Property rights with biological spillovers: when Hardin meets Meade

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Abstract

In an overlapping generations setup we address the issue of the optimal number of property rights to allocate over a natural resource when the goal is to maximize the stock of the natural resource at the steady state. We assume that the effect of the property rights regime on the evolution of the resource is twofold: through biological spillovers and through monitoring costs. Property rights are assigned to local communities, which can decide whether to cooperate or not. The outcome in the strategic setting is hence compared to the one in the cooperative setup. A fiscal policy able to decentralize the cooperative outcome is studied.

Keywords: overlapping generations, resource management, common pool resource, spatial interdependence, strategic behaviour, cooperative behaviour.

JEL Classification: H21, K11, Q20

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1 Introduction

That unlimited open access to public resources leads to the “tragedy of the commons” (see Hardin [16]) is a well-known result. Hardin conceptualizes the idea of the tragedy of the commons by means of a grazing example, assuming that the pasture is open to all and that there is no cooperation among the users. Each herder will therefore try to keep as many livestock as possible. Since the individual benefit for each herder of adding one animal is larger than the social cost of overgrazing, she will continue to add one more animal, which finally will bring ruin to all. In order to internalize the externality stemming from the over-exploitation of public resources, Hardin suggests two different options: either selling them off as private property, or keeping them as public property and allocating the right to enter them. However, there is an important, unanswered question in Hardin’s paper: how many property rights should be allocated?

Hardin’s paper has been used to explain resource degradation and led national governments to put restrictions on the local systems of resource management. For instance, since the mid-80s many developing countries in Africa, Asia and Latin America have introduced some forms of decentralized forest management (see Baland and Platteau [3]). Advocates of decentralization justify decentralization reforms on the grounds that the increased efficiency, equity and inclusion that should arise from decentralization result in better and more sustainable management. Despite these claims, many case studies suggest that decentralization efforts often ended up with deforestation or depletion of natural resources. Ostrom [23] shows that under certain conditions, when communities are given the right to self-organise they can democratically govern themselves to preserve the environment.

Therefore, it seems of some interest to investigate how the allocation of property rights advocated by Hardin as a solution to over-exploitation of public, natural resources can be conciliated with the empirical evidence that decentralization often led to ecological exhaustion. In other words, to move beyond Hardin’s theory, we need to identify key variables present or absent in particular settings, so as to understand successes and failures.

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1 It is important to stress since the beginning that Hardin [16] gets the idea for his basic argument that common property systems allow individuals to benefit at a cost to the community from the Oxford economist, the Rev William Forster Lloyd who in 1833 in his book *Two Lectures on the Checks of Population* writes: “If a person puts more cattle into his own field, the amount of the subsistence which they consume is all deducted from that which was at the command of his original, […] but if he puts more cattle on a common, the food which they consume forms a deduction which is shared between all the cattle, as well that of others as his own,stock”. Hardin takes Lloyd’s observation and transforms it by injecting the added ingredient of “tragic” inevitability: “Freedom in a commons brings ruin to all”. When Hardin writes his article, the consequences arising from the open access to a natural resource have already been analyzed by Gordon [15], in the field of the overexploitation of fisheries. Gordon’s article is more rigorous than Hardin’s, however Hardin’s article has become one of the most cited (and criticized) academic papers ever published. Part of the success of his article is that it uses pithy, nontechnical examples and a controversial argument concerning human population growth.

2 Decentralization is usually defined as a process by which more authority and control over resources are given to lower levels of government.
failures (Basurto and Omstrom [5]).

One element that seems to be important when analyzing resource management is the existence of spatial (or biological) externalities across property rights’ owners. Meade [21] introduces the concept of "positive externality" in the presence of a well-defined property rights regime: he imagines a beekeeper who lives next to an apple orchard. In the course of making honey, his bees provide a benefit in terms of pollination to the orchard owner next to him. At the same time, however, the production of additional apples would also make more nectar available for the bees, thus resulting in a greater production of honey. Meade’s example, in which the crops of the farmer depend on the number of bees owned by the beekeeper and vice versa, makes clear the existence of spillovers across property rights’ owners.\footnote{This concept has been further developed both in fish and forest management. In Datta and Mirman [11] and Fischer and Mirman [13], the growth rate of a natural resource depends, \textit{inter alia}, on how the resource is managed; and the \textit{common property regime} defined over the resource affects the evolution of the resource itself.}\footnote{While in forest management, mostly theoretical studies have suggested that individual forest landowners receive amenity benefits from adjacent stands (Bowes and Krutilla [6]; Swallow and Wear [27]; Amacher \textit{et al.} [1]). Recently, Vokoun \textit{et al.} [28] use a landowner survey data to examine incentives to cooperate concerning joint forest management and coordination of harvesting. Their results confirm that cross parcel externalities modeled in theoretical works do in fact exist. They also find spatial factors to be particularly important to induce landowners to participate in cooperative agreements.}

How can we take advantage of both Hardin’s theory on property rights and the existence of spatial spillovers amongst private owners (such that the boundaries of private titles and the boundaries of the impacts of resource use may not coincide) to understand successes and failures in resource management? In fact, we could assert that defining property rights and dividing the natural resource in fenced plots plays an important role in influencing the evolution of the resource itself. On the one hand it reduces the overall maintenance costs; but on the other hand it makes biological spillovers among plots less likely to occur, thus having a negative impact on the natural growth rate of the resource. The major issue is then to define the optimal number of plots that allows to maximize the stock of the natural resource. Of course, this analysis must take into account the sensitivity of the natural resource to the property rights regime defined over it. There are natural resources which are extremely reactive to the way the resource itself is split, so that splitting the resource is always detrimental for its natural evolution. We call this \textit{Meade effect}. On the contrary, there are other resources whose natural growth rate is affected to a smaller extent by the property rights regime defined over them. We call this \textit{Hardin effect}.

