Flying the Nest: Intergenerational Strategic Interaction, Coresidence, and Social Mobility

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Abstract: Southern Europe is characterized by high levels of intergenerational earnings persistence and high levels of coresidence between parents and their offspring. In contrast, Northern Europe exhibits low levels of earnings persistence and of coresidence. In light of this evidence, I develop a sequential game between parents and children over coresidence: richer parents with a preference for coresidence bribe their children to keep them at home, and therefore those children have more resources to invest in education, generating higher income persistence. Next, I analyze the quantitative importance of strategic interactions inside the family over living arrangements in generating intergenerational earnings persistence. I embed the game in an overlapping-generation model with heterogeneous agents, exogenous education, coresidence, and inter-vivos transfers. I show that about 7.5 percent of intergenerational earnings persistence in Italy can be explained through the mechanism proposed. Moreover, I show that a transfer of 400 euros per month would lower the coresidence rate to less than 50 percent, highlighting the importance of credit constraints to understand differences across Europe.

Keywords: Intergenerational Earnings Persistence, Coresidence, Overlapping Generation Models, Intergenerational Transfers.

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1 Introduction

Intergenerational persistence of the outcomes of parents and their children has long been a topic of interest for social scientists. A large part of the literature has focused on documenting the cross-country intergenerational elasticities and correlations of income, wealth, and education. The first column of Table 1 summarizes results from different studies on the topic. Estimated intergenerational persistence is generally found to be high in Southern European countries, namely Italy, Spain, Greece, and Portugal, where estimated intergenerational persistences is around 0.4, a level similar to the U.S. and the U.K. Estimated intergenerational persistence is instead low in Northern European countries such as Norway, Finland, Sweden, and Denmark, with numbers varying between 0.15 and 0.27. This means that in Southern Europe around 40 percent of the father’s earnings position, relative to the mean in his cohort, is transmitted to his son, while only about 20 percent of it is transmitted in Northern Europe. The second column of Table 1 documents a positive relationship between coresidence of young adults with their parents and intergenerational earnings persistence. In Northern Europe low earnings persistence is accompanied by a share of young adults (age 20-24) that coresides with the parents lower than a half, while in Southern European countries this share is close to 90 percent, and these countries have high intergenerational persistence of earnings. In light of this observation, in this paper I analyze how the decision process of the family over living arrangements affects earnings persistence.

Intergenerational earnings persistence is the definition for how much of the parent’s position in the earnings distribution is transmitted to the offspring. The measure of earnings persistence is by definition the coefficient of the regression of the logarithm of the son’s permanent earnings to that of the father’s:

\[ \log(y_{son}) = \beta_0 + \beta_1 \log(y_{father}) + \epsilon \]

where a \( \beta_1 \) coefficient of zero means that the earnings of father and son are not correlated, and a coefficient of one means that they are perfectly correlated. Permanent earnings are a measure of lifetime earnings. In order to measure permanent earnings it would be necessary to have a very long panel that follows the same household across generations. Given that such datasets are not available, several alternative measures have been developed that rely on the specificities of different datasets. This implies that measures of earnings persistence across countries, such as those reported in Table 1, should be compared with caution. The general stylized facts of low persistence in Northern Europe and high persistence in Southern Europe, however, are untouched by these considerations (See Holter (2012) for details on the difficult of comparison of different studies and Black and Devereux (2011) for a review of the estimation methods).
### Table 1: Intergenerational Earnings Elasticity and family arrangements

<table>
<thead>
<tr>
<th>Country</th>
<th>Estimated Earnings Elasticity</th>
<th>Share of youth living with parents (20-24)</th>
<th>Age of leaving parental home</th>
</tr>
</thead>
<tbody>
<tr>
<td>Denmark</td>
<td>0.15</td>
<td>0.44</td>
<td>22</td>
</tr>
<tr>
<td>Norway</td>
<td>0.17</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Finland</td>
<td>0.18</td>
<td>0.43</td>
<td>22</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.27</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Germany</td>
<td>0.32</td>
<td>0.73</td>
<td>25</td>
</tr>
<tr>
<td>Spain*</td>
<td>0.40</td>
<td>0.92</td>
<td>30</td>
</tr>
<tr>
<td>France</td>
<td>0.41</td>
<td>0.70</td>
<td>25</td>
</tr>
<tr>
<td>Italy**</td>
<td>0.43</td>
<td>0.92</td>
<td>30</td>
</tr>
<tr>
<td>USA</td>
<td>0.47</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>UK</td>
<td>0.50</td>
<td>0.57</td>
<td>24</td>
</tr>
<tr>
<td>Portugal</td>
<td>-</td>
<td>0.88</td>
<td>30</td>
</tr>
<tr>
<td>Greece</td>
<td>-</td>
<td>0.81</td>
<td>29</td>
</tr>
</tbody>
</table>


The issue of coresidence between parents and children has been given a lot of attention in the past decade. One strand of this literature looks at labor market and income risk as reason for living with parents: Kaplan (2012) looks at the possibility of moving back home and finds that it is an important insurance channel against labor market risk, and that it can help explain aggregate features common to the low-skilled youth such as high labor elasticity and a low savings rate. Fogli (2000) models coresidence as a response of the family to credit market imperfections. Becker et al. (2005) examine the role of income insecurity of both parents and children in the coresidence choice in a household with partially altruistic parents and pooled income and find that when parents are selfish, a first order stochastic dominant shift in the distribution of the parent’s income increases the probability that the child cohabits. Another strand of literature focuses instead on cultural differences: family ties vary across countries and there is evidence that they influence coresidence and other outcomes (Alesina and Giuliano, 2010). Giuliano (2007) examines second generation immigrants in the US and finds that the coresidence patterns mimic those of the countries of origin, she then argues that different rates for coresidence with children can be explained by cultural traits. Manacorda
and Moretti (2006) exploit an exogenous change in parental income in Italy and estimate an increase in coresidence deriving from it. They argue that this result indicates that Italian parents have a preference for coresidence and that this preference differs across countries. Rosenzweig and Wolpin (1993), in their paper on the role of inter-vivos transfers and family coresidence in the US, reach the conclusion that US parents have a preference for privacy.

Recent research has started to focus on the determinants and causes of social mobility, and the differences across countries. The possible determinants of intergenerational mobility fall mostly into two main categories: institutions and family. On one side there has been a focus on imperfect credit markets and educational policies. Restuccia and Urrutia (2004) argue that government policies directed towards early education would be more effective in reducing earnings persistence. They build an overlapping generations model with altruism in which parents invest in the education of their children, distinguishing between early and college education. Parents are fully altruistic and make all decisions for their children, without any room for strategic interactions in the family. They show that investments in early education are quantitatively more important to understand intergenerational earnings persistence than investment in college education. Holter (2012) investigates the role of differences in taxation and public expenditure in education for understanding cross-country differences in earnings persistence. He calibrates an intergenerational life-cycle model with human capital accumulation and government policies and finds that the impact of taxation on human capital accumulation can explain a large part of the variation in earnings persistence between the US and 10 other countries. Another strand of research deals with intergenerational transmission of ability and the role that nature and culture have in determining income possibilities. Different preferences and cultural traits in society can help account for the way different countries respond to incentives. Early work by Becker and Tomes (1994) notes how intergenerational transmission of ability is a result of nature and nurture and can also be influenced by “cultural” educational factors.\(^2\) This point has been

\(^2\)See Piketty (2000) for a survey of the theoretical literature on inequality and Black and Devereux (2011) on a survey on the empirical literature.
revived recently in the literature on culture and economics and on family ties. Alesina and Giuliano (2010) show that family ties are important in determining behavior and economic outcomes and argue that they can have a role in explaining cross-country differences; Guiso et al. (2006) call for more understanding of how culture and economic outcomes interact.

Most literature assumes that, after high school, young agents leave their parents’ house and take their own tertiary education decisions, given a resource transfer from altruistic parents. However, as documented in the third column of Table 1, the average age at which children leave the parental home across Europe is well above 20. I argue that college enrollment and living arrangements choices are contemporaneous and not independent. Hence, I focus on tertiary education as a mechanism of transmission of the position in the earnings distribution across generations, and I investigate the choice of undertaking college education together with the decision of moving away from the parental home. Moreover, the decision made by young adults about where to live affects their parents, hence I depart from unitary household models and model strategic interactions inside the family over living arrangements and college attendance.

I build a model in which credit constrained young agents have a preference for independence while parents can have a taste or distaste for coresidence. While young agents have to decide how much to invest in education and how much to work, parents can give them a transfer in order to induce them to accept the parents’ preferred living arrangement. Parents and children play a sequential game, the parent offers the child a menu of transfers contingent to coresidence status. The child chooses the parent’s preferred option if he is indifferent between living at home or move out. The model shows that, when parents have a preference for coresidence, they transfer resources to their child to keep him home. High income parents are able to keep their children home, while coresiding young agents have more resources to invest in education, from not having to pay living expenses and receiving a transfer. This mechanism acts to decrease income mobility. On the other hand, when parents have a preference for independence, they transfer resources to their offspring only
when they are not able to move out, and this dampens earnings persistence. To the best of my knowledge, this is the first attempt to analyze how strategic interactions inside the family on the joint coresidence and schooling decisions of youths affect the intergenerational transmission of income.

The mechanism described relies on the assumption of borrowing constraints. The assumption of a strong borrowing constraints for young adults is realistic for countries where student loans are not widespread, such as Southern European countries. I analyze a version of the model with assets and show that if young agents could borrow freely, then their education decision would be independent of the parental transfer and there would be no intergenerational effect. Hence, the strategic interaction over coresidence acts as a multiplier for the credit market: intergenerational elasticity is high when parents have a taste for coresidence and the access to credit by young agents (such as study loans) is limited.

Next, I study how much of the earnings persistence in Italy can be explained by the mechanism generated by differences in preferences. To do this, I construct a 6-period overlapping generations model that embeds the game over family arrangements. The model also features human capital accumulation in college, intergenerational transmission of ability, and an education system. I show the results of some quantitative exercises that use a version of the OLG model calibrated to Italian data with a simulated method of moments and using data from the Survey of Household Income and Wealth of the Bank of Italy. I show that up to 7.5 of the intergenerational earnings elasticity in Italy can be accounted for by preferences for coresidence. Credit constraints are pivotal for the mechanism described in the model, they are also an important factor to understand differences across Europe, since credit access is relatively easier in Northern Europe than in Southern Europe, especially for young adults. Hence, it is important to understand the relative importance of the main mechanisms of the model, parental preferences and credit constraints. In order to assess the impact of lowering credit constraint, I give a transfer to young adults and I analyze the effects of this transfer on coresidence patterns, education, and social mobility. I find that a transfer of 400 euros
lowers the coresidence rate to less than 50 percent. This underlines importance of credit constraints for the mechanism described and to understand differences across Europe.

