The race for polluting permits

Thierry Bréchet and Susana Peralta
International markets for tradable emission permits (TEP) co-exist with national energy taxation. A firm trading emission permits in the international market also pays energy taxes in its host country, thus creating an interaction between the international TEP-market and national energy taxes. In this paper we model that interaction in a framework of a perfectly competitive international TEP-market, where heterogeneous firms trade their TEP endowments. National governments set energy taxes non-cooperatively so as to maximize fiscal revenue from energy and profit taxes. We identify the driving forces behind Nash equilibrium taxes. We show how they depend on the total amount of TEPs in the market, on firms' TEP-endowment and on the number of participating countries. We also show how energy taxation varies with the introduction of the market on a previously unregulated world. Finally, we highlight the fact that the TEP-market does not achieve abatement cost efficiency, despite its being perfectly competitive.

**Keywords**: tradable permits, fiscal competition, Kyoto protocol

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1 CORE, Louvain School of Management and Chair Lhoist Berghmans in Environmental Economics and Management, Université catholique de Louvain, Belgium. E-mail: brechet@core.ucl.ac.be

2 Faculdade de Economia, Universidade Nova de Lisboa, Portugal, CEPR and CORE, Université catholique de Louvain, Belgium. E-mail: peralta@fe.unl.pt

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1 Introduction

 Tradable emission permits (TEPs) have became a major policy instrument with the entering into force of two large-scale international markets. On the one hand, a worldwide market is emerging for greenhouse gases under the Kyoto protocol. On the other hand, the European Union Emission Trading Scheme (EU-ETS) already organizes trading for permits on carbon dioxide emissions among industrial firms within the EU. Yet, while being involved in such markets, each country still remains responsible for its own fiscal policy, notably the one on energy fuels, the main source of carbon dioxide emissions. Thus, we may expect strategic behavior, since a country’s fiscal policy can indirectly affect the equilibrium TEP price and, consequently, the compliance cost of the firms hosted in that country. Intriguingly, this interplay between TEPs and national taxes has been largely disregarded in the literature. The purpose of this article is to show that international markets for TEPs, such as under the Kyoto protocol or the EU-ETS, may deeply reshape national energy taxation.

 There are two main reasons for countries to reshape their national energy taxes when joining an international TEP-market. Firstly, by accepting a supranational market-based instrument the countries get rid of emission regulation (since the global cap on emissions is set at the international level). However, they experience a cutback in their tax revenues since firms will have to reduce their energy consumption in order to decrease carbon emissions. Energy taxes representing a substantial source of revenue for most countries, reactions can be expected. Secondly, the firms involved in the market for TEPs bear an emission abatement cost and each country may be tempted to help the firms it hosts by reducing energy taxes.

 In this paper we show that the two previous arguments can be handled in a setting of fiscal competition. We consider heterogeneous firms located in a set of countries. Firms use fuels for production, and carbon emissions stem from fuels combustion. The emissions are regulated by a perfectly competitive international market for tradable permits. The permits are freely given to the firms. Energy taxes levied on fuel consumption are under the jurisdiction of national governments, whose objective function combines national firms’ profit and energy tax revenue.

 We begin by looking at the TEP-market equilibrium. We show that national energy taxes depress the TEP price: countries have market power on the TEP-market, despite the competitive behavior of the firms. Such a market power is a source of externalities amongst the countries.

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1The Kyoto protocol came into force in February 2004 and the first commitment period will cover the years 2008 - 2012.
2The EU-ETS started in January 2005 and is operational since then.
3In 2005 the share of taxes in oil prices borne by the industry was 4.3% in Belgium, 4.8% in Japan, 4.9% in the United States, 13.6% in Germany, 20.3% in the UK and 46.2% in Italy (source: International Energy Agency [16]). Energy tax revenue amounts to 5.2% of the total tax revenue on average, for the EU-15 (or 2.4% of GDP), ranging from 3.5% in the Netherlands and Belgium to 6.3% in the UK, 7.7% in Italy, and 8.4% in Portugal (source: Eurostat [12]).
4As an example, a permit price of 30$ per ton of carbon dioxide roughly doubles the price of coal for the industry.
countries. Firstly, the tax base of other countries varies because the cost of energy changes, the so-called fiscal externality. Secondly, the net payments from national to foreign firms also change because the TEP price varies, the so-called pecuniary externality. Both effects are known in the capital tax competition literature (see, e.g., Wilson [24] for a survey).

In the analysis of the tax competition game, we pinpoint the three effects which drive the strategic interaction amongst countries and ultimately determine equilibrium taxes. We show that the higher the energy consumption, the higher the country’s incentive to tax (tax base affect); if national firms’ TEP demand is very responsive to the energy cost, taxation is discouraged (sensitivity effect); finally, taxation is influenced by the net position of the country on the market for permits and by its market power (terms of trade effect).

We then use these three effects to analyze several related issues. Firstly, we restrict country asymmetry in a way that allows us to assume away the terms of trade effect. We use this framework to study the impact of the total amount of TEPs traded in the market, the number of participating countries and the very introduction of the TEP-market in a previously unregulated world. Moreover, this simplified framework allows for a detailed analysis of equilibrium existence. We then reintroduce the terms of trade effect and show that larger permit endowments lead countries to tax less. We also highlight the important strategic difference between importing and exporting countries. The former tax less than the latter, and it may even happen that a net permit exporting country ends up subsidizing energy consumption. Finally, we point to one important consequence of considering strategic tax setting: the international market for tradable permits does not lead to the minimization of total emission abatement costs. This contradicts the usual textbook wisdom and is due to the unequal Nash equilibrium tax rates which result from country heterogeneity itself (the result of firm asymmetry).

Since the pioneer works of Dales [9] and Montgomery [19] the literature on tradable permits has followed many directions, some of them being related to our subject of concern.\(^5\)

For many years now the literature has paid attention to the interaction between distortive taxation and optimal environmental policy. This has been popularized under the (sometimes fuzzy) concept of double dividend. Goulder [13] provides an authoritative taxonomy of this concept. Much progress in our understanding of this interaction has occurred. In particular, Babiker et al. [1] use a CGE model to show that the interplay between carbon policies and pre-existing taxes can differ markedly across countries, depending on the levels of prior distortive taxes in an economy. Notably, they argue that climate policies under consideration will likely not provide a weak double dividend in a number of European countries. This strand of literature, however, ignores the fact that country-level policies may react to the implementation of international climate policies.

Actually, the idea that country-level regulation may strategically interact with the market for TEPs has been little discussed in the literature\(^6\) or it has been addressed in indirect or implicit ways (see Cropper and Oates [8], Coggins and Swinton [6], Bui [3]). To our knowledge, the only paper which builds the bridge between TEP markets and tax compe-

\(^5\)Hence, all the literature comparing the merits of policy instruments (prices versus quantities) is out of the scope of our analysis. We do not compare instruments, we analyze the implications of adding a new instrument (tradable permits) on pre-existing ones (energy and carbon taxes).

\(^6\)For example, Oates’s book [20] on environmental policy and fiscal federalism disregards this issue.
tition is the one by Santore et al. [23]. These authors examine the strategic behavior of state-level utility regulators in the context of the US federal trading system on sulfur oxide emissions. State-level regulators act independently of the federal authority by imposing pollution penalties on their own utilities. Like in our paper, Santore et al. show that emissions trading is not cost-efficient under fiscal competition.

