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References
1. Introduction.

20-some years ago, Werner Hildenbrand and Alan Kirman concluded their classical monograph on *Equilibrium Analysis* with the statement (p. 239):

"There are no assumptions on the isolated individuals which will give us the properties of aggregate behaviour which we need to obtain uniqueness and stability. Thus we are reduced to making assumptions at the aggregate level which we cannot justify by the usual individualistic assumptions."

Making assumptions at the aggregate level is standard practice in macroeconomics. Yet, we shall argue here that General Equilibrium Theory (GET) is the framework best suited for an integrated treatment of micro- and macro-economics.

Our claim rests on one obvious extension to classical GET, namely incomplete markets. But that extension brings in other recent developments in GET: money, price rigidities and quantity constraints, overlapping generations. And it points to further desirable extensions.

2. Incomplete markets (GEI)

GET with incomplete markets – GEI – starts from the commonplace observation that many risks fail to be traded on markets (or otherwise). Yet, a fundamental result of classical GET, namely Borch’s theorem (1960), asserts that universal risk pooling is required for Pareto-efficiency, under quite general conditions. Missing risk markets may thus entail Pareto-inefficiency. As we shall see, they do so, generically and in a strong sense.

The GEI model, crude as it remains, is well suited to probe into that issue. Indeed, GEI focuses on missing markets while side stepping other potential sources of inefficiency. For a more realistic and richer framework, one turns to Temporary General Equilibrium (TGE), as we do below (section 7).

GEI starts from the classical Arrow-Debreu model with time and uncertainty (an event-tree), i.e. Chapter 7 of *Theory of Value*. Markets for contingent claims to future commodities (either explicit or implicit in asset markets) are incomplete. But it is assumed that (i) the spot markets operating at all date-events are competitive; and (ii) the prices that do or will prevail on these markets are known to all agents. This perfect foresight assumption goes well beyond rational expectations.

3. Generic constrained inefficiency of competitive equilibria

An important concept for normative analysis in GEI was introduced by Peter Diamond in 1967: Constrained Pareto Efficiency (CPE), i.e. Pareto efficiency over the set of allocations that are implementable through the existing markets.

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1 See Radner (1968) for an early existence theorem. Drèze et al. (2009) claim to have plugged the loophole in Radner’s work (excess supply of shares at zero price).

2 A complex issue for positive analysis in GEI with production concerns the decision criteria of firms: profit maximisation is not defined in absence of a complete set of prices for contingent commodities. There is no place here to review the extensive (and still growing) literature devoted to that issue. See Drèze et al. (2009, section 1) for a short overview.
It has proved feasible to spell out necessary and sufficient conditions for CPE in the standard GEI model – see Drèze (1974) for a 2-period model, extended by Bonnisseau and Lachiri (2004). That work leads in particular to a characterisation of firm decisions compatible with CPE: at every date-event, they must be *Pareto-efficient from the viewpoint of final shareholders of the firm at that date-event.*

Let competitive equilibria at which firms make decisions compatible with CPE be labelled "stockholders equilibria". Do they verify the two welfare theorems: (i) are they constrained Pareto efficient? and (ii) can every CPE allocation be implemented as a stockholders equilibrium, under suitable transfers?

The answers are clear: (i) **Generically** (say, in initial endowments), *competitive stockholders equilibria fail to be CPE* and (ii) with multiple goods or assets, **not every CPE can be sustained as a competitive stockholders equilibrium.**

### 4. Second-best price-wage rigidities

These general negative results invite the question: how can one improve upon competitive stockholders equilibria? Not by looking for different decision criteria of firms, since these are defined by *necessary conditions* for CPE; thus, by entertaining departures from competitive clearing of asset or commodity markets? The existence of possibilities along that line was illustrated, *inter alia:* for assets by Geanakoplos and Polemarchakis (1986); for commodities by Polemarchakis (1979) generalised in Herings and Polemarchakis (2005).

That work aims at generality, not operationality – inviting further attention to specific contexts. Of special interest for macro-economics is the investigation of labour markets, which can be pursued in two contexts: (i) intertemporal state-dependent contracts; and (ii) future hirings.