The rest of the paper is organized as follows: In Section 2, I present and show the implications of a simple game in which parents and children decide over living arrangement and human capital investment. In Section 3, I build a more detailed overlapping generations model that embeds the game between parents and children, that I will use to derive the quantitative implications for cross-country intergenerational mobility. In Section 4, I discuss the methodology and results of the quantitative exercise. Section 5 concludes.

2 A Game Over Living Arrangements

2.1 The Game

In this section, I present a model of a household in which the young and the old agent have to decide whether or not to live together. The two agents have different preferences over coresidence and will play strategically over living arrangements.

Consider a two-period lived household composed of a parent and a child. The parent dies at the end of the first period with probability one, while the child lives for two periods. Both agents derive utility from consumption according to a standard felicity function: \( u(\cdot) \) is increasing and strictly concave and satisfies the Inada conditions. The young agent has a taste for independence and derives disutility from coresidence with his parent, denoted by the preference parameter \( \psi^c \leq 0 \). The old agent instead derives utility or disutility from coresidence with the child according to the preference parameter \( \psi^p \), which can be higher or lower than zero, describing respectively a taste or a distaste for coresidence.\(^3\) Coresidence

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\(^3\)Why would young adults move out from their parents homes before having to form their own household? Household production has returns to scale, hence there is no efficiency reason for wanting to move out. If the child works and pools resources with his family, the parent’s utility is maximized with the child coresiding with them. If instead, the child is assumed to be a “parasite” living off his parent’s wealth, we can explain the fact that the parent would want the child to move out but not why the child would want to. Since there is no straightforward economic explanation for leaving the nest, it is common to assume that young agents have a taste for independence and gain utility from moving out (see for example Kaplan (2012) and Fogli
status is denoted by the dummy $I$, which takes value zero if the young agent moves out of his parent’s house and value one if he stays. The preferences of the young ($c$) and old ($p$) agents are:

$$U^c = u(c) + \psi^c I$$
$$U^p = u(c) + \psi^p I.$$  

(1)

Young and old agents are characterized by ability endowments $a^p$ and $a^c$, and they are each endowed with one unit of time. In the first period the young agent allocates his time between human capital investment and work for a fixed salary $\bar{y}$. Investment in human capital (education) $e$ is costly: the time cost of investing in human capital is $D$. In the second period of his life, the young agent works for a salary $y(a^c, e) = a^c f(e)$ that depends on his ability and on the human capital acquired when young. The human capital accumulation function $f(e)$ is increasing in schooling and strictly concave, so $f'(e) > 0$ and $f''(e) < 0$. If the young agent does not invest in human capital, so that $e = 0$, then in the second period, the wage is the same as the low fixed wage of period one, $f(0) = \bar{y}$. The old agent works for a salary $y(a^p)$ that depends on his own ability.

When a young agent lives with his parent, I assume that living expenses are completely covered by the parent. For simplicity, I also assume that the parent does not incur any additional cost to support the child.⁴ If the child moves out, he has to cover his own living expenses himself, which amount to $\ell$.

The old agent moves first. He offers to his offspring a menu of transfers contingent on

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⁴I assume that there are economies of scale in the household for providing living support, and I assume an extreme case of economies of scale. Living expenses for the old agents are not explicitly modeled, this does not change the results of the model.

(2000)). Moreover, Billari and Tabellini (2008) argue that “[t]he main finding is that a late transition to adulthood, measured by the date of leaving the parental home, is associated with lower income later in life. Of course, both income and transition to adulthood are jointly determined [...]”. 

⁴I assume that there are economies of scale in the household for providing living support, and I assume an extreme case of economies of scale. Living expenses for the old agents are not explicitly modeled, this does not change the results of the model.
the offspring decision whether to move out or not. He then chooses consumption residually.

\[
V^p(a^p, a^c) = \max_{c^p, T} [u(c^p) + \psi^p I]
\]
\[
c^p \leq y(a^p) - T
\]
\[
c^p \geq 0, \quad T \geq 0.
\]

Next, the child chooses consumption, schooling and coresidence:

\[
V^c(a^p, a^c) = \max_{c^c, e, I} [u(c^c) + \psi^c I + u(a^c f(e))]
\]
\[
c^c \leq (1 - e)\bar{y} - \ell(1 - I) - De + T
\]
\[
c^c \geq 0, \quad I \in \{0, 1\}.
\]

2.2 Solution and Equilibrium

Denote the game described above by \(G(a^p, a^c, \psi^p, \psi^c)\). The strategy of the old agent consists of his consumption \((c^p)\) and the transfer \((T)\). The strategy of the young agent consists of consumption \((c^c)\), coresidence choice \((I)\), and investment in human capital \((e)\).

The game is sequential. The choices are dependent on each other. The old agent chooses the optimal transfer in order to exert from his offspring his favorite living arrangement, but also takes into account the preference of the young agent for independence.

The timing is as follows:

1. The old agent moves first. He conjectures his offspring’s next move and offers him a contract \((T, I)\) that specifies the transfer and the coresidence status attached to it. He chooses the transfer optimally, and consumes residually. Denote his strategy with \(\delta_p(a^p, a^c, \psi^p, \psi^c|C) = c^p, T\), where \(C\) is the parent’s conjecture.

2. Having observed his parent’s offer, the young agent chooses optimal human capital investment \((e)\), consumption \((c^c)\), and a transfer-coresidence combination. I assume commitment, the young agent cannot deviate (leave his parent’s home after he gets
the transfer). Denote the young agent’s strategy with $\delta_c(a^p, a^c, \psi^p, \psi^c, \delta_p) = c^c, I, e$.

The Subgame Perfect Nash Equilibrium of the game $G(a^p, a^c, \psi^p, \psi^c)$ is such that $C = \delta_c$ and $\delta_p(\cdot|C) = \delta_p(\cdot|\delta_c)$, that is, that the parent’s beliefs about his offspring’s strategy are true.

**Proposition 1** There exists a Subgame Perfect Nash Equilibrium of the game $G$, and it is unique.

**Proof.** See appendix A.2 for a formal proof. The proof is based on showing that there is at most one crossing of the life-time utility of living at home and of living alone. Hence the equilibrium is unique. This condition, together with the fact that the father cannot exert resources from his child and the fact that he is going to transfer resources only if it is convenient for him, guarantees existence of the equilibrium.

The game is solved by backward induction, starting with the young agent’s problem. The young agent maximizes the present value of his lifetime income choosing consumption, coresidence and investment in human capital as from equation 3. The choice of coresidence is discrete and will be determined by his preferences, and the transfer that the old agent offers him. Let $\psi^c$, be the value of any additive utility deriving from coresidence status. Denote instead by $\tilde{\ell} = \ell - T$ any additional disutility deriving from coresidence, i.e. living expenditures in case of non coresidence minus the transfer. The first order conditions for the optimal choice of the young agent is:

$$u'((1-e)\bar{y} - \tilde{\ell} - De)(\bar{y} + D) = a^c f'(e) u'(a^c f(e))$$

The optimal investment in human capital is then denoted by $e(a^c, \tilde{\ell})$, and is a function of ability and of the additional disutility deriving from coresidence. Optimal human capital accumulation depends on ability, on the return to human capital and his costs, both direct ($D$) and indirect ($\bar{y}$), and is increasing in the resources he has, $\tilde{\ell}$. Note that, for intertemporal
elasticity of substitution smaller than one, which is consistent with the empirical evidence, the optimal level of investment is decreasing in ability level. This is a consequence of the zero borrowing constraint imposed on the agents.\textsuperscript{5} The agents want to smooth consumption and, if they are not able to borrow, to do so by studying less and working more in the first period. The higher the ability level the higher is the salary they get in either case in the second period, hence, the lower investment in human capital.

### 2.3 Different Parental Preferences

The outcome of the game depends on the preference for coresidence of the father combined with the taste for independence of the child. There are two cases:

A. The father gains utility from coresiding with his offspring ($\psi_p > 0$);

B. Both the father and the child prefer not to cohabit ($\psi_p \leq 0$).

In case A. the young agent will move out of his father’s house. This in true unless the preference of the young agent for independence is very low, or the living expenses to move out are too high. He then prefers coresiding with his father rather than moving out. In this case the old agent will transfer a positive amount to his offspring in order to help him move out. In case B. the old agent has a preference for coresidence but the young agent wants to move out. The father will then “bribe” his child choosing a transfer such to exert coresidence from him.

The problem of the old agent in both cases can be described as follows: Given the belief of the old agent about his offspring’s best response to his choice, he either offers a zero transfer or a transfer that makes his offspring indifferent between family arrangements. I assume that when the young agent is indifferent, he goes for the option that makes the old agent happier. The optimal transfer has to be feasible and convenient for the old agent, i.e.

\textsuperscript{5}See appendix A for a proof. For $\Pi E > 1$, given the zero borrowing constraint, optimal investment increases with ability.
the utility of the outcome achieved with the transfer has to be at least as large as without the transfer.

In the following subsections I analyze the two cases separately.

2.3.1 $\psi^p > 0$:

The old agent has a preference for coresidence with his offspring, and he will “bribe” him into coresiding. The old agent offers a zero transfer if the offspring moves out and a positive transfer if he stays. This transfer is such that the young agent is indifferent between staying in the parental home and moving out, given his choice of human capital investment. The old agent “bribes” his child as long as it is feasible for him and convenient. The old agent computes $\tilde{\ell}$ such that:

$$u((1 - e(a^c, \ell))\bar{y} - De(a^c, \ell) - \ell) + u(a^c f(e(a^c, \ell))) =$$

$$u((1 - e(a^c, \tilde{\ell}))\bar{y} - De(a^c, \tilde{\ell}) - \tilde{\ell}) + \psi^p + u(a^c f(e(a^c, \tilde{\ell})))$$

(5)

Denote the transfer that makes the young agent indifferent by $\tilde{T} = -\tilde{\ell}$, and the maximum transfer that is convenient for the old agent by $\check{T}$ such that $u(y(a^p) - \check{T}) + \psi^p = u(y(a^p))$.

Hence, the equilibrium transfer living arrangement pair will be:

$$(T, I)^* = \begin{cases} (\max\{0, \check{T}\}, 1) & \text{if } \max\{0, \check{T}\} \leq \check{T} \\ (0, 0) & \text{if } \max\{0, \check{T}\} > \check{T} \end{cases}$$

(6)

Note that if $\tilde{\ell} \geq 0$ the child prefers to live with the parent even without a transfer and the optimal transfer will trivially be zero.