As regards climate policy issues, the fact that actual TEP markets diverge from the standard textbook has been addressed recently by Babiker et al. [2]. In particular, these authors emphasize the fact that the gains from trading can be outweighed by secondary costs associated with prior tax distortions and market imperfections, providing an illustration with the CGE model EPPA. In the same spirit, Copeland and Taylor [7] use an international trade setting to show that the gains from trade can be ruined by terms of trade effects. This calls for further analyzing how markets for tradable emission permits may interplay with pre-existing policy instruments.

For that reason, the analysis carried out in our paper encompasses elements from both tax competition and TEP literatures in a comprehensive theoretical framework. Since countries tax energy and firms need permits to consume polluting energy, the TEP-market effectively creates a mobile tax base (the polluting permits). We model tax competition amongst countries which set energy taxes in a world where an international market for TEPs on carbon dioxide at the firm level (which parallels the Kyoto carbon market or the EU-ETS) is implemented. We replicate the basic property of such a setting, as Santore et al. [23], i.e. the cost-inefficiency stemming from asymmetric national taxes, but we go deeper in the analysis of the Nash equilibrium in taxes, thus providing a more comprehensive insight of that market for policy support. In particular, we highlight the importance of the net importing/exporting position of firms in a given country in the international TEP-market, which is reminiscent of the effect found in the capital tax competition literature by DePater and Myers [10] and Peralta and van Ypersele [22].

The remaining of the paper proceeds as follows. The next section sets out the model and preliminary results regarding the TEP-market. Section 3 discusses the main forces driving tax choices and discusses the existence of the Nash equilibrium. The effects of the total number of TEPs and the number of participating countries in equilibrium taxation are analyzed in Section 4, while section 5 is devoted to the comparison with autarky. We analyze the effect of firms’ TEP-endowment and discuss how it can influence the efficiency of the TEP-market in sections 6 and 7. Section 8 concludes. All proofs are relegated to the appendix.

2 The model

There are $F$ firms, indexed by $f$, located in $N$ countries, indexed by $c$. The set of firms is $I$, i.e., $I = \{1, 2, \ldots, F\}$ and the subset of firms locating in country $c$ is $I_c$. The number of firms in country $c$ is denoted $F_c$.


Naturally, $\bigcap_{c=1}^{N} I_c = \emptyset$.
2.1 Firms

Firms produce a consumption good under a constant returns to scale technology. We have in mind a short term analysis in which firms are immobile and use energy as the only variable input. By normalization, each unit of the good is produced using one unit of energy $e_{fc}$. The firm takes the good price, $\delta$, as given, and chooses the level of energy consumption that maximizes its profits. We may think of $\delta$ as the reservation price that consumers are ready to pay for the consumption good.\(^9\)

Energy combustion by firms causes pollution. By normalization, we assume that one unit of energy consumption yields one unit of pollution emission, which in turn corresponds to one polluting permit.\(^10\) Pollution is regulated with an international market for tradable emission permits (TEP) in which each firm is price-taker.\(^11\) We denote the price of permits by $\rho$. Each firm is endowed with $\bar{e}_f$ emission permits, so that it buys permits if it chooses to pollute more than $\bar{e}_f$, and sells a part of its endowment otherwise.\(^12\) Each country imposes an ad valorem national tax on the energy input, denoted $t_c$. The market for the energy input is perfectly competitive, with a price normalized to one. Hence, the tax-inclusive price of the energy input $e_{fc}$, of firm $f$ located in country $c$, which we shall call cost of energy, writes

$$p_c = \rho + t_c$$

The choice of $e_{fc}$ implies the following trade-off. By decreasing energy consumption, the firm decreases its energy tax bill, equal to $t_c e_{fc}$, and saves the (opportunity) cost of the emission permits, $\rho e_{fc}$. However, the firms’ technology is such that reducing emissions is costly. We model this feature using the pollution abatement cost function $C_f(e_{fc})$, such that $C'_f < 0$ and $C''_f > 0$.\(^{13}\) Firm $f$ located in country $c$ chooses the energy input, $e_{fc}$, to maximize its profit, given by

$$\Pi_{fc} = \delta e_{fc} - t_c e_{fc} - \rho(e_{fc} - \bar{e}_f) - C_f(e_{fc}),$$

Hence, $e_{fc}(p_c)$ is implicitly given by the first order condition,

$$\delta - C'_f(e_{fc}) = \rho + t_c \tag{1}$$

and we have that

$$\frac{de_{fc}}{dp_c} = e'_f = -\frac{1}{C''_f(e_{fc})} < 0,$$

\(^9\)Notice that if we model a downward sloping demand we must make an explicit hypothesis about the market structure of the final good market. This would complicate matters and take us away from our main motivation, i.e., the interaction between the market of tradable permits and energy taxation through their effect on firms.

\(^{10}\)For example, carbon dioxide emissions are strictly proportional to the carbon content of the fuels.

\(^{11}\)This is not a strong hypothesis, for the EU-ETS market covers more than 10,000 installations over 25 countries in the European Union.

\(^{12}\)We assume that TEP allocation is given for free. In the EU-ETS, free allocation must represent at least 95% of total allocation in the first phase (2005-2008), and at least 90% in the second phase (2008-2012). Assuming full or partial auctioning would not alter our results.

\(^{13}\)We further impose the following technical conditions: $C_f(0) \to \infty$, and $\lim_{e_{fc} \to 0} C''_f(e_{fc}) = 0$. 


hence, when the cost of energy increases, the firm pollutes less. In addition, we suppose that the demand for permits is (weakly) convex, that is,

$$\frac{d^2e_{fc}}{dp^2_c} \geq 0$$

This assumption corresponds to the intuitive property that, as the tax-inclusive price of emissions (i.e., the cost of energy) increases, emissions decrease, but at a decreasing rate.

### 2.2 Governments

The introduction of a market for tradable permits for a global pollutant sets a global emission ceiling, hence the environmental benefit is exogenous for each country and national taxes are no longer needed for regulating pollutant emissions. Moreover, since the consumer pays her reservation price of $\delta$ for each unit of the final good, the consumer surplus is equal to zero irrespective of the government’s fiscal choices. Hence, we concentrate on the interaction between the TEP-market and energy taxation, through the way they both affect firms’ profits. Admittedly, firms’ profits constitute a major concern of governments when it comes to fiscal decisions regarding energy. Such a concern was behind Sweden and Denmark’s July 2006 appeal to the European Commission for permission to cut their existing carbon taxes on the trading sector. After the implementation of the EU-ETS, those countries considered the co-existence of carbon taxes with emissions trading as double taxation, which could jeopardize their competitiveness. Moreover, many countries are used to subsidizing energy production or consumption. For example, the OECD survey on *Environmentally Harmful Subsidies* (OECD, [21]) reports calculations made by the U.S. Department of Energy’s Energy Information Administration (EIA). In the United States subsidies to primary energy in 1999 amounted to nearly USD 4 billion, of which 60% were tax expenditures.

