L_N} \right) + \delta L_S \ln \left( \frac{C_{SP}(\theta_N, \theta_S, 0)(0) + F(0) + C_{SR}(\theta_N, 0) - C_{SR}(\theta_N^e, 0)}{L_S} \right) \right]
\]

s.t. \( \theta_N \leq \theta_N^e \) and \( F(0) \geq 0 \).

\[
L = L_N \ln \left( \frac{C_N(\theta_N, \theta_S, 0)(0) - F(0)}{L_N} \right) + \delta L_S \ln \left( \frac{C_{SP}(\theta_N, \theta_S, 0)(0) + F(0) + C_{SR}(\theta_N, 0) - C_{SR}(\theta_N^e, 0)}{L_S} \right) + \mu F(0) + \phi (\theta_N^e - \theta_N)
\]

The first order conditions are

(C6) \( \frac{\partial L}{\partial \theta_N} = 0 \)
(C7) \( \frac{\partial L}{\partial F(0)} = 0 \)
(C8) \( \mu F(0) = 0 \)
(C9) \( F(0) \geq 0 \)
(C10) \( \pi \geq 0 \)
(C11) \( \phi (\theta_N^e - \theta_N) = 0 \)
(C12) \( \phi \geq 0 \)
(C13) \( \theta_N \leq \theta_N^e \)

Case 1: \( \theta_N = \theta_N^e \), \( F(0) = 0 \).

Then (C6) is equivalent to

\[
\left. \frac{L_N}{c_N(\theta_N, 0, 0)(0)} \frac{\partial c_N(\theta_N, 0, 0)(0)}{\partial \theta_N} + \frac{\delta L_S}{c_{SP}(\theta_N, 0, 0)(0)} \left[ \frac{\partial c_{SP}(\theta_N, 0, 0)(0)}{\partial \theta_N} + \frac{\partial c_{SR}(\theta_N, 0)(0)}{\partial \theta_N} \right] \right|_{\theta_N = \theta_N^e} = \phi > 0
\]

On can check that \( \frac{\partial c_{SP}(\theta_N, 0, 0)(0)}{\partial \theta_N} + \frac{\partial c_{SR}(\theta_N, 0)(0)}{\partial \theta_N} < 0 \) for all \( \theta_N \), so (C6) implies

\[
\left. \frac{\partial c_N(\theta_N, 0, 0)(0)}{\partial \theta_N} \right|_{\theta_N = \theta_N^e} > 0
\]

But from the proof of proposition 2, \( \theta_N^e \) is such that \( \frac{\partial c_N(\theta_N^e, 0, 0)(0)}{\partial \theta_N} < 0 \)

Thus \( \theta_N^e = \theta_N \), \( F(0) = 0 \) is not a solution.

Case 2: \( \theta_N = \theta_N^e \), \( F(0) > 0 \)

\[
\left. \frac{L_N}{c_N(\theta_N, 0, 0)(0) - F(0)} \frac{\partial c_N(\theta_N, 0, 0)(0)}{\partial \theta_N} + \frac{\delta L_S}{c_{SP}(\theta_N, 0, 0)(0) + F(0)} \left[ \frac{\partial c_{SP}(\theta_N, 0, 0)(0)}{\partial \theta_N} + \frac{\partial c_{SR}(\theta_N, 0)(0)}{\partial \theta_N} \right] \right|_{\theta_N = \theta_N^e} = \phi
\]

For the same reason as in Case 1, this cannot be a solution.

Case 3: \( \theta_N < \theta_N^e \), \( F(0) = 0 \)

Then (C6) is equivalent to :
\[ \frac{L_N}{C_N(\theta_N,0,0)(0)} \frac{\partial C_N(\theta_N,0,0)(0)}{\partial \theta_N} + \frac{\delta L_S}{C_S(\theta_N,0,0)(0) + C_S(\theta_N,0) - C_S(\theta_N^e,0)} \left[ \frac{\partial C_S(\theta_N,0,0)(0)}{\partial \theta_N} + \frac{\partial C_S(\theta_N,0,0)(0)}{\partial \theta_N} \right] = 0. \]

The solution to this equation is denoted \( \theta_N^c(\delta) \).