The old agent exerts coresidence from his offspring as long as it is convenient to him, i.e. as long as the utility he gains from “bribing” his offspring is higher than the utility he gains

\[ \text{The maximum transfer the father can afford to pay is } T = y(a^p) \text{ that, given the properties of the felicity function, is always smaller than } \check{T}. \text{ If if } \check{T} > 0 \text{ and } T > y(a^p) \text{ the parent's income is lower than the transfer needed to “bribe” the child, and the child will move out. If, instead } \check{T} < T < y(a^p) \text{ the parent can afford the transfer but it is not convenient for him to “bribe” the child.} \]
not doing it. This is expressed by the threshold transfer $\bar{T}$, and depends on the old agent’s wealth, positively related to his human capital and ability. Therefore, high ability old agents are able to “bribe” their offspring into coresidence, and coresiding families are characterized by high ability parents.

**Proposition 2** Given the ability of the child, there exists a threshold $\bar{a}^p$ of the father’s ability such that, if $a^p < \bar{a}^p$, then $T = 0$ and $I = 0$. The threshold $\bar{a}^p$ is increasing in the father’s preference parameter $\psi$.

**Proof.** Let $\bar{T}$ be such that $u(y(a^p) - \bar{T}) + \psi^p = u(y(a^p))$. It is straightforward to see that $\bar{T}$ increases with ability, since $y'(a) > 0$ and by concavity of the felicity function.

Given the child’s ability, if the income of the father is low, he cannot afford to exert coresidence from his child and there will be no coresidence. Parents with a higher income are able to keep their offspring at home. We know from above that the optimal investment in education is increasing in the resources available to the young agent. If a young agent moves out he will have to pay living expenses and will not receive a transfer. In contrast, if he coresides, he will have more resources to invest in education. Therefore, young agents who coreside with their parents invest more in education than those who do not. For a given ability, young agents with richer parents will coreside and study more. This generates persistence of earnings between parents and children and lowers intergenerational mobility.

The more utility the old agent gains from coresidence, the more he will be willing to transfer to his offspring to induce coresidence, given his own ability parameter and feasibility of the transfer. Hence, the stronger the parent feels about the child, the more people coreside and the stronger is the effect on income persistence. On the other hand, the stronger the young agent’s preference for independence, the more difficult it is for the old agent to bribe him, and the lower the effects of the strategic interaction inside the family on social mobility.
2.3.2 $\psi^p \leq 0$:

If both young and old agents derive utility from independence, they do not coreside, unless the utility that the young agent derives from coresiding is larger than the utility he obtains from not coresiding. This is true only if living costs more than compensate the disutility he derives from coresidence, $\psi^c$.

**Proposition 3** Define $\tilde{a}$ such that:

$$u((1 - e(a^c, 0, 1))\bar{y} - D e(a^c, 0, 1)) + u(\tilde{a} f(e(a^c, 0, 1))) + \psi^c =$$

$$u((1 - e(a^c, 0, 0))\bar{y} - D e(a^c, 0, 0) - \ell) + u(\tilde{a} f(e(a^c, 0, 0)))$$

(7)

*If the intertemporal elasticity of substitution is smaller than one, for all $a < \tilde{a}$, the young agent prefers to coreside with his parent, even if he has a preference for independence. Young agents with ability above $\tilde{a}$ do not cohabit and receive zero transfer.*

**Proof.** In appendix A, I prove that the lifetime utility from coresiding and that from moving out are increasing in ability and cross at most once. With $IES < 1$, the positive slope of lifetime utility is larger for a young agent living alone as compared to living at home. ■

As ability increases, the effect of the fixed cost of moving out on the value function of the young agent is smaller, so the agents that prefer to cohabit are the ones with low ability. The agents who want to cohabit are, in fact, the ones who cannot afford it, given their optimal choice of human capital investment.

The old agent then chooses a transfer to his offspring that compensates this utility differential, conditional on the offspring moving out, as long as he can afford to pay it and it is convenient for him. Hence, he chooses $\tilde{\ell}$ such that:

$$u((1 - e(a^c, \tilde{\ell}))\bar{y} - D e(a^c, \tilde{\ell}) - \tilde{\ell}) + u(a^c f(e(a^c, \tilde{\ell}))) =$$

$$u((1 - e(a^c, 0))\bar{y} - D e(a^c, 0)) + \psi^c + u(a^c f(e(a^c, 0)))$$

(8)
The transfer necessary to get the young agent to move out is denoted by $\tilde{T} = - (\tilde{\ell} - \ell)$. This transfer is smaller than the living expenses the child has to cover if he moves out. In fact, $\tilde{T}$ has to compensate these costs given the preference for independence of the young agents.

Furthermore, the maximum transfer the father would want to to pay is denoted by $\bar{T}$ such that $u(y(a^p) - \bar{T}) \geq u(y(a^p)) + \psi p$.7 The equilibrium combination of transfer and family arrangement then is:

$$(T, I) = \begin{cases} 
(max\{0, \tilde{T}\}, 0) & \text{if } max\{0, \tilde{T}\} \leq \bar{T} \\
(0, 1) & \text{if } max\{0, \tilde{T}\} > \bar{T}
\end{cases}$$ (9)

When it is convenient for the old agent, he transfers enough resources to the young agent to make him indifferent between staying and leaving. When, instead, the young agent has no interest in coresiding this transfer is trivially zero. If the transfer necessary to convince the young agent to move out is too high for the old agent, and his utility is maximized with a zero transfer and coresidence, different generations live together.

**Proposition 4** Assume that the child’s ability $a^c$ is smaller than the threshold $\tilde{a}$ such that the young agent prefers to cohabit. Given the child’s ability, let $\bar{a}^p(a^c)$ be the threshold of the parent ability such that $u(y(\bar{a}^p(a^c)) - T(a^c)) = u(y(\bar{a}^p(a^c))) + \psi p$. The threshold $\bar{a}^p$ is increasing in the father’s preference parameter $\psi p$.

Therefore, if $a^p < \bar{a}^p(a^c)$, then $I = 1$ and $T(a^c) = 0$, while if $a^p > \bar{a}^p(a^c)$, then $I = 0$ and $T(a^c) > 0$.

**Proof.** See proposition 2. ■

Low ability young agents are not able to afford moving out. Of these, the children of medium income families receive a positive transfer that help them move out, they move out and study more. Children of low income families instead coreside because their parents are

7See footnote 8 for details.
not able to help them. Hence, young agents from low income families coreside and do not pay living expenses, while young agents from medium income families move out and receive a transfer. This transfer is smaller than total living expenses since the young agent derives positive utility from independence.\(^8\) Young agents from high income families, instead, move out and pay their own living expenses. This mechanism generates mobility of earnings since young agents from low-mid income families have more resources to invest in human capital. The more disutility the father gets from coresidence, the more he is be willing to transfer to his child to help him, given his own ability parameter and feasibility of the transfer. Hence, the stronger the parent feels about the child, the less people end up coresiding. Moreover, the stronger the child’s preference for independence, the easier it is to help him move out.

To sum up, the simple model shows that the type of strategic interactions inside the family matter for persistence of earnings. If the agents have contrasting preferences, i.e. the parent prefers his child to cohabit with him, then he will bribe the child into coresiding with resources that will be used to invest in education. Since the wealthier parents can more easily afford the bribe, this mechanism increases persistence of earnings. In contrast, if both the parent and the children have a preference for independence, parents will transfer resources only when their offspring are not able to afford moving out by themselves. This acts as a sort of insurance mechanism for the low ability young agents and reduces income persistence.

### 2.4 Relaxing the Borrowing Constraint

Up to now I have kept the rather strict assumption that young agents cannot borrow nor save. I argued that it was reasonable since young adults are often shut out of the credit market, especially in Southern European countries. In this section I relax this assumption. Agents are now allowed to borrow up to a certain limit $\bar{d}$.

The parent’s problem remains unchanged as in equation 2. The young agent instead

---

\(^8\)see appendix A for a proof.
solves the following augmented maximization problem:

\[
V^c(a^p, a^c) = \max_{c_1, c_2, e, I, d} \left[ u(c_1) + \psi^c I + u(c_2) \right]
\]

\[
c_1 \leq (1 - e)\bar{y} - \ell(1 - I) - De + T + d
\]

\[
c_2 \leq a^c f(e) - Rd
\]

\[
d \leq \bar{d}
\]

\[
c_1 \geq 0, \quad c_2 \geq 0, \quad I \in \{0, 1\}.
\]

Where \(R\) is the interest rate of the asset.

Allowing savings and borrowing does not change the equilibrium definition of section 2.2. The Subgame Perfect Nash Equilibrium of the augmented game \(G(a^p, a^c, \psi^p, \psi^c, \bar{d})\) is such that \(C = \delta_c\) and \(\delta_p(\delta_c) = \delta_p\), where \(\delta_c = c_1^c, c_2^c, I, e, d\). Existence and uniqueness are proven following a similar fashion as the game without borrowing, see B for details of the proof.

Denote again by \(\tilde{\ell} = \ell - T\) any additional disutility deriving from coresidence, i.e. living expenditures in case of non coresidence minus the transfer. The young agent’s problem is now characterized by two first order conditions, for education investment and for borrowing:

\[
e : \quad -u'((1 - e)\bar{y} - \tilde{\ell} - De + d)(\bar{y} + D) + a^c f'(e)u'(a^c f(e) - Rd) = 0
\]

\[
d : \quad u'((1 - e)\bar{y} - \tilde{\ell} - De + d) - Ru'(a^c f(e) - Rd) \geq 0.
\]

If the borrowing constraint is not binding and the young agents are able to borrow as much as they need, optimal education is not increasing in the amount of resources the agent has but is only depending on ability and on the costs and benefits. From equations 11 in fact we have that optimal education is such that the following equality is satisfied:

\[
R(\bar{y} + D) = a^c f'(e)
\]

from where is clear that optimal education does not depend on \(\tilde{\ell}\) and is increasing in ability.
Hence, when I do not assume a constraint to borrowing or this constraint is not binding, the link between the parental transfer and education and hence social mobility is broken. The mechanism described in the above section depends on the inability of the young agent to borrow as much as he would need.

If the borrowing constraint \( \bar{d} \) is instead binding, the agent is not able to borrow as much as he needs and he faces a trade off between working and consuming more today, investing less in education and hence in future consumption, and investing more in education consuming more tomorrow and less today. Hence, optimal investment in education is increasing in the amount of resources the young agent has and the decision process over coresidence described above affects intertemporal elasticity of earnings. Given that the constraint is binding, relaxing the borrowing constraint increases the effect of resources on education. The intuition lies with the fact that the agent borrows and has to repay tomorrow, hence when he is given more resources he invests in education more to compensate the loss in tomorrow’s consumption given by the repayment.