(i) A useful starting point came with the work on *implicit labour contracts* of the mid-seventies, under which risks to labour productivity are shared between a risk-neutral firm and its risk-averse employees. A suggestive link to Borch’s Theorem is developed in Drèze (1993). In a 2-period GE model with quadratic preferences, efficient labour contracts call for state-dependent 2nd-period wages indexed partly on a price index and partly on real national income, with respective weights chosen by each worker. Interestingly, these labour contracts *reduce the risk premium* on asset markets.

This is one among several illustrations of the *indirect relevance of Borch’s reasoning under incomplete markets.* Other illustrations concern public pensions and the design of assets (including assets indexed on national income). These can again be studied in *streamlined GE models,* of which the quadratic (hence CAPM) approximation is an example.

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1. For two periods, see Drèze (1974), Hart (1975), Geanakoplos and Polemarchakis (1986), and the more general theorem in Geanakoplos et al. (1990). For more than 2 periods, the results hold *a fortiori.*
2. The ground had been laid for a study of such departures with the work on *equilibria with price rigidities and quantity constraints* of the early- and mid-seventies. See Grandmont (1977, section 4) or Herings (1996) for a presentation.
3. See, e.g., Azariadis (1987). An intellectual root of that work comes from the remark by James Meade (1972): “While property owners can spread their risks by putting small bits of heir property into a large number of concerns, a worker cannot put small bits of his effort into a large number of different jobs.”
(ii) But not all workers are covered by long-run labour contracts. (For instance, in the US, annual labour turnover in industry is 40%). Thus, labour contracts do not cover the risks associated with the terms of new hirings. Starting from Meade’s remark (in footnote 5), it is natural to ask whether wage rigidities cum unemployment benefits can entail risk-sharing benefits that outweigh the associated productivity losses.

One (positive) answer to that question is found in the paper by Drèze and Gollier (1993), which again rests on a streamlined GE model to derive: (i) a sufficient condition for inefficiency of the competitive equilibria; and (ii) a characterisation of the constrained efficient allocations, which generically rest on downward wage rigidities cum unemployment benefits.\(^6\)

We conclude that GET with incomplete markets must allow for micro-founded price/wage rigidities and the associated second-best policies.

5. Money

Price and wage rigidities can be either nominal or real. Dealing with nominal rigidities (the more frequent case) calls for extending GET to money.

The literature offers a number of alternative specifications of monetary GEI or TGE equilibria. On this occasion, it is natural to privilege that published in the Hildenbrand Festschrift edited by Debreu et al. (2001), namely Drèze and Polemarchakis (2001). It has the ancillary merit of being simple and transparent.

Money is the medium of exchange. It is produced at no cost by a (central) bank, which lends it to firms and households at one-period interest rates set by the bank. Under GET or GEI, that rate is event-dependent, but all rates are known to all agents from the start. The interest rates announced by the bank also apply to one-period safe bonds; i.e., there is full arbitrage between bank loans and bonds.

The transactions technology is a primitive, on par with the production technology. It is defined by a correspondence (uhc and convex-valued) from the space of prices and transactions to money holdings. (The timing of transactions within periods is ignored.)

In GET, competitive monetary equilibria exist under standard assumptions. The overall price level and the variability of inflation rates across successor date events are indeterminate and welfare-neutral. Monetary policy thus only affects average (expected) inflation rates. Of course, these features (especially the welfare neutrality) no longer hold under price rigidities, which however induce more interesting forms of indeterminacy.\(^7\)

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\(^6\) For other reasons why incomplete markets breed price rigidities, not necessarily second-best, reference could be made to fixed costs and to estimation of demand elasticities. Under periodic known variations of production levels, fixed costs are covered by the higher prices associated with full use of capacity (“peak-load pricing”). Under stochastic variations, the same approach remains valid if markets are complete, so that higher revenues in the full-use-of-capacity states can be transferred to other states. But that is no longer possible when markets are incomplete, giving firms an incentive to cover fixed costs in all states. And it is furthermore the case that incomplete markets entail limited information on demand elasticities, another source of price rigidities. See Drèze (1979b) or Drèze and Herings (2008).

