OPTIMAL MIX BETWEEN PAY AS YOU GO AND FUNDING IN PUBLIC PENSION SCHEMES

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Common research with :
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Motivation

- Increasing interest to mix pay as you go and funding techniques in pension
- Balance of state and private pensions
- This mix can be done even inside the social security scheme
- Risk management approach in finance, in insurance ... and ... in pension: integration of risks in the decision process

**Purpose**: theoretical justification of the diversification between PAYG and funding using portfolio theory arguments and choice of an optimal mix


Bilancini E. and D’Antoni A.:
The desirability of pay as you go pensions when relative consumption matters and returns are stochastic

Guigou J.D., Lovat B. and Schiltz J.:
Optimal mix of funded and unfunded pension systems

Thogersen O. and Cardoso P.:
Alternative risk sharing mechanism of social security
Public Finance Analysis, 66 (2), 134-152, 2010
Outline

- 1. The basic model
- 2. A model with 2 financial assets
- 3. Solution with 1 risky and 1 riskless asset
- 4. Future research
1. Basic Model

The Overlapping Generation Model (OLG Model):

Stylization tool in order to capture the *dynamic evolution* of population in time with a focus on equilibrium between active people and retirees.

**OLG Assumptions:**
- Agents have finite lives
- They live in two periods:
  - they are “young”, then “old”, then dead
  - when one generation becomes old, another young generation is born.
1. Basic Model

Model with 2 periods:

- $X_0$: Entry age
- $X_r$: Retirement
- $X_d$: Death

- Working period
- Retirement period
1. Basic Model

**Notations / deterministic model:**

\[ L(x, t) = \text{number of people aged } x \text{ at time } t \]
\[ \pi = \text{contrib. rate on salary (DC plan)} \]
\[ i = \text{financial rate of return} \]
\[ g = \text{rate of increase of salary} \]
\[ S(t) = \text{mean salary at time } t \]
\[ P(t) = \text{mean pension at time } t \]
\[ d = \text{demographic rate of increase} \]
\[ p_{x_0} = \text{survival probability between } x_0 \text{ and } x_r \]
1. Basic Model

Demographic evolution:

Retired and active population at time $t$:

$$L(x_r, t) = L(x_0, t-1) p_{x_0} = (L(x_0, t)/(1 + d)) p_{x_0}$$

- Retired population
- Active population
- Longevity risk
- Demographic effect
1. Basic Model

Comparison of the **replacement rate in pay as you go** and in **funding**:

\[
RR(t) = \frac{\text{pension}}{\text{salary}} = \frac{P(t)}{S(t)}
\]
1. Basic Model

Replacement rate in \textit{pay as you go}:

Actuarial equivalence between contributions and benefits paid both at time $t$:

$$L(x_r, t) \cdot P(t) = L(x_0, t) \cdot \pi S(t)$$

$$\Rightarrow RR = \frac{\pi}{p_{x_0}} (1 + d)$$
1. Basic Model

Replacement rate in **funding**:

Actuarial equivalence between present value of contributions and benefits for a fixed cohort:

\[ L(x_r,t)P(t) = L(x_r - 1,t - 1)\pi S(t - 1)(1 + i) \]

\[ RR = \frac{\pi}{p_{x_0}} \frac{1 + i}{1 + g} \]
1. Basic Model

Replacement rate – **diversification strategy**:

\[ a = \text{proportion of the contribution invested in funding} \]

\[ 1-a = \text{proportion in payg} \]

(with \(0 < a < 1\))

\[ \text{RR} = \frac{\pi}{p_{x_0}} \left\{ a \frac{1+i}{1+g} + (1-a)(1+d) \right\} \]

Same influence of longevity risk for payg and funding
1. Basic Model

Classical Portfolio theory:
- Optimal choice between stocks and bonds depending on the risk aversion of the investor.
- Bonds and Stocks have different risk profiles

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1. Basic Model

(a = proportion of the contribution invested in funding (control variable)

\[ RR(\omega) = \frac{\pi}{p_{x_0}} \left( a \left( \frac{1 + i(\omega)}{1 + g(\omega)} \right) + (1 - a)(1 + d(\omega)) \right) \]

\[ = \frac{\pi}{p_{x_0}} Z(\omega) \]

Assumption:
\[ p = \text{deterministic (no longevity risk)} \]

General distribution with dependency structure between:
- financial risk (i)
- demographic risk (d)
- inflation risk (g)
1. Basic Model

**Basic Random Variable:**

\[ Z = a F + (1 - a)D \]

= return of the mixed strategy

\[ D = 1 + d \; ; \; F = (1 + i)/(1 + g) \]

(2 positive random variables)

**Dependency assumption:**

- \( g \) and \( d \) independent (salary and demography)
- \( g \) and \( i \) dependent (salary and returns)
- \( d \) and \( i \) could be dependent! (returns and demography)
1. Basic Model

**Risk Management - Mean variance analysis:**

Optimization of the mean replacement rate but taking into account the risk through the variance.