3 The Quantitative Model

In this section, I describe an overlapping generations model with college and coresidence decisions and inter-vivos transfers in which I embed a version of the game described in section 2. The model is set to quantify the effects on cross-country differences in intergenerational mobility of different family structures. Households in this economy play a noncooperative sequential game in which they choose consumption, transfers, coresidence and human capital investment.

3.1 The Economy

The economy is populated by a large number of dynasties of fathers and sons. Every period, a mass one of young agents is born. Agents enter the economy at age 20 and exit
it at age 50 with certainty. They participate in the economy for six 5-years periods. At age 25, every agent has a child, who enters the economy when the parent is 45. There are no financial assets, and every agent is endowed with one unit of time each period.

In their first period in the economy, young agents can invest in human capital by choosing how much of their time endowment to spend studying. The remaining time, they work for a low fixed wage, independent of their human capital. From the second period on, they become full-time workers. Labor supply is inelastic. Full-time workers in every period earn a wage that depends on their human capital and on a life-cycle component.

Agents are not altruistic. However, they derive utility or disutility from coresidence with the other living member of the household. Parents can give an inter-vivos transfer to their child when he enters the economy.

Note that, at any given point in time, one third of the dynasties is composed of two agents and two thirds of only one agent, since the newborn has not yet entered the economy.

3.2 Preferences and Coresidence

I denote each 5-year period by \( t \) and age by \( k = 1, \ldots, 6 \). Agents derive utility from consumption, leisure, and coresidence with the other member of the dynasty. Preferences are of the form:

\[
U_t = \frac{(c^k_t)^{1-\sigma}}{1-\sigma} + \psi^k I_t \tag{13}
\]

Where \( I \in \{0, 1\} \) is a coresidence dummy, and \( c \) is consumption. The preference parameter \( \psi^k \) denotes the utility the agent gains from coresiding with the other member of the dynasty. \( \psi^k \) is indexed with the period: I restrict it to be negative for \( k = 1 \) to indicate psychic costs of young agents from coresidence with their parents and to be zero for all periods \( k = 2, \ldots, 5 \). The preference for coresidence parameter for agents with \( k = 6 \), i.e. parents, is not restricted and can take both positive and negative values, denoting a taste or a distaste for coresidence. Future utility is discounted by the factor \( \beta \).
There are housing costs in this economy: agents have to pay fixed living expenses $\ell$ every period. When a young agent lives with his father, I assume that living expenses are completely covered by the parent without any additional cost. If the young agent moves out, he has to cover living expenses himself.

### 3.3 Human Capital and Wages

Agents are heterogeneous in initial skills $a \in \bar{A}$ at age 20. Initial skills are a mix of an innate ability component and a human capital component acquired in high school, which I do not model. In the first period of economic life, I assume that initial human capital is equal to the initial skills, $h = a$. I assume that skills are imperfectly transmitted from parent to child according to a first order $A$-state Markov chain with transition matrix $\Gamma$, mean normalized to one and with $a_{\min} > 0$. The conditional probability of having a child of ability $a$ for a parent of ability $a'$ is denoted by $\pi(a|a')$.

Young agents at the beginning of their life decide to invest part of their time endowment in education and work the rest of the time for a fixed “college” wage $\bar{w}$. The time cost of education is denoted by $D$. The amount of human capital acquired depends on the student initial skill and on how much he decides to invest in college:

$$h_{t+1} = h_t + h_t\epsilon_t^\alpha$$  \hspace{1cm} (14)

Young agents become full time workers in their second life period and earn wage $w$ that

---

9I restrict the vector of states $\bar{A}$ and the element of the transition matrix $\Gamma$ such that the transmission of ability mimics a continuous AR(1) process

$$log(a') = \zeta log(a) + \epsilon \quad \epsilon \sim N(0, \sigma_a^2).$$

Tauchen (1986) describes a method to map a continuous AR(1) process into a discrete state first-order Markov chain with mean equal to one and $a_{\min} > 0$. Moreover, the process converges to a log-normal stationary distribution

$$h \rightarrow logN(0, \frac{\sigma_a^2}{(1 - \zeta^2)}).$$

such that the Markov process depends only on the parameters $\zeta$ and $\sigma_a$.
depends on the human capital investment they undertook in the previous period and on labor market experience \( \xi \). Labor market experience is equal to potential experience in absence of unemployment.

\[
w = h_{\gamma_0} e^{\gamma_1 \xi + \gamma_2 \xi^2 + \gamma_3 \xi^3 + \gamma_4 \xi^4} \tag{15}
\]

### 3.4 Household’s Problem

At the beginning of the period, the ability of both members of the household and the human capital of the parent are known to everyone. Uncertainty in this model is given by the future realization of initial skills, i.e. the initial skill of the current child’s future child and nephew to the current parent.

When the young agents are in their first period of economic activity, they play a game with their parents, who are in their last period of economic activity. The timing of action is as follows: the old agent chooses the optimal transfer to give to his offspring, then the young agent chooses the family arrangement, how much to invest in education, and how much to work. Finally, both agents consume residually. In all other periods of their lives, agents get their labor income and simply choose how much to consume.

Let \( t \) denote the time period, \( j \) the dynasty and \( k = 1, \cdots, 6 \) age. The state of dynasty \( j \) at time \( t \) remembers the investment in education and initial ability of all active members of the household \( x^j_t \):

\[
x^j_t = (a^{jk}_t, a^{j,k-5}_t, h^{jk}_t, h^{j,k-5}_t).
\]

where \( a^{jk}_t \) and \( a^{j,k-5}_t \) are initial abilities of the young agent \( (a^{jk}_t) \) and of the parent \( (a^{j,k-5}_t) \), and \( h^{jk}_t \) and \( h^{j,k-5}_t \) are the human capital of the child \( (h^{jk}_t) \) and of the parent \( (h^{j,k-5}_t) \), respectively. Note that the parent of the young agent born in period \( t \) was born five model periods before him, hence his age is \( t - 5 \). The economy is populated by one-agent dynasties and two-agents dynasties: in the first period of the life of a young agent, his parent is still alive and
age is 6. The old agent exits the economy at the end of his sixth period and the young agent lives four periods as the only active agent in the dynasty. At the beginning of his sixth period his own child enters the economy and again the dynasty is composed of two agents. Therefore, when an agent is of age $k = 2, \ldots, 5$, the state will only consist of his own human capital and initial ability. At the beginning of his sixth period his child enters the economy and the child’s initial ability is realized. The state then consists of both agents initial ability and human capital. Hence, for all $k = 2, \ldots, 5$, the state of dynasty $j$ at time $t$ is $x^j_t = (a^{jk}_t, \cdot, h^{jk}_t, \cdot)$.

Each dynasty can be in different stages: a one-agent dynasty or a two-agents dynasty. Each agent moves through three different stages in his life cycle: young adult/child, middle-aged adult, and old parent. Therefore, there are three maximization problems. For the time being suppress the superscript indexing the dynasty, and denote the value function of an agent of age $k$ at time $t$ by $V^k_t(x_t)$. There are three of these value functions: one for the young adult ($V^1_t(x_t)$), one for the middle-aged adult ($\{V^k_t(x_t)\}_{k=2}^5$), and one for the old parent ($V^6_t(x_t)$).

When the young agent enters the economy, he chooses consumption ($c^1_t$), coresidence ($I_t \in \{0,1\}$), and investment in human capital ($e^1_t$). The problem of a young agent is the following:

$$
V^1_t(a^1_t, a^6_t, h^1_t, h^6_t) = \max_{c^1_t, h^1_t} \left[ u(c^1_t) + \psi^k I_t + \beta \mathbb{E} V^2_{t+1}(a^1_{t+1}, \cdot, h^1_{t+1}, \cdot) \right]
$$

$$
c^1_t + \ell (1 - I_t) + De^1_t \leq (1 - e^1_t) \bar{w} + T_t
$$

$$
c^1_t \geq 0, \quad I_t \in \{0,1\}
$$

$$
h^2_{t+1} = h^1_t (1 + (e^1_t)^\alpha)
$$

From the second to the fifth period of life, the agents choose consumption ($c^k_t$), according
to the following Bellman problem:

\[
V^k_t(a^k_t, h^k_t) = \max_{c^k_t} \left[ u(c^k_t) + \beta \mathbb{E} V^{k+1}_{t+1}(a^{k+1}_{t+1}, h^{k+1}_{t+1}) \right]
\]

\[
c^k_t + \ell \leq w^k_t
\]

\[
c^k_t \geq 0, \quad k = 2, \ldots, 4
\]

\[
w^k_t = h^k_t \xi^k_t, \quad h^{k+1}_{t+1} = h^k_t
\]

(17)

\[
V^k_t(a^k_t, h^k_t) = \max_{c^k_t} \left[ u(c^k_t) + \beta \mathbb{E} V^{k+1}_{t+1}(a^{k+1}_{t+1}, a^{k+1}_{t+1+1}, h^{k+1}_{t+1}, h^{k+1}_{t+1+1}) \right]
\]

\[
c^k_t + \ell \leq w^k_t
\]

\[
c^k_t \geq 0, \quad k = 5
\]

\[
w^k_t = h^k_t \xi^k_t, \quad h^{k+1}_{t+1} = h^k_t
\]

At age \( k = 5 \) the middle-aged agent’s value function incorporates his expectations over his child’s ability, which will be revealed when the child enters the economy in the following period.

Finally, in the last period of his life, the agent is the parent of a child who is entering the economy. He has a taste for coresidence \( \psi^6 \) and he can transfer resources to him with an inter-vivos transfer \( T_t \). Moreover he chooses consumption \( (c^6_t) \). The Bellman problem is then:

\[
V^6_t(a^1_t, a^6_t, h^1_t, h^6_t) = \max_{c^6_t, T_t} \left[ u(c^6_t) + \psi^6 I_t \right]
\]

\[
c^6_t + \ell + T_t \leq w^6_t
\]

\[
c^6_t \geq 0, \quad T_t \geq 0
\]

\[
w^6_t = h^6_t \xi^6_t
\]

(18)

As noted before, the mechanism of the game relies on the assumption of borrowing constraints. If young agents could borrow freely, then their education decision would be
independent of the parental transfer, while if old agents could, then they would save in order to have more resources to transfer. While introducing savings at later ages would only strengthen the results, allowing the agents to borrow freely would break the link between the coresidence decision and intergenerational earnings persistence. However, the strong borrowing constraints for young adults is realistic for Southern European countries, where student loans are not widespread. More specifically, in the quantitative exercise I refer to Italy where student loans are now at the center of many proposals of reform of the University system, but have not yet been introduced nation-wide. Moreover, Italy has a strict credit market with very low loan-to-value ratios (see Jappelli and Pagano (1994)) that are particularly punitive for young adults. Hence my assumption of a zero borrowing constraints is reasonable.

