Pricing in Social Networks under Limited Information

Workshop on Networks in Economics and Finance
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The problem

**Monopolist:**
- faces a population of consumers
- aims at increasing its demand exploiting its (limited) information about the social network of its (current and perspective) customers.

**Consumers:**
- Some are not informed about the existence of the product
- Each of them has a certain number of social contacts → social network.

**Strategy:** To offer a reward to the current clients (informed by definition) so to induce them to activate their social networks and convince their peers to buy the product.

⇒ Referral Program
Motivation

Referral rewards are offered in many markets:

- Online storage services (e.g. Dropbox) reward consumers with extra space
- Online services extend the period of time of subscription (e.g. MMORPG) or money to spend within the service (e.g., Airbnb)
- Banks offer higher interest rate on the deposit or give valuable gifts (a bike, a Kindle, 100£,...)
- Many other markets: payment systems, online content providers, enterprise software solutions ....

An effective strategy:

- Mass media advertisements are not as trusted as discussions among customers (Katz and Lazarsfeld, 1955)
- Online social networks and ICT makes communication among consumers cheaper.
Research question

Given a network structure for the social interactions among consumers and study:

1. Optimality of introducing a referral program.
2. Welfare effects of the introduction of the reward (level of prices and profits).
3. The effect of network degree distribution on the results:
   - Random vs. Scale Free networks
   - The role of network density
Introduction and Motivation

Related Literature

The model

Results

Conclusions

Literature

- Network & Economics: Jackson (2005)
  - No Network externalities, Limited Information.

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Literature

- Network & Economics: Jackson (2005)
  - No Network externalities, Limited Information.
- **Word of Mouth Communication in Consumers networks**: Galeotti (2010), Galeotti & Goyal (2009), Campbell (2013)
  - Communication happens only under condition that enough incentives are provided.
Literature

- Network & Economics: Jackson (2005)
  - No Network externalities, Limited Information.
- **Word of Mouth Communication in Consumers networks**: Galeotti (2010), Galeotti & Goyal (2009), Campbell (2013)
  - Communication happens only under condition that enough incentives are provided.
- **Viral/Referral marketing**: Biyalogorsky et al. (2001), Leskovec et al. (2007), Schmitt et al. (2011), Tucker (2011)
  - Focus on pricing.
Demand Side

\[ N = \{1, 2, \ldots, n\}: \text{ set of myopic consumers indexed } i \]

- unit demand, reservation value \( r_i \sim U[0, 1] \)
- number of neighbors or degree \( k_i \sim f(k) \)
- possibly uninformed (proportion \( \beta \)) about the existence of the product.

Few remarks:

\((R1)\) The distributions of reservation value, degrees and information are independent of each other.

\((R2)\) Each consumer has private information on his own characteristics, while the distributions are common knowledge

\((R3)\) No externality from others’ consumption
Supply Side: Monopolist

- Sells a non-durable good.
- Sets prices in two periods.
- In the second period, he offers a per-friend unitary bonus $b$ to first period buyers who refer friends (willing to buy).
- Each new consumer can refer only one old consumer.

Normalizing marginal cost to 0 and inter-temporal discount to 1, the profit of the monopolist will be composed by:

$$\pi = \pi_1 + \pi_2 = p_1 D_1(p_1) + p_2 D_2^1(p_2) + (p_2 - b) D_2^2(b, p_2)$$

- profits by time-1 informed people
- profits by time-2 informed people
Timing of the model

0

\begin{itemize}
\item Period 1
\begin{itemize}
\item \( p_1 \) (Monopolist)
\item Purchase (early informed)
\end{itemize}
\item Period 2
\begin{itemize}
\item \( p_2, b \) (Monopolist)
\item Investment & Purchase (early inf.)
\item Purchase (inf. time 2)
\end{itemize}
\end{itemize}

Solution by backward induction

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Previously uninformed consumers:

- A share $\beta$ of a population of $n$
- They can buy only if they receive the information through the social network (prob. $\bar{\rho}$)
- Once informed, an agent buys if $r > p_2 \rightarrow P(.) = (1 - p_2)$
- Second period demand from newly informed agents is thus:

$$D_2^2 = n\beta(1 - p_2)\bar{\rho}$$

The probability of getting informed in the second period $\bar{\rho}$ is an increasing function of the number of first period buyers who speak about the product, $D_1^{inv}$.
Analytical Results

Investment decisions

Take a degree-$k$ old buyer. If he speaks about the product:

- Bears a (fixed) cost $C$
- Expects to enjoy some reward. The expected reward increases with the degree and decreases with the number of other people investing, $D_{1}^{Inv}$.

In expected terms:

$$\frac{\beta k(1 - p_2)b}{g(D_{1}^{Inv})}$$

- The expected net benefit is monotone in the size of the ego networks $\Rightarrow \exists$ A threshold of minimal degree for investment: $k$
Investment decisions (cont’d).

- **Investment condition:**
  \[
  \frac{k}{g(k)} \geq \frac{C}{\beta b(1 - p_2)} \quad \text{and} \quad \frac{k}{g(k)} < \frac{C}{\beta b(1 - p_2)} \quad \forall k < k
  \]

  \[\implies\]

- The number of investors $D_{1}^{\text{inv}}$ is simply given by the share of investors $\sum_{k \geq k} f(k)$ among old buyers.

- Accordingly, the probability that information is transmitted in a single interaction is given by $D_{1}^{\text{inv}} / n$.