Consequently we assume that the government maximizes a weighted sum of energy tax revenue and firms’s profits, the latter with a weight of $\gamma < 1$ in the objective function. The parameter $\gamma$ may be interpreted as the share of the firm owned by the country’s residents. Alternatively, it may be the profit tax rate, in which case the government is a revenue maximizer. The objective of the government of country $c$ is to choose the tax rate $t_c$ so as to maximize

$$U_c = \sum_{f \in I_c} \left( t_c e_{fc} + \gamma \Pi_{fc} \right), \quad (2)$$

where $0 < \gamma < 1$.

Each firm is characterized by a technology $C_f$ and a permit endowment $\bar{e}_f$. Country heterogeneity results from differences in the set of firms it hosts, namely: the number of firms ($F_c$), firms’ technology ($C_f(e_{fc})$, $f \in I_c$) and/or firms’ permit endowments ($\bar{e}_f$, $f \in I_c$).

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14 The assumption of revenue maximizing government is reasonable when residents care sufficiently about the public goods (see Kanbur and Keen, [18]). If the government maximizes a social welfare function with a redistributive objective then, under revenue constraints, it may be optimal to behave as a net revenue maximizer, if the welfare gains from higher net revenue are offset the losses due to a net revenue maximizing policy (see Chander and Wilde, [5]).

15 We further suppose that tax rates are bounded such that $t_c \in [\bar{t}, \bar{t}]$, with both bounds $\bar{t}$ and $\bar{t}$ finite.
For the purpose of our analysis it will be useful to restrain country heterogeneity. In particular we will make use of the following two definitions.

The first one is the counter-part in our TEP framework of the usual assumption of symmetric technology in capital tax competition models (based on a representative firm).

**Definition 1 (Technology-comparable countries)** Two countries $a$ and $b$ are technology-comparable if, for each firm in $I_a$, there is one and only one firm in $I_b$ with the same technology.

As already mentioned, terms of trade effects are one of the determinants of tax setting in our framework. In order to better understand the other effects at work and also to be more precise in the technicalities of the tax game it will prove useful to abstract from terms of trade in some parts of the analysis. With this objective in mind, we introduce the second definition.

**Definition 2 (Quasi-symmetric countries)** Two countries $a$ and $b$ are quasi-symmetric if they are technology comparable and the total permit endowment of firms in country $a$ is the same as in country $b$, i.e., $\sum_{f \in I_a} \bar{e}_f = \sum_{f \in I_b} \bar{e}_f$.

Notice that countries are not perfectly symmetric to the extent that the firms sharing the same technology do not necessarily have the same TEP-endowment. It is the total TEP-endowment that matters.

Given the central role played by the TEP-market in our analysis, it is useful to begin by analyzing some of its properties. We do so in the next subsection.

### 2.3 The market for tradable emission permits

It is out of the scope of this paper to discuss how tradable permits are allocated among firms.\(^{16}\) We thus take the allocation of permits to individual firms, $\bar{e}_f$, as exogenous. The emission cap in the global economy is then $\bar{E} = \sum_{f \in I} \bar{e}_f$.

Equilibrium in the TEP-market is defined as

$$\sum_{c=1}^{N} \sum_{f \in I_c} e_{fc}(p_c) = \sum_{f \in I} \bar{e}_f = \bar{E} \quad (3)$$

The following lemma states the existence and uniqueness of the equilibrium.

**Lemma 1** There exists a unique permit price $\rho(t, \bar{E})$ that clears the market. Furthermore, assuming that $\bar{E} < \sum_{c=1}^{N} \sum_{f \in I_c} e_{fc}(\bar{t})$, the equilibrium permit price is strictly positive.

\(^{16}\)For such an analysis, see e.g. Helm [14]. For the EU-ETS, national allocations plans have been decided at the country level, but in close coordination with the EU Commission. See Ellerman et al. [11] for an analysis of the whole procedure. See also the web site of the European Commission for up-to-date information on National Allocation Plans (http://ec.europa.eu/environment).
Notice that to ensure a positive permit price in equilibrium, it suffices to impose a sufficient restrictive global emission cap.

National energy taxes have an impact on this market since they influence national firms’ energy consumption. Totally differentiating (3), one obtains

$$\frac{d\rho(t, \bar{E})}{dt_C} = \rho_c = -\frac{\sum_{f \in I_c} e'_{fc}(p_c)}{\sum_{j=1}^{N} \sum_{f \in I_j} e'_{fj}(p_j)} < 0$$

(4)

The intuition works as follows. If a country increases the tax rate on the energy input, firms in that country reduce their energy consumption, thus demanding less permits. The aggregate demand for permits decreases, and, ceteris paribus, the price must decrease to clear the market. One should note that

$$\frac{de_{fc}}{dt_{fc}} = e'_{fc}(1 + \rho_c) = e'_{fc} \left( \sum_{j \neq c} \sum_{f \in I_j} e'_{fj}(p_j) \right) > 0$$

That is, if the tax in country $c$ increases, the TEP price decreases, but the cost of energy increases and the firms in the country pollute less. The lower TEP price leads to an increase in the emissions in other countries:

$$\frac{de_{fc}}{dt_j} = e'_{fc} \rho_j > 0$$

This means that a given country potentially has market power in the TEP-market, since it may influence the equilibrium price by changing its energy tax rate. To grasp the determinants of countries’ market power, it is useful to perform the following exercise. Suppose the demand for permits is iso-elastic, with the same elasticity for all firms in a given country. Let $\eta_c$ denote the elasticity in country $c$. Also, let $E_c = \sum_{f \in I_c} e_{fc}$. Then, we may rewrite (4) as

$$\rho_c = -\frac{\sum_{f \in I_c} \eta_{fc} e_{fc} e_{fc}}{\sum_{j=1}^{N} \sum_{f \in I_j} \eta_{fj} e_{fj} p_j} = -\frac{\eta_c E_c}{\sum_{j=1}^{N} \eta_j E_j}$$

The impact of a given country’s energy tax rate on the TEP price depends on two factors. On the one hand, the larger the number of firms and their demand for permits, the greater is the market power of the country. On the other hand, if firms react strongly to the cost of energy, they have a greater impact on the international market and the country has a higher market power. In the following section these effects will be further analyzed and we will show how they interact in equilibrium.

3 Preliminary results

This section is devoted to the formal analysis of the tax competition game. We begin by identifying the three effects which drive tax setting by national governments. We then proceed to the analysis of the existence of equilibrium. As it turns out, terms of trade effects make the existence analysis very cumbersome, so we chose to prove existence in the more restrictive context of quasi-symmetric countries.
3.1 Three key equilibrium effects

We solve for a Nash equilibrium in energy taxes. Country \( c \) chooses \( t_c \) so as to maximize \( U(t_c) \), given by (to save notation, we shall write \( p_c \) and \( \rho \) as a shorthand for \( \rho(t, \bar{E}) \), and \( p_c(t, \bar{E}) = \rho(t, \bar{E}) + t_c \), respectively)

\[
\sum_{f \in I_c} t_c e_{fc}(p_c) + \gamma [\delta e_{fc} - t_c e_{fc}(p_c) - \rho (e_{fc}(p_c) - \bar{e}_f) - C_f (e_{fc}(p_c))] 
\]

The first order condition writes

\[
(1 - \gamma) \sum_{f \in I_c} e_{fc}(p_c) + t_c \sum_{f \in I_c} \frac{de_{fc}(p_c)}{dt_c} - \gamma \rho_i \sum_{f \in I_c} (e_{fc}(p_c) - \bar{e}_f) = 0 
\]

The above expression identifies the determinants of tax setting by each individual country.