3.5 Equilibrium

History dependence is governed by the Markov chain, which has a stationary equilibrium. This, plus the existence and uniqueness of the one period game equilibrium, is sufficient to guarantee the existence and uniqueness of a stationary equilibrium of the overlapping generations model. See Appendix B for a discussion of the equilibrium of the one period game in the context of the OLG model.

3.5.1 Stationary Recursive Equilibrium

A Stationary Recursive Equilibrium in this economy is a set of value functions \( \{V^1(a^1, a^6, h^1, h^6), \{V^k(a^k, \cdot, h^k, \cdot)\}_{k=2}^5, V^6(a^1, a^6, h^1, h^6)\} \) and decision rules \( \{\delta^1, \{c^k\}_{k=2}^5, \delta^6\} \) for all dynasties \( j \), and a stationary distribution \( \mu(x) \) over the states, such that:

(a.) For all dynasties \( j \), the decision rule \( \{\{c^k\}_{k=2}^5\} \) solve the problem of the one-agent dynasties;
(b.) For all dynasties $j$, the decision rules $\{\delta^1, \delta^6\}^j$ solve the problem of the two-agents dynasties, i.e. there is a subgame perfect Nash equilibrium to the sequential game played by the two agents;

(c.) The stationary distribution is induced by the Markov chain that governs the initial ability distribution $(\Gamma, \bar{A})$ and the policy functions.

4 The Quantitative Exercise

4.1 Calibration

I set the parameters the model to represent relevant features of Italy. The strategy consists on fixing part of the parameters exogenously and then estimating the others using a set of moments obtaines from the Survey of Household Income and Wealth (SHIW) of the Bank of Italy.

4.1.1 Parameters Calibrated Outside the Model

The parameters that are set without solving the model are shown in Table 2. Following the empirical literature reviewed in Browning et al. (1999) I set the coefficient of relative risk aversion of the CRRA utility function equal to 2. The unit cost of human capital investment is set equal to the maximum tuition fee of the University of Roma La Sapienza, that amounts to 2000 euros per year. Living arrangement costs are then set to the average rent plus utilities for a student in Rome, 500 euros per month.

From the second period of their life, agents earn wage $w = h\gamma_0e^{\gamma_1x + \gamma_2x^2 + \gamma_3x^3 + \gamma_4x^4}$. I estimate the parameters $\gamma_1 - \gamma_4$ from wage data of the SHIW.

4.1.2 Parameters Calibrated Inside the Model

The seven remaining parameters are the preference parameters for coresidence $\psi^p$ and $\psi^c$, the human capital accumulation parameter $\alpha$, $\gamma_0$ from the wage equation, the wage
of college graduates $\bar{y}$, and the parameters from the AR(1) process of ability $\zeta$ and $\sigma_\epsilon^2$. I estimate them using moments for the age-range 20-50 that captures the age of the agents active in the model. The full set of moments and of parameters is shown in Table 3.

### Table 3: Parameters Calibrated Inside the Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
<th>Data Moment (SHIW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi^p$</td>
<td>0.05</td>
<td>parent’s taste for coresidence</td>
<td>share of students coresiding</td>
</tr>
<tr>
<td>$\psi^c$</td>
<td>$-2.361e^{-5}$</td>
<td>child’s taste for coresidence</td>
<td>share of non-students coresiding</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.35</td>
<td>$h = a + ae^\alpha$</td>
<td>$\sigma^2_w$</td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>10000×year</td>
<td>$w = h\gamma_0e^{\gamma_1x+\gamma_2x^2+\gamma_3x^3+\gamma_4x^4}$</td>
<td>average wage</td>
</tr>
<tr>
<td>$\bar{y}$</td>
<td>12000×year</td>
<td>$\bar{y}$</td>
<td>avg wage college students</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.4</td>
<td>$ln(a^c) = \zeta ln(a^p) + \epsilon$</td>
<td>intg elasticity of earnings</td>
</tr>
<tr>
<td>$\sigma_\epsilon^2$</td>
<td>0.63</td>
<td>$\epsilon \sim N(0, \sigma^2_\epsilon)$</td>
<td>share of college students</td>
</tr>
</tbody>
</table>

In order to estimate these parameters I use an exactly identified simulated method of moments with a diagonal weighting matrix. This means that I minimize the sum of squared deviations of the simulated moments from the model with the data moments in Table 3. Denote the vector of parameters to be estimated by $\theta = \{\psi^p, \psi^c, \alpha, \gamma_0, \bar{y}, \zeta, \sigma_\epsilon^2\}$, the vector of data moments by $m$ and the vector of simulated moments by $\hat{m}(\theta)$. Then the estimation strategy is as follows:

$$\hat{\theta} = \min_\theta (m - \hat{m}(\theta))'(m - \hat{m}(\theta)).$$  \hspace{1cm} (19)

In Appendix D you find details of the estimation algorithm. The quantitative model is very stylized and with few parameters. Nonetheless, as can be seen from Table 4, I am able to get close to matching the moments in the data. The variance of most of the empirical moment
is not known, hence it is not possible to compute standard errors for the estimation.

Table 4: Estimation Statistics

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Data Moments</th>
<th>Simulated Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>share of students coresiding</td>
<td>0.97</td>
<td>1.0</td>
</tr>
<tr>
<td>share of non-students coresiding</td>
<td>0.89</td>
<td>0.93</td>
</tr>
<tr>
<td>share of college students</td>
<td>0.39</td>
<td>0.48</td>
</tr>
<tr>
<td>average wage</td>
<td>23622×year</td>
<td>31992×year</td>
</tr>
<tr>
<td>avg wage college students</td>
<td>4326×year</td>
<td>4786×year</td>
</tr>
<tr>
<td>intg elasticity of earnings</td>
<td>0.43</td>
<td>0.4188</td>
</tr>
<tr>
<td>$\sigma^2_w$</td>
<td>20814×year</td>
<td>20644×year</td>
</tr>
</tbody>
</table>

4.1.3 Results

The first interesting aspect to investigate is how much of the intergenerational earnings persistence in Italy can be explained by the decision process over living arrangement. To understand what the model predicts I shut down the coresidence channel and I compare an economy in which no-one coresides to the calibrated economy. When no-one coresides all persistence derives from transmission of ability. As can be seen from Table 5, the mechanism described in the model can explain around 7.5 percent of the Italian intergenerational earnings persistence. Hence, 7.5 percent of persistence of earnings in the model derives from the decision process over living arrangements, that relies on family ties and on credit constraints. Since there is no asset market in the model, nor there are altruistic motives for intergenerational transfers, the importance of the mechanism can be overestimated in this exercise. Hence, this result has to be read as an upper bound of the influence of family ties on earnings persistence.

As shown in the introduction, there is a positive correlation between coresidence rates and intergenerational earnings persistence in Europe. The mechanism described in this
model relies both on family ties and on credit constraints. Both aspects can be important to understand differences in social mobility and coresidence patterns across Europe, and they go in the same direction. In order to understand the relative importance of credit constraints in the model I quantify the transfer to young adults necessary to lower coresidence rates to the level of Northern Europe. More specifically, I simulate a transfer that is given to every young agent in the economy, independently from his level of investment in education. Table 6 shows different transfer values and the effect they have on coresidence patterns in the model. I find that, as the transfers get larger, coresidence decreases. At first, agents invest more in education and coresidence rates stay high. As they start moving out of the parental home, their investment in education decreases, to then increase again once the living expenses are covered. A small transfer is used to invest in education, as the transfer gets larger agents start moving out using the transfer to cover the living costs. When the transfer gets high enough, they are able to both finance education and living arrangements. In order to decrease coresidence rates to Scandinavian levels, I need to give young agents a transfer of about 400 euros a month, which is comparable to the grant that college students receive in Sweden during their studies.\footnote{Other than receiving a grant of approximately 400 euros per month, Swedish students can opt for borrowing about twice as much from the study aid authority (Centrala Studiestödsnämnden).} Earnings persistence increases with the transfer as agents use it to invest more in education. This is due to the fact that parents need to give higher transfers in order to keep the children home, since they are endowed with an external source of resources.

An important part of credit access for young adults is determined by access to student aid. While Universities in both Southern and Northern Europe have low tuition costs, Scandinavian countries have more generous universal student aid programs (composed of grants and loans) than Southern Europe. Scandinavian student aid programs subsidize consumption and living expenses of students, and virtually consist in a negative cost of studying. Student aid is contingent on enrollment (part-time or full-time) and has some mild merit requirements. One third of the aid is in form of grant, and two thirds in form of...
Table 6: Coresidence Patterns with Universal Transfers to Young Agents

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>200×year</th>
<th>400×year</th>
<th>1000×year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Coresidence rate</td>
<td>0.96</td>
<td>0.93</td>
<td>0.93</td>
<td>0.85</td>
</tr>
<tr>
<td>Share of Workers Coresiding</td>
<td>0.93</td>
<td>0.67</td>
<td>0.08</td>
<td>0</td>
</tr>
<tr>
<td>Share of Students Coresiding</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Share of Students (e ≥ 0.5)</td>
<td>0.48</td>
<td>0.77</td>
<td>0.92</td>
<td>0.85</td>
</tr>
<tr>
<td>Elasticity of Earnings</td>
<td>0.42</td>
<td>0.43</td>
<td>0.42</td>
<td>0.42</td>
</tr>
<tr>
<td>Share of persistence generated</td>
<td>0.076</td>
<td>0.12</td>
<td>0.13</td>
<td>0.22</td>
</tr>
</tbody>
</table>

<table>
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<th></th>
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<th>4800×year</th>
<th>6000×year</th>
<th>12000×year</th>
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<tr>
<td>Total Coresidence rate</td>
<td>0.60</td>
<td>0.42</td>
<td>0.26</td>
<td>0.20</td>
</tr>
<tr>
<td>Share of Workers Coresiding</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Share of Students Coresiding</td>
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<td>1.00</td>
<td>1.00</td>
<td>0.50</td>
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<tr>
<td>Share of Students (e ≥ 0.5)</td>
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<td>0.42</td>
<td>0.26</td>
<td>0.4</td>
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<td>0.40</td>
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<td>Share of persistence generated</td>
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student loan with an income contingent repayment scheme. The universal transfer described above is a good approximation of this system in the context of the model. I abstract from the discrete choice of education and hence cannot make predictions on the level of enrollment. However, the results above can be interpreted as the behavior of students enrolled in tertiary education who receive the grant and decide how much time to invest in their education: as the grant increases students increase the time they invest in education towards their optimal level, and eventually use the remaining grant to move out from home. When the borrowing constraint is lifted students first reach their optimal education level and then acquire privacy. Constrained students use the possibility of staying at home with their parents to direct scarce resources to education. Hence, limits to access to credit can result in higher rates of coresidence of young students with their parents rather than in lower education.