\(^7\) More recent unpublished work of Drèze (2009) investigates the implications of introducing an (exogenous) timing of transactions within periods. The bank then receives continuously non-interest-bearing deposits and
The extensive indeterminacy of nominal variables reflects an incomplete model specification, leaving room for a positive theory of inflation and calling for a specification of short-run dynamics.\(^8\) It also calls for a policy instrument suitable to anchor expectations about the price level.\(^9\)

6. Multiple equilibria

One property of constrained equilibria that matters a lot for macroeconomics is their potential multiplicity.

That possibility was raised in a seminal paper by John Roberts (1987). More general results appear in Herings (1996), Drèze (1997) then Citanna et al. (2001).\(^10\)

To illustrate, the main theorem in Citanna et al. goes as follows: in a standard GE model, let the prices of a subset of commodities be given, and let the supply of these commodities be subject to endogenous upper bounds; there exists a continuum of supply-constrained equilibria with arbitrary severe rationing of supplies.\(^11\) The explanation is simple: if \(L\) prices are given, \(L-1\) relative prices are fixed; but \(L\) rationing constraints are introduced, leaving one degree of freedom, which corresponds to the overall level of rationing for these commodities, or to the level of flexible prices relative to the \(L\) fixed prices.

Multiple equilibria do, of course, open the door to macroeconomic fluctuations (to output gaps). And they place new demands on the modelling of expectations, better pursued in the TGE framework.

7. Temporary General Equilibrium (TGE)

TGE was initially defined by Sir John Hicks in 1939, but the relevant modern reference for GET is the seminal paper by Jean-Michel Grandmont (1974).

TGE is the only framework suitable to study situations where some of the agents active at future dates are not present or represented on today’s markets. We shall expand on that theme in section 10.

But TGE is also the framework arising when the perfect foresight assumption of GEI is relaxed. The natural alternative is rational expectations, which is much weaker.

\(^1\) Issues continuously overdraft facilities. Deposits earn no (or very low) interest. Distinct interest rates apply to bank loans and bonds, with the latter not exceeding the former. But interest rates on bonds are now determined by market clearing. And they are the relevant ones for inflation. In the standard GE model, all inflation rates become indeterminate.

\(^2\) Some GE work related to short-run dynamics is found in the stability analysis of tâtonnement (or non-tâtonnement) processes with price rigidities and quantity constraints; see Drèze (1991, 1999) or Herings et al. (1999).

\(^3\) “Taylor rules”, that are claimed to pin down price levels uniquely, rest on an assumption of bounded inflation; but the assumption is not documented, in line with the “tailor principle” according to which it simplifies life to “tailor your assumptions to desired results”. The natural instrument to curb spiralling inflation is credit constraints, as used in Europe in the early eighties.

\(^4\) These results stand in apparent conflict with earlier work by Laroque and Polemarchakis (1978). The explanation rests with model specification and assumptions.

\(^11\) More precisely: the least severe (across commodities) supply constraint may range from zero to a non-binding level.
Under multiple market equilibria tomorrow (a continuum?), rationality does not place stringent restrictions on expectations: any density on the set of potential realisations deserves attention.\(^\text{12}\)

A suggestive characterisation of multiple equilibria under nominal rigidities is stated in an unpublished paper of Drèze (2009) who obtains, in a quite general TGE model, a result also valid for GEI, namely: "let \(u\) denote the average degree of underutilisation of resources (the so-called "output gap") and \(i\) denote the rate of inflation in the first period; for every \(A\in \mathbb{R}^+\), there exists a supply-constrained equilibrium with \((1 – u) (1 + i) = A\)." In the absence of rigidities, one falls back on the indeterminacy result of Drèze-Polemarchakis for the overall price level, hence initial inflation. With rigidities, one obtains a generalised Phillips curve, in the spirit of the above-mentioned theorem by Citanna et al..

Again, we conclude to the need of extending the model to short-run dynamics, and to the role of a policy instrument anchoring expectations about activity levels.