As \( \frac{\partial C_S(\theta_N,0,0)(0)}{\partial \theta_N} + \frac{\partial C_S(\theta_N,0,0)(0)}{\partial \theta_N} < 0 \), \( \theta_N^c(\delta) \) is such that \( \frac{\partial C_N(\theta_N^c(\delta),0,0)(0)}{\partial \theta_N} > 0 \).

The higher \( \delta \), the lower \( \frac{\partial C_N(\theta_N,0,0)(0)}{\partial \theta_N} \). Thus, \( \theta_N^c(\delta) \) decreases with \( \delta \).

(C7) \( \Rightarrow \) \( \mu = \frac{L_N}{C_N(\theta_N,0,0)(0)} - \frac{\delta L_S}{C_S(\theta_N,0,0)(0) + C_S(\theta_N,0,0)(0)} \)

(C10) requires \( \delta \leq \frac{L_N}{C_N(\theta_N^c(\delta,0,0)(0))/L_N} \)

Case 4: \( \theta_N < \theta_N^e, F(0) > 0 \)

Then (C7) \( \Rightarrow \) \( F(0) = \frac{\delta L_S C_N(\theta_N,0,0)(0) - L_N \left[ C_S(\theta_N,0,0)(0) + C_S(\theta_N,0,0)(0) - C_S(\theta_N^e,0,0)(0) \right]}{L_N + \delta L_S} \)

Replacing \( F(0) \) simplifying, (C6) becomes:

\[ \frac{\partial C_N(\theta_N,0,0)(0)}{\partial \theta_N} + \frac{\partial C_S(\theta_N,0,0)(0)}{\partial \theta_N} + \frac{\partial C_S(\theta_N,0,0)(0)}{\partial \theta_N} = 0 \Leftrightarrow \frac{\partial Y_N(0)}{\partial \theta_N} + \frac{\partial Y_S(0)}{\partial \theta_N} = 0. \]

One can check that the unique solution to this equation is \( \theta_N = 0 \).

(C9) requires \( \delta \geq \frac{L_N}{C_N(\theta_N^c(\delta,0,0)(0))/L_N} \) \( \equiv \delta \)

Let’s show that the North’s problem has a unique solution for all \( \delta \).

First, one can check that for \( \delta = \frac{C_S(\theta_N^c(\delta,0,0)(0) + C_S(\theta_N^c(\delta,0,0)(0) - C_S(\theta_N^e,0,0)(0))/L_S}{C_N(\theta_N^c(\delta,0,0)(0))/L_N} \), \( \theta_N^c(\delta) = 0 \).

Second, as \( \theta_N^c(\delta) \) is decreasing with \( \delta \), \( \theta_N^c(\delta) \geq 0 \Leftrightarrow \delta \leq \delta \)

Third, as \( \theta_N = 0 \) is the solution to

\[ - \left[ \frac{\partial C_S(\theta_N,0,0)(0)}{\partial \theta_N} + \frac{\partial C_S(\theta_N,0,0)(0)}{\partial \theta_N} \right] = \frac{\partial C_N(\theta_N,0,0)(0)}{\partial \theta_N}, \]

\( \theta_N \geq 0 \Leftrightarrow - \left[ \frac{\partial C_S(\theta_N,0,0)(0)}{\partial \theta_N} + \frac{\partial C_S(\theta_N,0,0)(0)}{\partial \theta_N} \right] \geq \frac{\partial C_N(\theta_N,0,0)(0)}{\partial \theta_N} \)

Suppose \( \delta \geq \delta \Leftrightarrow \theta_N^c(\delta) \leq 0 \).

Then \( \theta_N^c = 0 \) and \( F^c(0) = \frac{\delta L_S C_N(0,0,0)(0) - L_N \left[ C_S(0,0,0)(0) + C_S(0,0,0)(0) - C_S(\theta_N^e,0,0)(0) \right]}{L_N + \delta L_S} \) is a solution.

Let’s show that \( \theta_N^c = \theta_N^c(\delta) \) and \( F^c(0) = 0 \) is not a solution.