The decision problem can be written as:

\[
\min_a \ Var\ Z \quad \text{subject to} \quad E(Z) = Z_0
\]

Utility framework: for a fixed \( \gamma > 0 \) (risk aversion):

\[
\max_a \ U(Z) = \max_a \left( E(Z) - \frac{\gamma}{2} \cdot Var\ Z \right)
\]
2. A Model with two financial assets

Introduction of 2 possible Financial assets:

\[ F = \beta \cdot F_1 + (1 - \beta) \cdot F_2 \]

( for instance a stock index and a bond index)

The global portfolio with PAYG and funding is now:

\[ Z = a \cdot \beta \cdot F_1 + a(1 - \beta) \cdot F_2 + (1 - a) \cdot D \]
2. A Model with two financial assets

Or:

\[ Z = x_1 \cdot F_1 + (a - x_1) \cdot F_2 + (1-a) \cdot D \]

Where:

1 - \( a \) = part in PAYG

\( x_1 \) = part in the first financial asset

Purpose:

maximise:

\[ U(Z) = E[Z] - \frac{\gamma}{2} Var[Z] \]
2. A Model with two financial assets

**Expectation:**

\[
E[Z] = a \cdot \beta \cdot E[F_1] + a (1 - \beta) \cdot E[F_2] + (1 - a) \cdot E[D]
\]

**Variance:**

\[
\text{Var}[Z] = a^2 \cdot \beta^2 \cdot \text{Var}[F_1] + a^2 (1 - \beta)^2 \cdot \text{Var}[F_2] + (1 - a)^2 \cdot \text{Var}[D] \\
+ 2a^2 \beta (1 - \beta) \text{Cov}[F_1, F_2] \\
+ 2a (1 - a) \beta \text{Cov}[F_1, D] + 2a (1 - a) (1 - \beta) \text{Cov}[F_2, D]
\]
2. A Model with two financial assets

**PROPOSITION:**

The optimal solution is given by:

\[
\begin{align*}
\text{Optimal part in funding:} & \\
 a^* = & \frac{\text{var}(F_1 - F_2).m_1 - \text{cov}(F_1 - F_2, F_2 - D).m_2}{\text{var}(F_1 - F_2).\text{var}(F_2 - D) - (\text{cov}(F_1 - F_2, F_2 - D))^2} \\
\text{Optimal part in the first financial asset inside the funding part:} & \\
\beta^* = & \frac{x_1^*}{a^*} = \frac{\text{var}(F_2 - D).m_2 - \text{cov}(F_1 - F_2, F_2 - D).m_1}{\text{var}(F_1 - F_2).m_1 - \text{cov}((F_1 - F_2, F_2 - D).m_2}
\end{align*}
\]
2. A Model with two financial assets

With:

\[ m_1 = \text{var}(D) - \text{cov}(F_2, D) + \frac{1}{\gamma} E(F_2 - D) \]

\[ m_2 = \text{cov}(F_2 - F_1, D) + \frac{1}{\gamma} E(F_1 - F_2) \]

**Remark**: the diversification between PAYG and funding is effective if and only if:

\[ 0 < a^* < 1 \]

If we forbid any short selling of financial assets:

\[ 0 \leq \beta^* \leq 1 \]
3. Solution with one risky and one riskless asset

Generalization of the classical portfolio theory in presence of a riskless asset but adding now the possibility to invest in PAYG (« demographic asset »)

\[
\begin{align*}
EF_2 &= R = (1 + r) < EF_1 \\
\text{var } F_2 &= 0 \\
\text{cov}(F_1, D) &= 0
\end{align*}
\]

r is a (real !!!) risk free rate
3. Solution with one risky and one riskless asset

PROPOSITION:

- Optimal part in PAYG:

\[
1 - a = \frac{1}{\gamma} \cdot \frac{(ED - (1 + r))}{\text{var } D} = \frac{1}{\gamma} \cdot \lambda_D
\]

The optimal part in PAYG is independent of the parameters of the existing risky asset on the market; it depends only on the risk premium of the demographic return w.r.t. the risk free rate.
3. Solution with one risky and one riskless asset

PROPOSITION:

- Optimal part in the risky asset:

\[
x_1 = \frac{1}{\gamma} \cdot \frac{(EF_1 - (1+r))}{\text{var}F_1} = \frac{1}{\gamma} \lambda_F
\]

- Optimal part in the risk less asset:

\[
a - x_1 = 1 - \frac{1}{\gamma} (\lambda_D + \lambda_F)
\]
3. Solution with one risky and one riskless asset

Diversification condition between funding and PAYG:

\[ 0 < a < 1 \]

\( a < 1 \) : OK if: \( ED > R \)

(If PAYG has a lower mean return than the real risk-free rate but has a positive variance, it is surely not optimal!)

\( a > 0 \) : OK if: \( \gamma > \lambda_D \)
3. Solution with one risky and one riskless asset

No short selling of the riskless asset:

\[ 0 < a - x_1 < 1 \]

\[ a - x_1 < 1 : \text{OK if } \text{ED} > \text{R} \]

\[ 0 < a - x_1 : \text{OK if } \gamma > \lambda_D + \lambda_F \]
3. Solution with one risky and one riskless asset

**NUMERICAL ILLUSTRATION:**

Scenario 1:

- risk free rate: 1.5%
- risky asset:
  - mean return: 5%
  - volatility: 20%
- demography:
  - mean return: 2%
  - volatility: 5%
3. Solution with one risky and one riskless asset

Scenario 1:

![Graph showing the performance of assets under different scenarios.](image)
3. Solution with one risky and one riskless asset

NUMERICAL ILLUSTRATION:

Scenario 2:

- risk free rate: 1.5%
- risky asset:
  - mean return: 5%
  - volatility: 20%
- demography:
  - mean return: 2%
  - volatility: 10%
3. Solution with one risky and one riskless asset

Scenario 2:

![Graph showing investment scenarios]
Further research

• Generalization of the mean variance principle (utility)

• Introduction of the longevity risk and analysis of correlation with the other risk factors

\[ RR(\omega) = \frac{\pi}{p_{x_0}(\omega)} \left( a \left( \frac{1+i(\omega)}{1+g(\omega)} \right) + (1-a)(1+d(\omega)) \right) \]

• Multi period models? Continuous time optimization (stochastic optimal control techniques)
THANK YOU

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