The model predicts that easing the credit constraint to young agents in Italy would decrease coresidence rates to the Scandinavian levels, hinting that the role of credit access is pivotal to understand coresidence. However, the effect of coresidence on social mobility depends on the shape of the decision process and is not affected by credit constraints.
5 Conclusions

In this paper, I develop a sequential game between parents and children over living arrangements. Agents decide whether the child will live with his parent or whether he will move out. I show that if parents gain utility from coresiding with their offspring, parents give the offspring a transfer in order to keep them home. The children that decide to coreside have more resources to invest in human capital, both from the transfer they receive and from not having to pay living expenses. High income parents are more likely to be able to afford the transfer and to coreside with their children. Hence, children of high income parents invest more in education, and this mechanism increases earnings persistence.

Next, I study the importance of the mechanism generated by differences in preferences to understand earnings persistence. To do this, I construct a 6-period overlapping generations model that embeds the game over family arrangements. The model also features human capital accumulation in college, intergenerational transmission of ability, and an education system. I show the results of quantitative exercises that use a version of the OLG model calibrated to Italian data with a simulated method of moments and using data from the Survey of Household Income and Wealth of the Bank of Italy. First, I show that the model predicts that at most 7.5 percent of intergenerational earnings persistence in Italy can be explained by the decision process over living arrangements. Second, in order to assess the importance of credit constraints for the mechanism described, I simulate a transfer to the young adults in the model. I find that a universal transfer of around 400 euros per month directed to all young adults independently of their education decreases coresidence rates to Scandinavian levels.

The assumption of zero borrowing constraints is important for the mechanism described in this paper. Future research in this area will be directed in further understanding the role of education policies and of credit markets in the joint decisions of parents and children over coresidence and education. Particularly interesting will be to analyze the role of the housing market in shaping early adulthood decisions and the dependence of young adults from their
parents.

In this paper I show that intertemporal earnings elasticity can be affected by the strength of family ties. An empirical exercise that looks at earnings elasticity across second generation immigrants from different countries would provide further evidence on the relevance of this mechanism.

In conclusion, the decision process over living arrangements can have consequences for intergenerational earnings persistence. This mechanism hinges on both family ties and credit constraints. Future research will be directed to further understand the role of family ties and intergenerational transfers to shape economic outcomes, and to analyze the incentives behind credit market and student aid policies for young agents.
References


Appendices

A Solution of the Game

A.1 The young agent’s problem

For given choice of residence, study the optimal human capital investment of a young agent with ability $a$. As in the main text, let $\psi^c$, be the value of any additive utility deriving from coresidence status. Denote instead with $\tilde{\ell} = \ell - T$ any additional disutility deriving from coresidence, i.e. living expenditures in case of non coresidence minus the transfer.

The optimal human capital investment satisfies the young agent’s first order condition:

$$ F \equiv -u'((1 - e)\bar{y} - De - \tilde{\ell})(\bar{y} + D) + u'(af(e))af'(s) = 0 \quad (A.1) $$

Let optimal human capital investment be denoted by $e(a, \tilde{\ell})$. The second-order condition implies that the first-order condition is decreasing in human capital investment:

$$ \frac{\partial F}{\partial e} < 0 \quad (A.2) $$

Human capital varies negatively (positively) with ability if the intertemporal elasticity of substitution is smaller than one, $\eta(c) < 1$ (larger than one, $\eta(c) > 1$).\(^{11}\) Implicit differentiation of A.1 yields

$$ \frac{\partial e}{\partial a} = -\frac{\frac{\partial F}{\partial a}}{\frac{\partial F}{\partial e}} $$

\(^{11}\)In a more general case, Lochner and Monge-Naranjo (2011) show that, for borrowing constraints larger than zero, human capital investment varies negatively with ability if IES is less than one, but an IES greater than one is a necessary but not sufficient condition for human capital investment to vary positively with ability.
and, by equation A.2, we have

\[ \text{sign} \left( \frac{\partial e}{\partial a} \right) = \text{sign} \left( \frac{\partial F}{\partial a} \right) \]

Hence:

\[
\frac{\partial F}{\partial a} = u'(a^e f(e))f'(e) + u''(a^e f(e))a^e f'(e)f(e) = \\
= u'(a^e f(e))f'(e) \left[ 1 - \frac{1}{\eta(a^e f(e))} \right]
\]

where \( \eta(c) \equiv -\frac{u'(c)}{cu''(c)} \). Therefore:

\[
\frac{\partial e}{\partial a} = \begin{cases} 
< 0 & \text{if } \eta(c) < 1 \\
> 0 & \text{if } \eta(c) > 1 
\end{cases}
\]

Human capital varies negatively with \( \bar{\ell} \). Implicit differentiation of equation A.1 yields:

\[
\frac{\partial e}{\partial \bar{\ell}} = \frac{\partial F}{\partial \bar{\ell}}
\]

and, again by equation A.2, we have

\[ \text{sign} \left( \frac{\partial e}{\partial \bar{\ell}} \right) = \text{sign} \left( \frac{\partial F}{\partial \bar{\ell}} \right) \]

Hence:

\[
\frac{\partial F}{\partial \bar{\ell}} = u''((1 - e)\bar{y} - De - \bar{\ell})(\bar{y} + D) < 0
\]

A.2 Existence and Uniqueness of the SPNE

Lifetime utility is decreasing in \( \bar{\ell} \) and, as of consequence, increasing in the transfer. Substitute the young agent’s optional choice of human capital investment, \( e(a, \bar{\ell}) \), into the
utility function and take the derivative:

\[
\frac{\partial}{\partial \tilde{\ell}} \left\{ u((1 - e(a, \tilde{\ell}))\bar{y} - D e(a, \tilde{\ell}) - \tilde{\ell}) + u(a^c f(e(a, \tilde{\ell}))) + \psi^c \right\} = \\
= \left\{ -u'((1 - e(a, \tilde{\ell}))\bar{y} - D e(a, \tilde{\ell}) - \tilde{\ell})(\bar{y} + D) + u'(a^c f(e(a, \tilde{\ell})))a^c f'(e(a, \tilde{\ell})) \right\} \frac{\partial e(a, \tilde{\ell})}{\partial \tilde{\ell}} + \\
+ u'(a^c f(e(a, \tilde{\ell})))f(e(a, \tilde{\ell})) = \\
= u'(a^c f(e(a, \tilde{\ell})))f(e(a, \tilde{\ell})) > 0
\]

where the last equality invokes equation A.1.

The parent can choose \( \tilde{\ell} \) in both possible living arrangements such the the child is indifferent between living with him or moving out. With this, I have established at most one crossing of the lifetime utility of living alone and of living at home, when varying \( \tilde{\ell} \). Given that the parent cannot extract resources from the child, this insures uniqueness of the transfer. The optimal transfer is also restricted by the fact that the father cannot exert resources from his offspring and from the feasibility constraint

\[
u(y(a^p) - T) - u(y(a^p)) \pm \psi^p \geq 0
\]

This insures the existence of an optimal transfer, while single crossing insures uniqueness.

### A.3 Single crossing condition

Lifetime utility increases with ability. Substitute the young agent’s optional choice of human capital investment, \( e(a, \tilde{\ell}) \), into the utility function and take the derivative:

\[
\frac{\partial}{\partial a} \left\{ u((1 - e(a, \tilde{\ell}))\bar{y} - D e(a, \tilde{\ell}) - \tilde{\ell}) + u(a^c f(e(a, \tilde{\ell}))) + \psi^c \right\} = \\
= \left\{ -u'((1 - e(a, \tilde{\ell}))\bar{y} - D e(a, \tilde{\ell}) - \tilde{\ell})(\bar{y} + D) + u'(a^c f(e(a, \tilde{\ell})))a^c f'(e(a, \tilde{\ell})) \right\} \frac{\partial e(a, \tilde{\ell})}{\partial a} + \\
+ u'(a^c f(e(a, \tilde{\ell})))f(e(a, \tilde{\ell})) = \\
= u'(a^c f(e(a, \tilde{\ell})))f(e(a, \tilde{\ell})) > 0
\]
where the last equality invokes equation A.1.

The positive slope in equation A.3 is larger (smaller) for a young agent living alone as compared to living at home if \( \eta(c) < 1 \) (\( \eta(c) > 1 \)), in fact:

\[
\frac{\partial}{\partial \ell} \left\{ u'(a^c f(e(a, \tilde{\ell}))) f(e(a, \tilde{\ell})) \right\} =
\]
\[
= \left\{ u'(a^c e(a, \tilde{\ell})) f'(e(a, \tilde{\ell})) + u''(a^c e(a, \tilde{\ell})) a f'(e(a, \tilde{\ell})) f(e(a, \tilde{\ell})) \right\} \frac{\partial e(a, \tilde{\ell})}{\partial \ell} =
\]
\[
= \begin{cases} 
  > 0 & \text{if } \eta(c) < 1 \\
  < 0 & \text{if } \eta(c) > 1
\end{cases}
\]

where the inequalities follow from the earlier result that \( \frac{\partial e}{\partial \ell} < 0 \).

With this, I have established at most one crossing of the life-time utility of living alone and of living at home, when varying ability.

### A.4 Study of the transfer when \( \psi^p \leq 0 \)

As discussed above, for some parameter values, the optimal transfer is such that the young agent is indifferent between living at home and moving out:

\[
V(a, T, 0) + \psi^c \equiv + u((1 - e(a, T, 0))\bar{y} - De(a, T, 0) - \ell + T)(\bar{y} + D)
\]
\[+ u(a^c f(e(a, T, 0))) + \psi^c = u((1 - e(a, 0, 1))\bar{y} - De(a, 0, 1))(\bar{y} + D)
\]
\[+ u(a^c f(e(a, 0, 1))) \equiv V(a, 0, 1)
\]

If \( \psi^c = 0 \), then to maintain the above equality I need that \( T = \ell \). Since \( \psi^c < 0 \) instead, then \( V(a, T, 0) < V(a, 0, 1) \). This is true only if \( T < \ell \).

Therefore, a young agent who lives alone invests less in human capital as compared to if he had lived at home. The reason is because net living expenditures are larger when living alone. When the father has a preference for coresidence, then he will bribe his child and a
positive transfer will be associated with living at home. When, vice versa, his preference is for independence, the transfer he will eventually give to the child is smaller than living expenses, hence this result holds.