\( \theta_N^c(\delta) \) is defined by

\[ \frac{C_S(\theta_N^c(\delta,0,0)(0) + C_S(\theta_N^c(\delta,0,0)(0) - C_S(\theta_N^e,0,0)(0))/L_S}{C_N(\theta_N^c(\delta,0,0)(0))/L_N} = - \delta \left[ \frac{\partial C_S(\theta_N,0,0)(0)}{\partial \theta_N} + \frac{\partial C_S(\theta_N,0,0)(0)}{\partial \theta_N} \] \frac{\partial C_N(\theta_N,0,0)(0)}{\partial \theta_N} \right] \leq \delta \]

as \( \theta_N^c(\delta) \leq 0 \Leftrightarrow - \left[ \frac{\partial C_S(\theta_N,0,0)(0)}{\partial \theta_N} + \frac{\partial C_S(\theta_N,0,0)(0)}{\partial \theta_N} \right] \leq \frac{\partial C_N(\theta_N,0,0)(0)}{\partial \theta_N} \)
Thus $\delta \geq \frac{c_{SP}(\theta_S^N(\delta),0,0)(0) + c_{SR}(\theta_S^N(\delta),0)(0) - c_{SR}(\theta_S^N(\delta),0)(0))}{c_{N}(\theta_S^N(\delta),0,0)(0)/L_N}$. 

Suppose that $\delta < \delta$. ($\Rightarrow \theta_N^c(\delta) \geq 0$)

Then $\theta_N^c = 0$ and $F^c(0) = \frac{\delta L_S c_N(0,0,0)(0) - L_N [c_{SP}(0,0,0)(0) + c_{SR}(0,0,0)(0) - c_{SR}(\theta_N^e,0)(0)]}{L_N + \delta L_S}$ is not a solution.

Let’s show that $\theta_N^c = \theta_N^c(\delta)$ and $F^c(0) = 0$ is a solution.

$\theta_N^c(\delta)$ is defined by

$$\left(\frac{c_{SP}(\theta_S^N(\delta),0,0)(0) + c_{SR}(\theta_S^N(\delta),0)(0) - c_{SR}(\theta_S^N(\delta),0)(0))}{c_{N}(\theta_S^N(\delta),0,0)(0)/L_N} \right) = - \frac{\frac{\partial c_{SP}(\theta_S^N,0,0)(0)}{\partial \theta_N} + \frac{\partial c_{SR}(\theta_S^N,0)(0)}{\partial \theta_N}}{\frac{\partial c_{N}(\theta_S^N,0,0)(0)}{\partial \theta_N}} \geq \delta$$

as $\theta_S^N(\delta) \geq 0$ and therefore $\left[\frac{\partial c_{SP}(\theta_S^N,0,0)(0)}{\partial \theta_N} + \frac{\partial c_{SR}(\theta_S^N,0)(0)}{\partial \theta_N}\right] \geq \frac{\partial c_{N}(\theta_S^N,0,0)(0)}{\partial \theta_N}$.

Thus $\delta < \frac{c_{SP}(\theta_S^N(\delta),0,0)(0) + c_{SR}(\theta_S^N(\delta),0)(0) - c_{SR}(\theta_S^N(\delta),0)(0))}{c_{N}(\theta_S^N(\delta),0,0)(0)/L_N}$. 

So the solution to the North’s problem is

$I^c(0) = c_{SR}(\theta_N^c,0,0)(0) - c_{SR}(\theta_N^e,0)(0)$ and

$$\theta_N^c = 0, F^c(0) = \frac{\delta L_S c_N(0,0,0)(0) - L_N [c_{SP}(0,0,0)(0) + c_{SR}(0,0,0)(0) - c_{SR}(\theta_N^e,0)(0)]}{L_N + \delta L_S}$$

if $\delta \geq \delta$. 

$$\theta_N^c = \theta_S^N(\delta), F^c(0) = 0$$ if $\delta < \delta$.

**Proof of Proposition 4**

The utility of the northern citizens is obviously higher with an additional instrument. The southern rich are indifferent as their participation constraint is binding. We only have to show that southern poor are better when the contract is used by the North.

First, one can check that $\delta < \delta$.

If $\delta < \delta$, let’s show that $c_{SP}(\theta_N^c(\delta),0,0)(0) + I^c(0) > c_{SP}(\theta_N^e(\delta),0,0)(0)$

$$\Leftrightarrow c_{SP}(\theta_N^c(\delta),0,0)(0) + c_{SR}(\theta_N^c(\delta),0)(0) - c_{SR}(\theta_N^e(\delta),0)(0) > c_{SP}(\theta_N^e(\delta),0,0)(0)$$

$$\Leftrightarrow c_{SP}(\theta_N^c(\delta),0,0)(0) + c_{SR}(\theta_N^c(\delta),0)(0) > c_{SR}(\theta_N^e(\delta),0)(0) + c_{SP}(\theta_N^e(\delta),0,0)(0)$$

Which is satisfied because $c_{SP}(\theta_N,0,0)(0) + c_{SR}(\theta_N,0)(0)$ is decreasing with $\theta_N$ and $\theta_N^c(\delta) < \theta_N^e(\delta)$.