- The tax base effect: the higher the energy consumed by the firms in country \( c \), the higher the country’s incentive to tax. This is weighted down by \( \gamma \), for an increase in the energy tax rate decreases firms’ profits;

- The sensitivity effect: given that the tax base decreases with the tax rate, taxation is discouraged (when the tax rate is positive);

- The terms of trade effect: it depends both on the aggregate net exporting position of the country’s firms and on its market power.

It is interesting to notice that the tax base and sensitivity effects actually sum up to a Laffer curve, modified by \((1 - \gamma)\). Throughout the paper we shall make the natural hypothesis of concavity of the modified Laffer curve, i.e.,

**Assumption 1 (Concavity of the modified Laffer curve)**

\[
\frac{d}{dt_c} \left( (1 - \gamma) \sum_{f \in I_c} e_{fc}(p_c) + t_c \sum_{f \in I_c} \frac{de_{fc}(p_c)}{dt_c} \right) \leq 0
\]

Being slightly lose on the terminology, we say that a country exports permits if the (equilibrium) net demand for permits by the country’s firms is negative, i.e., \( \sum_{f \in I_c} (e_{fc}(p_c) - \bar{e}_f) < 0 \). We define a permit-importing country analogously. Notice that a permit-exporting country receives a transfer from importing ones, amounting to the net sales of permits by the country’s firms (discounted by \( \gamma \)). Since this transfer is increasing in \( \rho \) and the country can manipulate \( \rho \) using its energy tax, we may expect exporting countries to set lower taxes than importing ones (or even subsidize energy). This strategic manipulation of \( \rho \) is in both cases amplified by the market power of the country.
Besides its impact on the terms of trade effect, the market power of the country also influences the sensitivity one. To see this, notice that

\[
\frac{de_{fc}(pc)}{dt_c} = (1 + \rho_t) \epsilon'_f(p_c)
\]

Hence, a higher market power lowers the sensitivity of the tax base to the tax rate, for the TEP price absorbs a greater share of the tax increase.

Given (5), we have that the Nash equilibrium energy tax of country \(c\) is given by the following implicit function,

\[
\hat{t}_c = \frac{(1 - \gamma) \sum_{f \in I_c} \hat{e}_{fc}(\hat{p}_c) - \gamma \hat{p}_c \sum_{f \in I_c} (\hat{e}_{fc}(\hat{p}_c) - \bar{e}_f)}{(1 + \hat{p}_c) \sum_{f \in I_c} |\hat{e}'_{fc}(\hat{p}_c)| (1 + \hat{p}_c) |\hat{E}_c|} = \frac{(1 - \gamma) \hat{E}_c - \gamma \hat{p}_c (\hat{E}_c - \bar{E}_c)}{(1 + \hat{p}_c) |\hat{E}_c|} \tag{6}
\]

Where the \(\hat{x}\) denotes equilibrium value of the variable \(x\), and the following notation is used for convenience:

\[
\hat{E}_c = \sum_{f \in I_c} \hat{e}_{fc}(\hat{p}_c), \quad \bar{E}_c = \sum_{f \in I_c} \bar{e}_f, \quad \text{and} \quad \hat{\bar{E}}_c = \sum_{f \in I_c} \hat{e}'_{fc}(\hat{p}_c)
\]

We show in the Appendix that equilibrium taxes as implicitly defined by (6) are finite.\(^{17}\)

The type of strategic interaction amongst countries will play an important role in our analysis. Not surprisingly, the degree of strategic complementarity or substitutability between tax rates depends on the three key effects. Recall that taxes are strategic complements (resp. substitutes) if a given country’s optimal reaction to the other’s tax increase is to increase (resp. decrease) its tax rate. Suppose, say, country \(b\) increases its tax rate; then, the net TEP price declines, so the TEP demand in country \(a\) grows. Hence, the tax base effect is a source of strategic complementarity, while the sensitivity effect is a source of strategic substitutability. As regards the terms of trade effect, we must ignore the second order effect on \(\rho_{tc}\) which we cannot a priori sign. An increase in the home tax base implies that importing countries import more, while exporters export less. In both cases, they face an incentive to tax more heavily, so that the terms of trade effect is another source of strategic complementarity. Interestingly, the analysis of the quasi-symmetric case in the next subsection shows that strategic substitutability can arise, even if the sensitivity effect is limited by the assumed concavity of the modified Laffer curve.

3.2 Equilibrium with quasi-symmetric countries

The simultaneity of the three effects presented above makes the analysis of equilibrium existence cumbersome. This difficulty is usual in tax competition models with terms of trade effects; see, e.g., Peralta and van Ypersele [22]. In their paper about equilibrium existence in tax competition games, Laussel and Le Breton [17] have to suppose that capital owners are absent, so as to avoid terms of trade effects. Another way to avoid the terms of trade effect is

\(^{17}\)Hence, equilibrium taxes are between \(\bar{t}\) and \(\bar{t}\) for every country \(c\), with the two bounds appropriately defined.
to concentrate on quasi-symmetric countries (Definition 2). In this framework, Assumption 1 is sufficient to ensure the existence of a second-order locally consistent equilibrium (2-LCE). This is a weaker concept of equilibrium, where players use the second-order Taylor expansion of their payoff function as means to evaluate potential deviations, such that the equilibrium strategy profile is robust to small deviations by players, i.e., in the neighborhood of the 2-LCE strategy (see Bayindir-Upmann and Ziad [4] for a first usage of this equilibrium concept in tax competition games). Moreover, given the continuity of payoff functions, if a Nash equilibrium exists, then it coincides with the 2-LC equilibrium (Bayindir-Upmann and Ziad [4]).

Let us define the elasticities of the tax base ($\eta_{Ec}$) and the sensitivity ($\eta_{Ec}$) effects relative to the cost of energy, $p_c$, as follows,

$$\eta_{Ec} = -\frac{d \hat{E}_c}{dp_c} \frac{\hat{p}_c}{\hat{E}_c}$$

and

$$\eta_{Ec} = -\frac{d \hat{E}_c}{dp_c} \frac{\hat{p}_c}{\hat{E}_c}$$

We show in the Appendix that, under the set of equilibrium taxes, Assumption 1 is equivalent to the following,

**Assumption 2** The demand for permits evaluated at the equilibrium taxes respects

$$\frac{\eta_{Ec}}{\eta_{Ec}} \geq \frac{1 - \gamma}{2 - \gamma}$$

We must impose an upper bound on the elasticity of the sensitivity effect, $\eta_{Ec}$, relative to the elasticity of the tax base, $\eta_{Ec}$. In other words, when the cost of energy increases, the tax base shrinks (disincentive to tax) and it becomes less sensitive (an incentive to tax). If this latter effect is too strong relative to the former, the payoff function (the modified Laffer curve) is not locally concave.

As a preliminary result, the following lemma characterizes equilibrium in a setting of quasi-symmetric countries.