Let’s go back to the case when both agents have a preference for independence. Suppose that \( \eta(c) < 1 \). From the section above, I know that the slope of the value function is positive, and it is larger for a young agent living alone as compared to living at home. Then there is single crossing and there is one threshold of ability of the young agent \( \bar{a}^c \) below which young agents need a transfer from the parent in order to move out. The transfer needed to make a child with \( a < \bar{a}^c \) os decreasing in ability.

The transfer \( T \) makes the child indifferent between leaving home and living at home. Define:

\[
G \equiv + u((1 - e(a, \ell - T))\bar{y} - De(a, \ell - T) - \ell + T) + u(a^c f(e(a, \ell - T))) + \\
- u((1 - e(a, 0))\bar{y} - De(a, 0)) - u(a^c f(e(a, 0))) - \psi^c = 0
\]

Implicit differentiation of \( G \) yields:

\[
\frac{\partial T}{\partial a} = - \frac{\partial G}{\partial a} = \frac{\partial G}{\partial \tilde{\ell}}
\]

where, again, \( \tilde{\ell} = \ell - T \).
The derivative $\frac{\partial G}{\partial \tilde{\ell}}$ is strictly negative:

$$\frac{\partial G}{\partial \tilde{\ell}} = u'(1 - e(a, \tilde{\ell}))\bar{y} - D e(a, \tilde{\ell}) - \tilde{\ell}) \left\{ - (\bar{y} + D) \frac{\partial e(a, \tilde{\ell})}{\partial \tilde{\ell}} - 1 \right\}$$

$$+ u'(a e(e(a, \tilde{\ell})))a e' e(a, \tilde{\ell}) \frac{\partial e(a, \tilde{\ell})}{\partial \tilde{\ell}}$$

$$= \left\{ - u'((1 - e(a, \tilde{\ell}))\bar{y} - D e(a, \tilde{\ell}) - \tilde{\ell}) (\bar{y} + D) + u'(a e(e(a, \tilde{\ell})))a e' e(a, \tilde{\ell}) \right\} \frac{\partial e(a, \tilde{\ell})}{\partial \tilde{\ell}}$$

$$- u'((1 - e(a, \tilde{\ell}))\bar{y} - D e(a, \tilde{\ell}) - \tilde{\ell})$$

$$= - u'((1 - e(a, \tilde{\ell}))\bar{y} - D e(a, \tilde{\ell}) - \tilde{\ell}) < 0$$

where the last equality invokes equation A.1.

Hence,

$$\text{sign} \left( \frac{\partial T}{\partial a} \right) = \text{sign} \left( - \frac{\partial G}{\partial a} \right)$$

where

$$\frac{\partial G}{\partial a} = - u'((1 - e(a, \tilde{\ell}))\bar{y} - D e(a, \tilde{\ell}) - \tilde{\ell}) (\bar{y} + D) \frac{\partial e(a, \tilde{\ell})}{\partial a}$$

$$+ u'(a e(e(a, \tilde{\ell}))) \left[ f(e(a, \tilde{\ell})) + a e' e(a, \tilde{\ell}) \frac{\partial e(a, \tilde{\ell})}{\partial a} \right]$$

$$+ u'((1 - e(a, 0))\bar{y} - D e(a, 0)) (\bar{y} + D) \frac{\partial e(a, 0)}{\partial a}$$

$$- u'(a e(e(a, 0))) \left[ f(e(a, 0)) + a e' e(a, 0) \frac{\partial e(a, 0)}{\partial a} \right]$$

$$= + u'(a e(e(a, \tilde{\ell}))) f(e(a, \tilde{\ell}) - u'(a e(e(a, 0))) f(e(a, 0))$$

where the last equality invokes equation A.1.

Given $\tilde{\ell} > 0$, this difference is strictly positive. In fact, take the derivative of
\( u'(\alpha^c f(e(a, \tilde{\ell})))f(e(a, \tilde{\ell})) \) with respect to \( \tilde{\ell} \).

\[
\frac{\partial}{\partial \tilde{\ell}} \left[ u'(\alpha^c f(e(a, \tilde{\ell})))f(e(a, \tilde{\ell})) \right] = u''(\alpha^c f(e(a, \tilde{\ell})))\alpha^c f(e(a, \tilde{\ell})) f'(e(a, \tilde{\ell})) \frac{\partial e(a, \tilde{\ell})}{\partial \tilde{\ell}} + \\
u'(\alpha^c f(e(a, \tilde{\ell}))) f'(e(a, \tilde{\ell})) \frac{\partial e(a, \tilde{\ell})}{\partial \tilde{\ell}}
\]

\[
= f'(e(a, \tilde{\ell})) \frac{\partial e(a, \tilde{\ell})}{\partial \tilde{\ell}} \left[ u''(\alpha^c f(e(a, \tilde{\ell})))\alpha^c f(e(a, \tilde{\ell})) + u'(\alpha^c f(e(a, \tilde{\ell}))) \right] \\
= f'(e(a, \tilde{\ell})) \frac{\partial e(a, \tilde{\ell})}{\partial \tilde{\ell}} u'(\alpha^c f(e(a, \tilde{\ell}))) \left[ 1 - \frac{1}{\eta(c)} \right] > 0
\]

Hence \( \frac{\partial T}{\partial a} < 0 \) for the range where the parent gives a positive transfer to his offspring to move out. The optimal transfer the parent pays is decreasing with the child’s ability, it is easier to “bribe” a high ability child.

### A.5 Study of the transfer when \( \psi^p > 0 \)

Proceeding as above, define:

\[
G \equiv u((1 - e(a, -T))\bar{y} - De(a, -T) + T) + u(\alpha^c f(e(a, -T))) + \\
- u((1 - e(a, \ell))\bar{y} - De(a, \ell) - \ell) - u(\alpha^c f(e(a, \ell))) - \psi^c = 0
\]

Implicit differentiation of \( G \) yields:

\[
\frac{\partial T}{\partial a} = -\frac{\partial G}{\partial a} \\
\frac{\partial G}{\partial T}
\]

where, again, \( \tilde{\ell} = \ell - T \).

The derivative \( \frac{\partial G}{\partial T} \) is strictly positive:

\[
\frac{\partial G}{\partial T} = u'((1 - e(a, T))\bar{y} - De(a, T) + T) \left\{ -(\bar{y} + D) \frac{\partial e(a, T)}{\partial \tilde{\ell}} + 1 \right\} \\
+ u'(\alpha^c f(e(a, T)))\alpha^c f'(e(a, T)) \frac{\partial e(a, T)}{\partial T} \\
= u'((1 - e(a, T))\bar{y} - De(a, T) + T) > 0
\]

where the last equality invokes equation A.1.
Hence,
\[
\text{sign} \left( \frac{\partial T}{\partial a} \right) = \text{sign} \left( -\frac{\partial G}{\partial a} \right)
\]
where
\[
\frac{\partial G}{\partial a} = -u'((1 - e(a,T))\bar{y} - De(a,T) + T)(\bar{y} + D)\frac{\partial e(a,T)}{\partial a} \\
+ u'(a^c f(e(a,T))) \left[ f(e(a,T)) + a^c f'(e(a,T)) \frac{\partial e(a,T)}{\partial a} \right] \\
+ u'((1 - e(a,\ell))\bar{y} - De(a,\ell) - \ell)(\bar{y} + D)\frac{\partial e(a,\ell)}{\partial a} \\
- u'(a^c f(e(a,\ell))) \left[ f(e(a,\ell)) + a^c f'(e(a,0)) \frac{\partial e(a,\ell)}{\partial a} \right] \\
+ u'(a^c f(e(a,T)))f(e(a,T)) - u'(a^c f(e(a,\ell)))f(e(a,\ell))
\]

where the last equality invokes equation A.1.

Given \( T > -\ell \), as shown above, this difference is strictly positive, and hence \( \frac{\partial T}{\partial a} < 0 \) for the range where the parent gives a positive transfer to his offspring to move out. The optimal transfer the parent pays is decreasing with the child’s ability, it is easier to “bribe” a high ability child.

B Solution of the Game with Borrowing

B.1 The young agent’s problem

For given choice of residence, study the optimal human capital investment of a young agent with ability \( a \). Again, denote instead with \( \tilde{\ell} = \ell - T \) any additional disutility deriving from coresidence, i.e. living expenditures in case of non coresidence minus the transfer. The young agent is allowed to borrow up to a limit \( \bar{d} \).
The first order conditions of the young agents are:

\[ e : \quad -u'((1 - e)\bar{y} - \tilde{\ell} - De + d)(\bar{y} + D) + af'(e)u'(af(e) - Rd) = 0 \]
\[ d : \quad u'((1 - e)\bar{y} - \tilde{\ell} - De + d) - Ru'(af(e) - Rd) \geq 0. \]

If the borrowing constraint is not binding and the young agents are able to borrow as much as they need, the first order conditions become:

\[ e : \quad R(\bar{y} + D) = af'(e) \]
\[ d : \quad u'((1 - e)\bar{y} - \tilde{\ell} - De + d) - Ru'(af(e) - Rd) = 0. \]

Taking derivatives of the first order conditions it’s clear that optimal education does not depend on \( \tilde{\ell} \) and is increasing in ability. Moreover, optimal debt is increasing in both ability and \( \tilde{\ell} \). The derivative of the optimal education investment \( e(a) \) with respect to ability is:

\[ \frac{\partial e(a)}{\partial a} = -\frac{f'(e(a))}{af''(e(a))} > 0. \]

The derivatives of optimal debt are:

\[ \frac{\partial d(a, \tilde{\ell})}{\partial a} = -\frac{-u''(c_1)(\bar{y} + D)e'(a) - Ru''(c_2)[af'(e(a))e'(a) + f(e(a))]}{u''(c_1) + R^2u''(c_2)} > 0 \]
\[ \frac{\partial d(a, \tilde{\ell})}{\partial \tilde{\ell}} = -\frac{-u''(c_1)}{u''(c_1) + R^2u''(c_2)} > 0. \]

If the borrowing constraint \( \bar{d} \) is instead binding, the agent borrows up to the limit. He faces a trade off between working and consuming more today, investing less in education and hence in future consumption, and investing more in education consuming more tomorrow and less today. Hence, optimal investment in education is increasing in the amount of resources the young agent has and in his ability, denoted human capital investment by \( e(a, \tilde{\ell}) \).