8. Incomplete markets breed demand volatility

As also noted in Drèze (2001), changes in the degree of uncertainty embodied in the agents’ expectations (in particular about tomorrow’s multiple equilibria) are apt to result in output gaps – whether the change be for better or for worse!

We know from the theory of savings – e.g. Drèze and Modigliani (1972) or Sandmo (1970) - that increased uncertainty is apt to trigger additional savings by households. And we know from the theory of investment – e.g. Dixit and Pindyck (1994) – that investments should only be implemented when the net present value of the resulting profit stream is greater than the value of an option to carry out the same investment at a later date. Every new prospect of further information is thus apt to promote postponement of investment: the associated cost is of second order, the gain is of the first order! These two effects compound each other towards opening a macroeconomic gap between savings and investment, resulting from the limited predictability of information flows.

9. Taking stock

So far, we have noted that, in GEI:

- competitive stockholders equilibria generically fail to be constrained efficient,
- second-best wage rigidities cum unemployment benefits permit improving Pareto-wise on competitive labour markets,
- price/wage rigidities are apt to entail existence of a continuum of constrained equilibria with a range of inflation rates and output gaps,
- aggregate demand and the relation of savings to investment are volatile, these endogenous (economic) uncertainties being just as relevant as exogenous uncertainties.

The same features apply to TGE.

\(^{12}\) Accordingly, formulations such as that in Balasko (2003), where the density on future realisations is largely independent of current signals, does not seem appropriate.
It follows that GET allocations and equilibria display features typically associated with macroeconomics – unemployment, inflation and output gaps - calling for appropriate second-best policies – unemployment benefits, monetary and fiscal policy.

The definition of these second-best policies does not flow automatically out of the general model: more precise specifications are needed, in particular the short-run dynamics linking successive allocations and the formation of rational expectations. This is not surprising; the same remark applies to second-best policies linked to other deviations from the Arrow-Debreu assumptions, like externalities, non-convexities (fixed costs or increasing returns) and imperfect competition. But our conclusion suggests that there is no need to develop an independent framework to study macroeconomic policies: GET with incomplete markets provides a suitable framework.

Spelling out macroeconomic implications of GET thus opens the avenue towards a unique consistent framework for micro-and-macro economics.13

Beyond this momentous methodological conclusion, and time permitting, we wish to address briefly a final theme – even if we must limit ourselves to methodological considerations without quoting (yet!) published results.

10. Intergenerational risk sharing14

Intergenerational risk sharing, whereby the income loss resulting from an unfavourable outcome today is partly shifted forward to future generations through borrowing, is not always available to individual agents: if the debt burden falls on their heirs, the latter (when they exist!) may refuse the inheritance. Government debt is less subject to that limitation and thus provides the natural vehicle for intergenerational risk sharing (as illustrated vividly by the financing of wars – or of the 2008-09 recession).

Consider a community whose endowment is subject to a shock: a stationary zero-order Markov process entails a positive or negative fixed shock with given (say equal) probabilities. Let each of N successive generations adjust its consumption by (1/N)-th of the shock it experiences and shift the balance (positive or negative) forward. In a stationary state, each generation will adjust its consumption by the mean of N shocks, so that the variance of its consumption is divided by N. 15

But there are pitfalls. As the number of generations and N tend to infinity, the cumulative balance of past shocks transferred to a new generation is unbounded. That is, for any positive real number R, the probability that the cumulative balance of unabsorbed past shocks (i.e. the

13 25 years ago, Drèze (1986) opened his Presidential Address to the EEA with the remark “Recurrently, I dream about students at the University of Nirvanah, taking up different subjects – like microtheory, macrotheory, welfare economics, business cycles and economic policy – all taught within the same methodological framework and fitting nicely together, like the pieces of a jig-saw puzzle, to form a coherent picture”. No longer a pipe-dream, perhaps; but where will the University of Nirvanah locate?

14 The branch of the economic literature where that topic is addressed squarely is the so-called overlapping generations (OLG) theory; see Geanakoplos and Polemarchakis (1991) for a survey.

15 This elementary example is discussed further in Gordon and Varian (1988).
outstanding debt) exceeds R is positive.\textsuperscript{16} And the probability that a future generation will have incentives to renge on the scheme is always positive.