**Lemma 2** Under Assumption 2, a second-order locally consistent equilibrium (2-LCE) exists with all countries setting the same energy tax rate. Moreover, if a Nash equilibrium exists, then it coincides with the 2-LCE.

When countries are quasi-symmetric, no country is either a net exporter or a net importer of permits. Hence, they face exactly the same incentives to tax and set the same tax rates in equilibrium. It is important to recall that the agents in the TEP-market are the asymmetric firms, which consume different quantities of permits, in general different from their endowments. Hence, they trade permits in equilibrium. The equality of equilibrium taxes also implies equal tax bases which, used in (4), yields $\rho_{tc} = -1/N$. Using (6) and taking into account that the terms of trade effect vanishes, we obtain the implicit expression for equilibrium taxes when countries are quasi-symmetric,

$$\hat{t}_c = \frac{(1 - \gamma) \hat{E}_c}{(1 - \frac{1}{N}) |\hat{E}_c|}$$
The relationship between the elasticities of the tax base and the sensitivity effect drives not only the local concavity of the equilibrium, it also determines the strategic complementarity (or substitutability) among taxes, as well as the stability of the equilibrium. We show in the Appendix that

**Lemma 3** When \( \eta_{Ec} > \eta_{Ec} \), taxes are strategic complements, and the equilibrium is stable. When \( \frac{1}{2-\gamma} \eta_{Ec} < \eta_{Ec} < \eta_{Ec} \), taxes are strategic substitutes, and the equilibrium is stable if and only if \( \eta_{Ec} > \frac{1}{3/2-\gamma} \eta_{Ec} \).

If we recall our previous discussion on the determinants of strategic substitutability, and the fact that the terms of trade effect is absent from the quasi-symmetric countries setup, the intuition for the above result becomes straightforward. The optimal reaction of a given country to the other’s tax increase is to increase its tax rate if the variation of the tax base is stronger than that of its sensitivity. In that case, taxes are strategic complements. It is interesting to notice that we may have strategic substitutability even when the elasticity of the sensitivity effect is bounded to ensure concavity of the payoff functions.

### 4 Emission cap, number of countries and energy taxes

Having studied the existence on the 2-LC equilibrium, we may use our setup with quasi-symmetric countries to analyze the interplay between energy taxes and the market for tradable emission permits.

#### 4.1 Emission cap

The fundamental feature of the TEP-market is the global emission cap in the economy. The traditional wisdom about TEP-market is that a strengthening of the global emission cap unambiguously increases the price of permits, hence the cost of energy, in equilibrium. Does this still hold under strategic taxation?

One preliminary comment is in order. Suppose one fundamental of the model varies, changing the Nash equilibrium in taxes. Then, the equilibrium emission level per firm \( \hat{e}_{fc} \) will either increase, decrease, or remain constant, for all the firms in a given country \( c \). This is a consequence of (1) and the fact that all firms face the same \( p_c \). Firms’ heterogeneity comes from different energy consumption levels; however, they all vary in the same direction.

The following proposition deals with the effect on equilibrium taxes if the market becomes more constraining, that is, if the total amount of permits is reduced.

**Proposition 1** Strengthening the global emission cap increases the costs of energy. Furthermore, if taxes are strategic complements, energy taxes decrease while the TEP price increases; if taxes are strategic substitutes, energy taxes increase.

When the emission cap becomes more stringent, firms in each country will consume less permits in equilibrium, so its cost must increase. The shrunk tax base is a disincentive to tax, while the sensitivity effect is an incentive to tax. The tax rate variation depends on which of the effects dominates, which also drives the strategic complementarity or substitutability among taxes.
4.2 Number of countries

Let us now turn to an enlargement of the TEP-market. One or more countries may join the market. This is not a mere abstraction in our setting, since we are dealing with a market which is implemented amongst a set of countries and, logically countries outside this market may decide to join at some point.\textsuperscript{18} Regarding the new firms’ TEP-endowment, we impose the (empirically and politically reasonable) constraint that the market cannot become less constraining, in the sense of an increase in the average TEP-endowment per firm. Hence, new firms receive at most the (pre-existing) average endowment. We now check what happens when the firms receive the exact average endowment. If one is interested in a decrease in the average TEP-endowment, one may apply Proposition 1.

**Proposition 2** Suppose the number of countries increases. If the average TEP-endowment per firm is kept constant, energy taxes decrease, the TEP price increases and the cost of energy does not vary.

The intuition for this result rests solely on the sensitivity effect. Given that each individual country has a lower market power, the tax base is more sensitive, thus discouraging taxation.

We conclude this section by presenting the following corollary, which highlights the fact that, in both previous propositions (in the former under strategic complementarity only), the TEP price varies more than it would if taxes were not to react to TEP-market changes. We call this the amplifying effect of strategic energy taxation.

**Corollary 1 (Amplifying effect of strategic energy taxation)** If taxes are strategic complements, the TEP price reacts more to a resizing of the TEP-market under strategic taxation than if countries would not react by changing tax rates.

5 Comparison with autarky

Having understood the interaction of the TEP-market with energy taxation, we now turn to the analysis of the likely impact of introducing the market on a previously unregulated world. In the absence of a TEP-market, the optimal energy consumption by the firms is implicitly given by $\delta - C_f(e_{fc}) = t_c$. Countries maximize (2), with $\Pi_{fc} = \delta e_{fc} - t_c e_{fc} - C_f(e_{fc})$. The first order condition is

$$(1 - \gamma) \sum_{f \in I_c} e_{fc}(t_c) + t_c \sum_{f \in I_c} \frac{de_{fc}(t_c)}{dt_c} = 0 \quad (7)$$

using the fact that in the absence of the TEP-market $de_{fc}/dt_c = de_{fc}/dp_c =$, the autarkic energy tax $t_c^a$ is given by

$$t_c^a = (1 - \gamma) \frac{E_{fc}^a}{E_{fc}^c}$$

\textsuperscript{18}For example, this may represent the impact of the EU enlargement on the ETS.
where $E_a^c$ denotes total emissions under autarky (analogously for $E_c^a$).

Comparison of (7) with (5) identifies the most obvious consequence of introducing the market: the terms of trade effect shows up as an important determinant of energy taxation. We abstract from this effect by focusing on quasi-symmetric countries. The following proposition looks at the effect of introducing a non-constraining TEP-market in the sense that $E_a^c = \bar{E}$.

**Proposition 3** Suppose a non-constraining TEP-market is introduced. Then, the costs of energy remain constant and energy taxes go up.

Introducing a non-constraining TEP-market keeps the tax base at the same level. However, the sensitivity of the tax base declines, thanks to the partial absorption of the tax increase by the TEP-price. In other words, the cost of energy varies one-to-one with the tax rate in autarky, while it varies less under a TEP-market. This effect is responsible for a tax increase. The so-called run to the bottom result of the tax competition literature is sometimes interpreted as a decrease in the tax rate when borders are open, i.e., the tax base becomes mobile. Proposition 3 show that the exact opposite happens in our setting.

We may now combine the results in Propositions 1 to 3 into Figure 1 which relaxes the assumption of the non-constraining market.