As above, the second-order condition implies that the first-order condition is decreasing.
in human capital investment:
\[
\frac{\partial F}{\partial e} < 0
\]

Human capital varies negatively with ability if the intertemporal elasticity of substitution is smaller than one, \(\eta(c) < 1\). As specified in a footnote above, an IES greater than one is a necessary but not sufficient condition for human capital investment to vary positively with ability. As above, after implicit differentiation of the first order condition, we have that

\[
\text{sign} \left( \frac{\partial e}{\partial a} \right) = \text{sign} \left( \frac{\partial F}{\partial a} \right)
\]

Hence:

\[
\frac{\partial F}{\partial a} = u'(a f(e) - R\bar{d}) f'(e) + u''(a f(e) - R\bar{d}) a f'(e) f(e) = u'(a f(e) - R\bar{d}) f'(e) \left[ 1 + \frac{a f(e) u''(a f(e) - R\bar{d})}{u'(a f(e) - R\bar{d})} \right]
\]

We know that \(\eta(c) \equiv -\frac{u'(c)}{cu''(c)}\). Hence, if \(\eta(c) < 1\):

\[
\frac{a f(e) u''(a f(e) - R\bar{d})}{u'(a f(e) - R\bar{d})} > \frac{(a f(e) - R\bar{d}) u''(a f(e) - R\bar{d})}{u'(a f(e) - R\bar{d})} \equiv -\frac{1}{\eta(a f(e) - R\bar{d})} > 1
\]

Hence

\[
\frac{\partial e}{\partial a} = \begin{cases} < 0 & \text{if } \eta(c) < 1 \\ ? & \text{if } \eta(c) > 1 \end{cases}
\]

Human capital varies negatively with \(\tilde{\ell}\). Implicit differentiation of the first order condition yields:

\[
\frac{\partial e(a, \tilde{\ell})}{\partial \tilde{\ell}} = -\frac{u''(c_1)(\bar{y} + D)}{u''(c_1)(\bar{y} + D)^2 + u''(c_2) a^2 f'(e(a, \tilde{\ell}))^2 + u'(c_2) a f(e(a, \tilde{\ell}))} < 0
\]
Given that the constraint is binding, relaxing the borrowing constraint increases the effect of resources on education. Differentiating the above expression for \( \bar{d} \) yields:

\[
\frac{\partial \kappa(a, \tilde{\ell})}{\partial \tilde{\ell}} \frac{\partial \tilde{\ell}}{\partial \bar{d}} = \Phi \frac{\Phi}{\left[ u''(c_1)(\bar{y} + D)^2 + u''(c_2)a^2 f'(e(a, \tilde{\ell}))^2 + u'(c_2)af(e(a, \tilde{\ell})) \right]^2}
\]

where

\[
\text{sign} \left( \frac{\partial \kappa(a, \tilde{\ell})}{\partial \tilde{\ell}} \right) = \text{sign} \left( \Phi \right)
\]

and after some reshuffling

\[
\Phi = -[(\bar{y} + D)u'''(c_1)[u''(c_2)a^2 f'(e(a, \tilde{\ell}))^2 + u'(c_2)af''(e(a, \tilde{\ell}))] + u''(c_1)(\bar{y} + D)[Ru'''(c_2)a^2 f'(e(a, \tilde{\ell}))^2 + Ru''(c_2)af''(e(a, \tilde{\ell}))] > 0
\]

### B.2 Further details

Existence and uniqueness of the SPNE as well as the single crossing condition follow the same proof as in A.2 and in A.3. Relaxing the borrowing constraint is straightforward and leads to almost identical equations.

### C Equilibrium of the one-period Game

Each period there are a young and an old agent of a third of the dynasties of the economy that play a sequential game \( G^j(\bar{x}_t) \). The state variables of \( G^j(\bar{x}_t) \) is denoted by the preference for coresidence parameters \( \psi^1_t \leq 0 \) and \( \psi^6_t \) and by the state \( x^j_t = (a^{j1}_t, a^{j6}_t, h^{j1}_t, h^{j6}_t) \). The strategy of the old agent is composed by his consumption \( (c^{j6}_t) \), and the transfer conditional on the coresidence status \( (T_t) \). The strategy of the young agent is college attendance \( (s^{j1}_t) \), consumption \( (c^{j1}_t) \), human capital investment \( (e^{j1}_t) \), and coresidence status \( (I_t) \).

Specifying the timing of the game in this setting is important in order to avoid multiplicity of equilibria. The timing of the game is as follows:
1. At the beginning of the period, the ability \((a_t^{j1})\) of the child is revealed.

2. Having observed the state \(x_t\), the parent offers a transfer \(T_t\) to his child conditional on the child choosing the coresidence status. I assume that the parent can observe coresidence and that there is no commitment problem.

3. The parent will then also choose consumption \((c_t^{j6})\). Given the parent’s offer, the child then decides coresidence status \((I_t)\), human capital investment \((e_t^{j1})\), and consumption \((c_t^{j1})\).

The first to move is the parent, having observed the state, the old agent conjectures the young agent’s move, and maximizes his utility. Denote the parent’s conjecture over his child’s decision rule by \(C_t\). The parent’s decision rule is then:

\[
\delta_0(\tilde{x}_t|C_t) = (c_t^{j6}, T_t(I_t = 0), T_t(I_t = 1))
\]

where \(T_t(I_t = 1)\) and \(T_t(I_t = 0)\) are the inter-vivos transfers the parent offers to his child in case of coresidence and non-coresidence. The young agent then observes the state and his father’s decision \(\delta_0\), and maximizes utility choosing his own strategy, i.e. consumption, human capital investment, and coresidence status, to which is attached a transfer.

\[
\delta_1(\tilde{x}_t; \delta_0) = (c_t^{j1}, e_t^{j1}, I_t).
\]

The Subgame Perfect Nash Equilibrium of the game is such that \(C_t = \delta_1\).

**Proposition 5** For each \(\tilde{x}_t \in \tilde{X}\), there exists a Subgame Perfect Nash Equilibrium, and it is unique.

**Proof.** Start from the old agent’s problem. The utility function is concave, and adding a linear element in \(I\) shifts it but does not change concavity. The constraint set is trivially convex and compact. In fact, the old agent chooses \(T\) according to his budget constraint,
the non-negativity constraints on consumption and transfer, his feasibility constraint, and setting the transfers such that the young agent is indifferent between the two options (this transfer is unique). Given the assumption that, when indifferent, the young agent chooses the old agent’s preferred option, the constraint set for the old agent pins down one transfer and is then convex and compact. The old agent’s problem has a unique solution and his value function is concave.

Given this result, let’s turn to the young agent’s problem. Again, the utility function is concave and bounded and the linear shift caused by coresidence does not affect concavity. If coresidence were perfectly divisible, the value function would be naturally concave. However, as Rogerson (1988) points out, the only value functions that matter are those associated with $I = 1$ and $I = 0$, that are unique and concave since the budget constraints associated are convex. Hence, the policy functions for both options are well defined and unique.\footnote{The objective function is strictly concave and the constraint set is convex. Hence, Kuhn-Tucker conditions are necessary and sufficient to find a maximum, and the solution is unique. Furthermore, the value function is strictly concave.}

The optimal transfer-coresidence status pair is then determined uniquely. ■

D Numerical Solution of the Model and Estimation

D.1 Solution Algorithm

1. Take parameters as given.

2. Use Adda-Cooper algorithm to generate the Markov Chain for ability.


4. Set distribution $Mu$ over the grid as uniform.

5. Guess expected derivative of the value functions, $EV'$.

6. Start iteration $it$, for every gridpoint:
(a) Find optimal $e$ when no coresidence, $e^I=0$:

- the FOC on education is increasing in $e$: calculate the FOC value for the extremes $e = 0$ and $e = 1$ controlling for negative consumption (use the expected value functions);
- if there is an internal solution iterate to find it;
- if there is no internal solution, flag it;
- calculate the value function when no coresidence $V^0$.

(b) Find optimal $e$ and $T$ when coresidence, $e^I=1$ and $T^I=1$:

- Find optimal education when the transfer is zero; calculate the value function when coresidence $V^1$.
- Compare $V^1$ and $V^0$:
  - If $(V^0 - V^1) \leq 0$ then coresidence is optimal even with zero transfer, set $T^* = 0$. Jump to (c).
  - If $(V^0 - V^1) > 0$, find the maximum transfer the parent can afford, $\bar{T}$.
  - Find optimal education when the transfer is $\bar{T}$; calculate the value function when coresidence $V^1$.
  - Compare $V^1$ and $V^0$:
    * If $(V^0 - V^1) \geq 0$ then the transfer needed is higher than what the parent can afford, it is not feasible to exert coresidence, $I = 0$. Jump to (c).
    * If $(V^0 - V^1) < 0$, the optimal transfer is between zero and $\bar{T}$, iterate to find it.

(c) Update the $EV'$s:

- when optimal $h$ falls between two gridpoints, update the $EV'$s splitting the mass according to the relative distance from the two gridpoints;
• use a dumping factor to update the $EV$’s.

(d) Update the distribution $Mu$:

• for every gridpoint, distribute the mass of agents in $Mu_{it-1}$ to $Mu_{it}$ according to the transition matrix of the Markov chain and the optimal $e$;

• when optimal $h$ falls between two gridpoints, update the distribution splitting the mass according to the relative distance from the two gridpoints.

(e) Check if $Mu_{it} = Mu_{it-1}$, if yes stop, if not back to 6.

7. Simulate the model.

D.2 Simulation

• Calculate moments from the distribution $Mu$.

• Calculate intertemporal elasticity:

  – Simulate the Model

    * Simulate the Markov Chain with Adda-Cooper method:

    * Attach optimal human capital to each point given ability

    * Calculate wages

    * OLS: calculate $\beta_1$ from $\log(y^{child}) = \beta_0 + \beta_1 \log(y^{parent}) + \epsilon$

D.3 Estimation Algorithm

Given the values assigned to the parameters calibrated externally, the other parameters are estimated through a Simulated Method of Moments, as following:

• Define data moments, group in a vector (see Table 3).

• Define weighting matrix as $W=\text{eye}(N)$ where $N=$number of moments.
• Define parameter space for the search as a N-dimension grid, S is the number of grid-points for every parameter.

• Order parameters/moments in vector M.

• Guess initial parameter values.

• Begin search.

  1. Search over first parameter
     - for each grid value, solve and simulate the model
     - get moments in a vector Msim
     - calculate the criterion (M-Msim)'W(M-Msim)
     - choose the gridpoint minimizing the criterion

  2. Repeat for all parameters

  3. Solve and simulate the model with all new parameter values

  4. Get moments in a vector Msim

  5. Calculate the criterion (M-Msim)'W(M-Msim)

  6. if criterion=0 stop, else update parameter values and back to point 1.