If $\delta < \delta < \delta$, let’s show that $c_{SP}(\theta_N^c(\delta),0,0)(0) + I^c(0) > c_{SP}(\theta_N^e,0,0)(0) + F^e(0)$.

$$\Leftrightarrow c_{SP}(\theta_N^c(\delta),0,0)(0) + c_{SR}(\theta_N^c(\delta),0)(0) - c_{SR}(\theta_N,0)(0) > c_{SP}(\theta_N^e,0,0)(0) +$$

$$\frac{\delta L_S c_N(0,0,0)(0) - L_N c_{SP}(0,0,0)(0)}{L_N + \delta L_S}$$

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\[ \Leftrightarrow C_{SP}(\theta_N^c(\delta), 0, 0, 0) + C_{SR}(\theta_N^c(\delta), 0)(0) > C_{SP}(\theta, 0) + \frac{\delta L_S}{L_N + \delta L_S} C_{SP}(\theta, 0, 0) + \]
\[ \frac{\delta L_S}{L_N + \delta L_S} C_N(\theta, 0, 0) \]

As \( \theta_N^c(\delta) < \theta \), global output is higher for \( \theta_N^c(\delta) \Rightarrow C_N(\theta_N^c(\delta), 0, 0)(0) + C_{SP}(\theta_N^c(\delta), 0, 0)(0) > C_N(\theta, 0, 0)(0) + C_{SP}(\theta, 0, 0)(0) + C_{SR}(\theta, 0)(0) \)

Then, it is sufficient to show that
\[ \frac{L_N}{L_N + \delta L_S} C_N(\theta, 0, 0)(0) + \frac{L_N}{L_N + \delta L_S} C_{SP}(\theta, 0, 0)(0) - C_N(\theta_N^c(\delta), 0, 0)(0) > 0 \]

As \( \theta \) maximizes \( C_N(\theta_N, 0, 0)(0) + C_{SP}(\theta_N, 0, 0)(0) \),
\[ \frac{L_N}{L_N + \delta L_S} C_N(\theta, 0, 0)(0) + \frac{L_N}{L_N + \delta L_S} C_{SP}(\theta, 0, 0)(0) > \]
\[ \frac{L_N}{L_N + \delta L_S} C_N(\theta_N^c(\delta), 0, 0)(0) + \frac{L_N}{L_N + \delta L_S} C_{SP}(\theta_N^c(\delta), 0, 0)(0) \]

Then we are left to show that
\[ \frac{L_N}{L_N + \delta L_S} C_{SP}(\theta_N^c(\delta), 0, 0)(0) > \frac{\delta L_S}{L_N + \delta L_S} C_N(\theta_N^c(\delta), 0, 0)(0) \]
\[ \Leftrightarrow \delta < \frac{C_{SP}(\theta_N^c(\delta), 0, 0)(0)/L_S}{C_N(\theta_N^c(\delta), 0, 0)(0)/L_N}, \text{ which is implied by } \delta < \frac{\delta}{\delta} \]

If \( \delta \geq \frac{\delta}{\delta} \), let’s show that \( C_{SP}(0, 0, 0)(0) + F^c(0) + I^c(0) > C_{SP}(\theta, 0, 0)(0) + F^e(0) \).
\[ \Leftrightarrow C_N(0, 0, 0)(0) + C_{SP}(0, 0, 0) + C_{SR}(0, 0)(0) > C_N(\theta, 0, 0)(0) + C_{SP}(\theta, 0, 0)(0) + C_{SR}(\theta, 0)(0) \]

which is satisfied because global output is maximized for \( \theta_N = 0 \).
References


Kanbur, R., 2006. The Economics of International Aid. In Handbook of the Economics of Giving, Altruism and Reciprocity Vol. 2; Editors: Kolm, S.-C., Ythier, J. M.