### 6 Permit endowments and energy taxes

The analysis carried out so far has focused on quasi-symmetric countries i.e. the impact of terms of trade effects on equilibrium taxes was left out. This simplification allowed us to gain insights about the market fundamentals ($\bar{E}$ and $N$) and the effect of introducing the market.
We are now going to analyze the terms of trade effect by focusing on technology comparable countries (see Definition 1). As discussed above, the sufficient second order condition for equilibrium existence becomes cumbersome so we proceed by assuming the concavity of the payoff functions, i.e.,
\[
\frac{d^2U_c}{dt^2} \leq 0, \forall c
\]
that ensures equilibrium existence.

We can state the following result.

**Proposition 4** Suppose payoff functions are concave and a Nash equilibrium in energy tax rates exists. Then, if the total TEP-endowment of firms in country \( a \) is lower than in country \( b \), and tax rates are strategic complements, country \( a \) sets a higher energy tax than country \( b \). Moreover, a TEP-importing country sets a higher energy tax than a TEP-exporting one.

The intuition behind Proposition 4 rests on the terms of trade effect, which is an incentive to tax for an importer, and a disincentive to tax for an exporter. Since the country with the least endowed firms will (at a given tax rate) either import more, or export less, it has an incentive to set a higher tax rate. The comparison between importing and exporting countries depends on the sign of the terms of trade effect: positive for exporters, and negative for importers. For this result it is crucial that governments care for firms’ profits, i.e., \( \gamma \) must be bounded away from zero, for otherwise the net importing position of firms in the international TEP-market is irrelevant for the government.

Notice that when \( \gamma \) is sufficiently high, the terms of trade effect is very important for the government, and an exporting country may actually subsidize energy. This is always the case in the limit case where \( \gamma = 1 \) and the country does not care about energy fiscal revenue, only about firms’ profits.

According to Proposition 4, country ranking in terms of energy taxes is the inverse of the total firms’ TEP-endowment ranking. The same argument as above allows us to answer one related question, namely, how does the equilibrium change if we switch the endowments of the firms in two different countries? We have the following result.

**Corollary 2** Suppose a reallocation of TEP endowments such that firms in country \( a \) get more permits and those in country \( b \) less, the total amount being kept unchanged. Then the energy tax decreases in country \( a \) and increases in country \( b \), if taxes are strategic complements.

Note that this result goes through independently of the distribution of firms’ endowments in each country. All that matters is the change in the aggregate TEP-endowment of the country.

### 7 Cost-inefficiency of the market for tradable permits

In setting up a market for permits, the aim is to curb pollution in a *cost-efficient* way. This property of TEPs is widely put forward in the literature (since Montgomery [19]) and
generally used as an argument by its advocates (see e.g. IEA [15]). This property, stated without strategic fiscal interactions, is clearly questionable in our more realistic setting. The allocation of pollution emission is cost-efficient if, for a given quantity of total pollution, the aggregate emission abatement cost is minimized. Formally, a cost-efficient pollution abatement is the solution of the following problem

\[
\min_{\{e_{fc}\}_{f \in I}} \sum_{f \in I} C_f(e_{fc}) \quad \text{s.t.} \quad \sum_{f \in I} e_{fc} = \bar{E}
\]

Hence, cost-efficiency is attained when \( C'_f(e_{fc}) = \kappa \), \( \forall f \in I_c \), where \( \kappa \) is a finite negative number. We show that, in the case of an international market with fiscal spillovers, this property does not hold anymore. Recalling that the set \( I_c \) is different among countries, in terms of the number of firms, their technologies and TEP-endowment, we state the following result.

**Proposition 5** When countries set energy taxes non-cooperatively, equilibrium taxes are generically different. Hence, an international market for tradable emission permits is not cost-efficient.

Proposition 5 relies only on the non-cooperative behavior of asymmetric countries. The asymmetry amongst countries stems from firm heterogeneity, the very reason why one would like to implement a TEP-market as a means to achieve cost efficiency in the first place. This result shows that the efficiency gains of introducing a market are not realized if the power to tax energy inputs is left to the national initiative. This is a worrisome result, in that empirical evidence confirms strong cross-country differences in energy taxation, which may be taken as evidence of asymmetries amongst firms locating in each country. This asymmetry will likely lead countries to set different taxes under the international TEP-market. Moreover, the relative importance of energy tax revenue in some countries makes it unlikely that all countries would be ready to give up their fiscal autonomy in this matter.

It is important to notice that country asymmetry per se does not imply asymmetric taxation, rather it is the non-cooperative behavior that does so. If countries were to cooperate to minimize total abatement cost they would set equal tax rates. If, instead, they had the objective of maximizing fiscal revenue, they would set \( t_c = \bar{t}, \forall c \). This is the usual result in models of fiscal competition where the tax base is inelastic internationally, which the case here since \( \sum_{c=1}^{N} \sum_{f \in I_c} e_{fc} = \bar{E} \).

## 8 Conclusion

With the background of the establishment of several environmental agreements at the international level, we analyze the interaction between an international market for tradable polluting permits and national energy taxes. The TEP-market introduces a mobile tax base between countries (the polluting permits), hence it creates room for fiscal competition.

19\(^{19}\)Indeed, suppose that there are no energy taxes, i.e., \( t_c = 0 \) in all countries. Then the choice of the energy input by the firms is such that \( \delta - C'_f(e_{fc}) = \rho \) and cost-efficiency is achieved.

20\(^{20}\)Unless we restrict country asymmetry, as in Lemma 2.
Our analysis highlights the effects that drive tax choices: the tax base, the elasticity and the terms of trade effects. The interplay among these effects allows us to characterize how equilibrium energy taxes depend on the fundamentals of the market: the total emission cap, the number of countries which join the market, and firms’ TEP-endowments. We also look at the impact of introducing the market on a previously autarkic world. Some of our results hinge on the nature of the strategic interaction amongst countries, i.e., whether taxes are strategic complements or substitutes. We show that this depends on the relative elasticities of the tax base and elasticity effects.

Increasing the restrictiveness of the market (i.e., decreasing the emission cap) leads to a higher costs of energy, while taxes may either increase or decrease, depending on whether they are strategic substitutes or complements, respectively. On the other hand, if new countries join the market, then the cost of energy goes up while energy taxes decrease unambiguously. The introduction of the market has the opposite effect on energy taxation, i.e., countries increase their energy taxes because the TEP price partially absorbs tax increases, thus decreasing tax base sensitivity. Looking at firms’ TEP-endowment amounts to analyzing the impact of the terms of trade effect. We show that, under strategic complementarity, the country with the more endowed firms sets a lower energy and that importers tax energy more heavily than exporters. One interesting result of strategic taxation is that there is a amplifying effect on the TEP price, which in all cases varies more than what it would in the absence of taxes (when taxes are strategic complements).

Finally, considering strategic taxation leads us to question the usual argument in favor of an international TEP-market, namely, that it achieves emission reduction in a way that minimizes total abatement cost. This implies that the TEP-market only leads to cost effectiveness if it is accompanied by a tax harmonization policy. However, the political difficulties of implementing such an harmonization policy are at the very source of the creation of the international TEP-market.