- The expect number of competitors and the probability of receiving the information can then be calculated.
Analytical Results

**Bonus & Price Setting**

- Anticipating investment and purchase decisions, the monopolist sets price and unitary bonus to maximize:

\[ p_2 D_2^1(p_2) + (p_2 - b) D_2^2(k) \]

subject to the investment condition

⇒ Results

- It is always profitable to set \( p_2^* = \frac{1}{2} \)
- Targeted some cutoff \( k^* \), the monopolist will set the bonus so that the inframarginal consumers are left with no surplus:

\[ b^* = \frac{2C_g^*}{\beta k^*} \]

- This target is simply the \( k \), maximizing profit when the price and the bonus above are chosen
1st Period Price Setting

The monopolist sets price $p_1$ so to maximize the intertemporal profit.

- 1st period: Traditional margins vs demand trade-off
  \[ \pi_1(p_1) = n (1 - p_1)(1 - \beta)p_1 \]

- 2nd period: again trade-off margins vs demand, i.e.
  - on $g$: A higher price weakens competition among speakers, resulting in a lower bonus and thus a higher margin
  - on $\bar{\rho}$: Less potential investors means less investment, and thus less information (demand)
  - Specifically:
  \[ \pi_2(p_1) = \left[ \frac{\beta}{4} - \frac{Cg^*(D_{1}^{\text{inv}}(p_1))}{k^*} \right] n\bar{\rho}^* (D_{1}^{\text{inv}}(p_1)) \]
Analytical Results

Summary of Results

- Except when the cost of investment is too high or the proportion of uninformed negligible the execution of the program is optimal.

- **Optimal Strategy**: To fix a low price in the first period \( (p_1) \) so to extend the basin of potential investors. Then increase the price \( (p_2 > p_1) \) and use the bonus so to diffuse the information.

- **With respect to the benchmark** (where no program is run): each consumer is better off.

- **Between Periods**: Informed consumers can be better or worse off depending on their popularity. The non informed agents are always better off.

- Levels of prices and bonuses depend on the structure of the social network.
Once incentives are fixed, only the agents with a sufficiently high degree invest. **Which degree is targeted?** This depends from the structure of the network. We study two classes of networks:

- **Random Network** \(\Rightarrow\) Binomial distribution:

  \[
  f(k) = \binom{n-1}{k} \lambda^k (1 - \lambda)^{n-1-k};
  \]

  where \(\lambda \in (0, 1)\) is the probability of each link to exist.

- **Scale Free Network** \(\Rightarrow\) Power Law:

  \[
  f(k) = \frac{1/k^\gamma}{\sum_{k \in \mathbb{N}} (1/n^\gamma)}
  \]

  where \(\gamma \in (2, 3)\) for social networks.
Scale free examples: $\gamma = 2.2$ (left) and $\gamma = 2.6$ (right)

$$f(k) = \frac{1/k^\gamma}{\sum_{k\in N} (1/n^\gamma)}$$

(1) Increasing $\gamma$ the average node becomes less connected.
(2) There are individuals much more connected than the average.
Effects of the Network: Numerical Results

Targeted Degree (random vs scale-free)

random network: effect of $\lambda$ and of $\beta$ on $k^*$

scale-free network: effect of $\gamma$ and $\beta$ on $k^*$
Effects of the Network: Numerical Results

Targeted Degree (random vs scale-free)

random network: effect of $\lambda$ and of $\beta$ on $k^*$

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**The phase transition**

Profit for optimal $p_1$ as function of chosen cutoff $k$

![Graph showing the phase transition](image)

- Max $\pi (\gamma=2.6)$
- Max $\pi (\gamma=2.5)$

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Fixed degree distributions

Once incentives are fixed, only the agents with a sufficiently high degree invest. **Which degree is targeted?** This depends from the structure of the network. We study two classes of networks:

- **Random networks**: The half most popular part of the population invests, regardless of the network density.

- **Scale-free Networks** (with power-law degree distribution):
  - either **each** informed agent invests (i.e., the monopolist focus uniquely on *extending the demand*)
  - or **only** the most popular agents invest (i.e., the monopolist concentrates uniquely on *maximization of margins*).

⇒ Optimal choice essentially depends from the network density.
Summing up

**What we do** In presence of uninformed consumers, a monopolist aims at expanding demand exploiting his (limited) knowledge about consumers’ social network.

- He offers a reward to old buyers.
- Some of them convince peers to buy enjoying these rewards.

**Summary of the results**

- Except very specific cases, reward is optimal.
- Price for old customers is lowered by the introduction of the bonus. No effect on the price for uninformed consumers.
- Average degree is targeted in random networks, regardless of density.
- In power law distribution with a sharp phase transition exists between "margin dominated" strategy and "demand dominated strategy"
Further Developments

**Industrial Organization side:**

1. Oligopolistic Competition in Prices.
2. Entry Model.
3. Optimal Choice of Advertisement: comparison between *network based* and *mass media* advertisement.

**Network Side:**

1. Local Information of consumers about their social neighborhood.
2. Information Percolation: Using concepts of percolation in the network (like Campbell 2013) to introduce more elaborate "structure" in the network (clustering, assortativity,....).
Merci
Thank You
QUESTIONS?
Optimal Bonus (scale-free)

random network: effect of $\lambda$ and of $\beta$ on $b^*$

scale-free network: effect of $\gamma$ and of $\beta$ on $b^*$

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1st period Price

random network: effect of $\lambda$ and of $\beta$ on $p_1^*$

scale-free network: effect of $\gamma$ and of $\beta$ on $p_1^*$

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