Transferring that illustration to macroeconomic shocks and the public debt opens up a number of issues that remain to be incorporated in GET. Indeed, fiscal policy can play two roles beyond pure (i.e. distributionally neutral) intergenerational risk sharing: it may implement intergenerational redistributive transfers; and it may contribute to reducing current output gaps (Keynesian stimulation) or inflationary pressures, and their variances.\textsuperscript{17}

In order to approach simultaneously these three aspects of fiscal policy, it is imperative to introduce \textit{public capital} in the model. The combinations of the current fiscal stance, the adjustments to the level of the public debt and the adjustments to the stock of public capital together open varied possibilities. Indeed, debt-financed public investment, being neutral from the viewpoint of intergenerational redistribution, offers an autonomous instrument of Keynesian stimulation or inflation control. That instrument may be adapted to current features (like the extent of unused capacities) not directly taken into account by intergenerational risk sharing.\textsuperscript{18} Whereas additions to public capital at unchanged levels of public debt, or additions to the public debt at unchanged stocks of public capital, implement intergenerational redistribution.

Finally, normatively desirable overall public debt levels should be defined with reference to both the value of public investments and the (possible) desire to correct the direction and size of private intergenerational transfers.\textsuperscript{19}

We are here raising more questions than we can answer; but one conclusion – similar to that stated in section 9 above - stands out clearly: \textbf{fiscal policies combining a concern for intergenerational risk sharing with current output/inflation control and intergenerational redistribution emerge naturally in GET.} In other words: GET encompasses naturally macroeconomics analysis!

\textsuperscript{16} Such concern lies at the root of the EU “Maastricht rules”, which stipulate that the public debt of a member nation should not exceed 60\% of its GDP while its annual budget deficit should not exceed 3\% of GDP. Of course, these rules are crude. In particular, no distinction is made between debt and deficits originating in current public expenditures and those corresponding to public investments. The implicit public debt resulting from unfunded pension rights is not included. And no justification is given for the chosen percentages.

\textsuperscript{17} In line with footnote 7 above, we refrain from expanding here on the “monetary policy” dimension of fiscal policy – a dimension that does of course deserve full attention. With a variety of debt instruments (nominal, real or indexed on real income), monetary policy can take new forms, like debt swaps. But ground for caution is brought out the substantial departures from arbitrage between nominal and indexed government bonds in the US; see Fleckenstein et al. (2011).

\textsuperscript{18} Of course, implications for private capital formation (“crowding out”) must be assessed as well.

\textsuperscript{19} Incomplete markets entail the prospect of excessive private transfers to heirs by households not holding their assets in the form of annuities.
QUESTIONS.

For the roundtable on May 21st, we propose to start from the following questions, hereby addressed to all participants. We may possibly address questions in a different order, and in conjunction with further questions raised by Herakles. And we should start, of course, with any clarifications that may be needed.

A. Do you agree with the four points listed in the first paragraph of section 9 (p.6)? If not, we should clarify to what extent, and why.

B. Do you agree with the implication that GET with incomplete markets provides a suitable framework to integrate microtheory and macrotheory?

If not, why? If yes, do you regard that conclusion as important: (i) for economic research; and (ii) for the teaching of economics – from undergraduate to PhD levels? Important: in what way(s)?

C. Going back to the opening quotation from Hildenbrand-Kirman (1988):

(i) isn't the case that very similar challenges arise today at the level of relating GEI-TGE to the models currently used by macroeconomists – like the New Keynesian Phillips Curve or DSGE?

(ii) after 1988, a number of us felt that "GET was dead"; does the work reviewed here suggest otherwise, after all? And could we claim that the integration of microtheory and macrotheory defines the current liveliness of GET – perhaps with a highly uncertain quality of life?

D. Acknowledging that GEI and TGE are technically demanding theories, how should the theorists communicate with policy-oriented, applied and empirical (macro)economists?

E. How could one go about writing a technically soft yet rigorous manual of economic theory encompassing microtheory and macrotheory?


Or is such an attempt premature, pending further extensions regarding short-run dynamics, expectations, fiscal and monetary policies?
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