One could envisage extending the model in several directions. Introducing a final good market is likely to discourage energy taxation (in that this causes a further distortion) in a symmetric way across countries, so that our results would remain qualitatively valid. Such an extension would call for a rewriting of the countries’ payoff function. The three effects highlighted in our analysis would still appear and would drive the results in a qualitatively similar manner. Finally, one could think of introducing a market for the energy input. If the market is perfectly competitive, our results go through unchanged. If instead it is an imperfectly competitive market, then the tax effects are likely to be mitigated. Indeed, say, a tax increase leads to a reduction in the demand for energy, hence decreasing its equilibrium price. This absorbs a part of the tax increase and leads to a mitigation of its effects, hence a lower incentive for the strategic use of taxes on behalf of the countries.

References


Appendix

Proof of Lemma 1 Recall the equilibrium condition on the market for TEPs (3). Since the left-hand side of (3) is strictly decreasing in $\rho$, for each tax vector $t = \{t_1, \ldots, t_N\}$, and the global permits supply $\bar{E}$ being given, there exists a unique permit price $\rho(t, \bar{E})$ satisfying (3). To see that $\rho(t, \bar{E}) > 0$, notice that

$$\frac{d\rho(t, \bar{E})}{dt_c} = \rho_{t_c} = -\frac{\sum_{f \in I_c} e_{fc}(p_c)}{\sum_{j=1}^{N} \sum_{f \in I_j} e_{fj}(p_j)} < 0$$

hence the lowest equilibrium $\rho(t, \bar{E})$ arises when $t_c = \bar{t}, \forall c$. This fact, together with the assumption that $\sum_{c=1}^{N} \sum_{f \in I_c} e_{fc}(\bar{t}) > \bar{E}$ ensures $\rho(t, \bar{E}) > 0$ when $t_c = \bar{t}, \forall c$, hence it must also be positive for any other possible tax vector. □

Proof that tax rates as defined by (6) are finite From (6), it follows that, in general, when $I_c \neq I_d$ for $c \neq d$, then $\hat{t}_c \neq \hat{t}_d$. In order to ensure finite equilibrium tax rates, we impose the following conditions on the cost function: $C_f(0) \to \infty$, to ensure that the demand for permits has a slope of 0 when $p_c$ goes to $\infty$, and $\lim_{e_{fc} \to 0} C''_f(e_{fc}) = 0$.

Suppose, to the contrary, that:

- $\hat{t}_c \to -\infty$. Then, country $c$ absorbs all the permit supply, i.e., $\sum_{f \in I_c} e_{fc} = \bar{E}$ and we have that $\sum_{f \in I_c} e'_{fc} = -\phi$, with $\phi > 0$ and finite. In addition, all the other countries $j$ have $\sum_{f \in I_j} e'_{fj} = 0$ and $\sum_{f \in I_j} e'_{fj} \to 0$, therefore $\rho_{t_c} \to 1$. Hence, (5) reads

$$(1 - \gamma)\bar{E} + \hat{t}_c(1 + 1)(-\phi) + \gamma \left(\bar{E} - \sum_{f \in I_c} \bar{e}_f\right) > 0,$$
contradicting the fact that $\hat{t}_c$ is an equilibrium tax.

- $\hat{t}_c \to \infty$. Then, demand for permits in country $c$ falls to $\sum_{f \in I_c} \hat{e}_{fc} = 0$ and $\sum_{f \in I_c} C_f(\hat{e}_{fc}) \to \infty$. Hence, the country’s utility level is given by
  \[
  \sum_{f \in I_c} \hat{t}_c 0 + \gamma \left( \alpha 0 - t_c 0 + \rho \bar{e}_f - \infty \right) \to -\infty
  \]
  and the country cannot be optimizing. It is trivial to check that it can do better with a tax $t_c = 0$ and a finite demand for permits (hence, a finite abatement cost).

Hence, it is trivial to chose $\underline{t}$ and $\bar{t}$ to ensure that equilibrium taxes belong to the interval $[\underline{t}, \bar{t}]$. □

Proof of Lemma 2 We proceed in two steps. First, we show that all countries set the same tax rate in equilibrium; then, we show that Assumption 1 ensures the existence of a 2-LCE.

(i) Suppose countries do not set the same tax rate in equilibrium and take two countries $a$ and $b$ with $\hat{t}_a \neq \hat{t}_b$. Then, $\hat{p}_a \neq \hat{p}_b$ and, for each pair of firms with technology $f$, we have that $\hat{e}_{fa} \neq \hat{e}_{fb}$, implying $\hat{E}_a \neq \hat{E}_b$. Since $\hat{E}_a = \hat{E}_b$, it follows that $\hat{E}_a - \hat{E}_a \neq \hat{E}_b - \hat{E}_b$. Using the fact that $\sum_{c=1}^{N} \hat{E}_c - \hat{E}_c = 0$, it follows that there exist at least two countries $a$ and $b$ such that $\hat{E}_a - \hat{E}_a < 0 < \hat{E}_b - \hat{E}_b$, implying $\hat{E}_b > \hat{E}_a$, or $\hat{t}_b < \hat{t}_a$. Since quasi-symmetric countries are also technology-comparable, Proposition 2 and the fact that Assumption 1 is equivalent to Assumption 2 in the case of quasi-symmetric countries together imply that the TEP-exporter country $a$ sets a lower energy tax than the TEP-importer country $b$, and we reach a contradiction.

(ii) Equality of equilibrium taxes implies that $\hat{E}_c = \hat{E}_d, \forall c, d$, and equilibrium in the permit market then ensures that $\hat{E}_c = \hat{E}_d = \hat{E}/N, \forall c, d$. Under the set of equilibrium taxes, the first order condition for each country $c$ reads
  \[
  (1 - \gamma)\hat{E}_c + \hat{t}_c \hat{E}_c (1 - \frac{1}{N}),
  \]
  from which
  \[
  \hat{t}_c = \frac{(1 - \gamma)\hat{E}_c}{(1 - \frac{1}{N}) |\hat{E}_c|}. \tag{10}
  \]
  Using the fact that $\hat{p}_{t_c} = -1/N$ under the equilibrium tax vector, the second order condition reads
  \[
  \left( 1 - \frac{1}{N} \right) \left( (2 - \gamma)\hat{E}_c + \hat{t}_c \frac{d\hat{E}_c}{d\hat{t}_c} \right),
  \]
  which, evaluated at $\hat{t}_c$ as given by (10), and after straightforward manipulation, yields
  \[
  \left( 1 - \frac{1}{N} \right) \frac{\hat{E}_c}{\hat{p}_c} \left( -(2 - \gamma)\eta_{E_c} + (1 - \gamma)\eta_{E_c} \right)
  \]
where
\[
\eta_{E_c} = -\frac{d\hat{E}_c}{dp_c} \hat{p}_c \hat{E}_c \geq 0 \quad \text{and} \quad \eta_{\ell_c} = -\frac{d\hat{E}_c}{dp_c} \hat{p}_c \hat{E}_c \geq 0
\]
which is negative under Assumption 2. □

**Proof of Lemma 3**  Taxes are strategic complements at the Nash equilibrium if
\[
d^{2}U_c = -\frac{1}{N} \left( (1 - \gamma)\hat{E}_c + \ell_c \frac{d^2\hat{E}_c}{d\hat{p}_c} \left( 1 - \frac{1}{N} \right) \right) = \frac{1 - \gamma}{N} \hat{E}_c \left( \eta_{E_c} - \eta_{\ell_c} > 0, \right.
\]
otherwise they are strategic substitutes.

An equilibrium is stable if reaction functions are contractions, i.e., \(\sum_{j \neq c} \left| \frac{\partial \ell_j}{\partial \hat{E}_j} \right| \leq \phi, \forall c\), for some \(\phi < 1\). Now,
\[
\sum_{j \neq c} \frac{\partial \ell_c}{\partial \hat{E}_j} = -\left( N - 1 \right) \frac{d^2U_c}{d^2\hat{E}_c} \frac{d^2U_c}{d^2\hat{p}_c} = \frac{\eta_{E_c} - \eta_{\ell_c}}{2 - \gamma \eta_{E_c} - \eta_{\ell_c}}, \quad (11)
\]
where the denominator is positive under Assumption 2. When taxes are strategic complements \((\eta_{E_c} - \eta_{\ell_c} > 0)\), the fact that \((2 - \gamma)/(1 - \gamma) > 1\) ensures that expression in (11) is below unity. When taxes are strategic substitutes, the numerator is negative so that reaction functions are contractions when
\[
\eta_{\ell_c} - \eta_{E_c} < \frac{2 - \gamma}{1 - \gamma} \eta_{E_c} - \eta_{\ell_c}
\]
and simple algebra yields the lemma. □

**Proof of Proposition 1**  Notice that
\[
\frac{d}{dp_c} \left( \frac{\hat{E}_c}{\hat{E}_c} \right) = \frac{\hat{E}_c d\hat{E}_c / dp_c - \hat{E}_c d\hat{E}_c / dp_c}{(\hat{E}_c)^2} = 1 - \frac{\eta_{E_c}}{\eta_{\ell_c}}
\]
Hence, the ratio \(\hat{E}_c / \hat{E}_c\) is increasing in \(\hat{p}_c\) if taxes are strategic complements; otherwise, it is decreasing. The ratio \(\hat{E}_c / |\hat{E}_c|\) is decreasing if and only if \(\hat{E}_c / \hat{E}_c\) is increasing.

Given that \(\hat{E}_c = \hat{E} / N\), when \(\hat{E}\) decreases, \(\hat{E}_c = \hat{E}_c\) decreases, implying that the price of energy in each country must increase. By (12), if taxes are strategic complements, \(\hat{E}_c / |\hat{E}_c|\) decreases, hence \(\hat{\ell}_c\) decreases. The fact that \(\hat{\ell}_c\) decreases and \(\hat{p}_c\) increases implies that \(\rho\) increases. An analogous reasoning applies to the case of strategic substitutes. □

**Proof of Proposition 2**  Supposing that \(\hat{E} / N\) rests unchanged, i.e., the endowment per firm does not change, then \(\hat{E}_C / |\hat{E}_c|\) remains constant and so does \(\hat{p}_c\). The only effect on \(\hat{\ell}_c\) comes through \(N\),
\[
\frac{d\hat{\ell}_c}{dN} = \left( 1 - \gamma \right) \frac{1}{|\hat{E}_c|} \frac{d}{dN} \left( \frac{1}{1 - \gamma} \right) < 0
\]
Moreover, since \(\hat{p}_c\) is constant, \(\hat{\rho}\) increases. □
Proof of Proposition 3  Let denote $t^a_c$ the autarky tax, and analogously for other equilibrium variables. The fact that the market is not constraining implies that $\bar{E} = \sum_{c=1}^{N} E^a_c$, and obviously $E^a_c = \hat{E}_c$, so that

\[ t^a_c = \left( 1 - \frac{1}{N} \right) \hat{t}_c < \hat{t}_c \square \]

Proof of Proposition 4

(i) For any two countries $a$ and $b$ and taking the set $\hat{t}_{i\notin \{a,b\}} = \{ \hat{t}_i, i \notin \{a,b\} \}$ as given, we may rewrite (5) for countries $a$ and $b$

\[ \Phi(\hat{t}_a, \bar{E}_a, \hat{t}_b, \hat{t}_{i\notin \{a,b\}}) = (1 - \gamma) \bar{E}_a + \hat{t}_a \bar{E}_a (1 + \hat{\rho}_a) - \gamma \hat{\rho}_a \left( \bar{E}_a - \bar{E}_b \right) = 0 \]
\[ \Phi(\hat{t}_b, \bar{E}_b, \hat{t}_a, \hat{t}_{i\notin \{a,b\}}) = (1 - \gamma) \bar{E}_b + \hat{t}_b \bar{E}_b (1 + \hat{\rho}_b) - \gamma \hat{\rho}_b \left( \bar{E}_b - \bar{E}_a \right) = 0 \]

Now take $\bar{E}_a < \bar{E}_b$, and suppose that $\hat{t}_a < \hat{t}_b$. Then, we have that $\hat{E}_a > \bar{E}_b$ and, by definition of the Nash Equilibrium, $\Phi(\hat{t}_a, \bar{E}_a, \hat{t}_b, \hat{t}_{i\notin \{a,b\}}) = 0$. Now using, successively, concavity of the payoff functions, strategic complementarity, and, lastly, the fact that $\partial \Phi(t_c, \bar{E}_c) / \partial \bar{E}_c < 0$ we may write

\[ 0 = \Phi(\hat{t}_a, \bar{E}_a, \hat{t}_b, \hat{t}_{i\notin \{a,b\}}) > \Phi(\hat{t}_a, \bar{E}_a, \hat{t}_b, \hat{t}_{i\notin \{a,b\}}) > \Phi(\hat{t}_b, \bar{E}_a, \hat{t}_a, \hat{t}_{i\notin \{a,b\}}) > \Phi(\hat{t}_b, \bar{E}_a, \hat{t}_a, \hat{t}_{i\notin \{a,b\}}) = 0 \]

And a contradiction is reached.

(ii) Suppose that $a$ is the importing and $b$ the exporting country, such that $\hat{E}_a - \bar{E}_a > 0 > \bar{E}_b - \bar{E}_b$. Notice that from $\Phi(\hat{t}_a, \bar{E}_a, \hat{t}_b, \hat{t}_{i\notin \{a,b\}}) = 0$ one has

\[ (1 - \gamma) \bar{E}_a + \hat{t}_a \bar{E}_a (1 + \hat{\rho}_a) < 0 \]

and from $\Phi(\hat{t}_b, \bar{E}_b, \hat{t}_a, \hat{t}_{i\notin \{a,b\}}) = 0$ one has

\[ (1 - \gamma) \bar{E}_b + \hat{t}_b \bar{E}_b (1 + \hat{\rho}_b) > 0 \]

The assumption that $\hat{t}_a < \hat{t}_b$, together with Assumption 1, imply that $(1 - \gamma) \bar{E}_a + \hat{t}_a \bar{E}_a (1 + \hat{\rho}_a) > (1 - \gamma) \bar{E}_b + \hat{t}_b \bar{E}_b (1 + \hat{\rho}_b)$, and we reach a contradiction.\square

Proof of Proposition 5  Using (6) and (1), it is immediate that cost-efficiency is not attained.\square