We address this issue in an overlapping generations (OLG) framework: we assume that a central government is entitled to assign property rights over a natural resource: at
each time \( t \) the resource is divided into \( D_t \) plots. Each plot is assigned to a community that must manage it. Within each community, at each time \( t \) a representative young and a representative old agent exist: the young agent harvests the resource, while the old agent owns the capital. Both the natural resource and the capital are used for production. We compare the result obtained in the strategic setting to that arising in a cooperative framework. We show that the gain from cooperation is always remarkable. A fiscal policy able to decentralize the cooperative outcome has then been studied and some policy implications have been stressed. It has also been shown that cooperation alone is not enough to maximize the stock of the resource in the steady state. However, assuming that the number of property rights is fixed, by means of a subsidy and for any given \( D \) it is possible to replicate in the strategic setup the maximum level of the natural resource reached in the cooperative framework. This article is organized as follows: in Section 2 we sketch the model. In Section 3 we analyze the extraction problem in a Cournot-Nash framework. In Section 4 we characterize the same problem in a cooperative framework. Section 5 describes the fiscal policy needed to decentralize the cooperative outcome. Section 6 concludes.

2 The model

We assume that a central government is entitled to assign property rights over a natural resource. At each time \( t \), the stock of the resource is split amongst \( D_t \geq 1 \) plots. The enforced property rights regime is assumed to influence the evolution of the resource through two different channels. First of all, the way the resource is split affects the natural growth rate: the higher is the number of plots, the less likely are the biological spillovers amongst plots and the lower is the growth rate of the resource. Second, the higher the number of plots, the lower are maintenance (monitoring) costs. The interplay of these two effects, combined with agents’ decisions, will shape the evolution of the resource at the steady state. According to the way a resource reacts to the interplay of these effects, we can define the Meade effect and the Hardin effect: in the first case, the resource is extremely reactive to the enforced property rights regime, so that splitting it into plots is always detrimental for the evolution of the resource itself. In the second case, the resource is less reactive to the way property rights are allocated.\(^5\)

The proper management of a renewable resource must be based on the knowledge of its population dynamics, which can be furthered through the use of mathematical models. The evolution over time of a renewable resource is generally modeled through the use of a differential equation using a logistic growth function with simple density-dependence. Despite being a good, stylized approximation of population dynamics of renewable resources, a number of factors which influence actual growth patterns are ignored in this model, including the age structure of the resource, random influences, as

\(^5\)Copeland and Taylor [10], in a paper where they study the degree to which countries escape the tragedy of the commons, define as Hardin economies those countries that have limited enforced power relative to their overcapacity and always exhibit de facto open access in steady state.
well as the spatial structure of the resource (see Perman [25]). According to a logistic function specification, everything else being equal, an area being twice as big as another one should have a population of twice the size, i.e. the population density should be the same, which need not be true in reality (see Andrén [2]).

In this work we will use a different specification. Let $X_t$ be the stock of the renewable resource at time $t$, and $D_t \geq 1$ the number of plots at time $t$. Each plot is then assigned to a community that has property rights over it and must manage it. Communities are assumed to be identical. The rule governing the natural resource dynamics is given by

$$X_{t+1} = [X_t(1 + b(D_t)) - Y_t]^\alpha$$

where $Y_t$ is total harvesting at time $t$, $b(D_t)$ is the growth rate of the natural resource at time $t$, which depends negatively on the number of plots at time $t$, $D_t$, and $\alpha \in (0, 1)$.

Equation (1) departs from differential equations using logistic growth function in order to include the spatial structure of the resource when studying its dynamics. The spatial structure of natural resources refers to the relative spatial arrangements of patches into which the resource is split and interconnection between them (Baskent and Jordan [4]). Since the mid-70s the ecological literature has pointed out the important role played by the spatial structure of renewable resources in conservation strategies. In 1975 the ecologist Jared Diamond [12] analyzes the problem of choosing a size and shape of natural reserves that would minimize the loss of species. He lists several considerations, including economic and social ones, and among the rules proposed is that single large preserves would be better at preserving species than a set of smaller, separate reserves of the same total area. If subdivision cannot be avoided then corridors should be left to connect areas so that populations could move between them. In 1980, the World Conservation Strategy [17] reproduces Diamond’s suggested principles for nature reserve design, with the general recommendation that a large reserve is better than a small one. In the '90s the concept of spatial structure and landscape ecology have been widely used to explain biodiversity loss. In particular, habitat fragmentation has been seen, through its three major components (namely, loss of the original habitat, reduction in habitat patch size, and increasing isolation of habitat patches), as a cause of the decline in biological diversity within the original habitat (see Wilcox [29] and Wilcox and Murphy [30]). The negative effect of fragmentation on the growth of the resource, as assumed by the cited literature, it is taken into account by Assumption 1

Assumption 1 $b(D_t) : R^+ \rightarrow R^+$ is $C^1$. $b'(D_t) < 0, \forall D_t \geq 1$.

Assumption 1 states that, at each time $t$, $b(D_t)$ is decreasing in the number of plots: it reflects the idea that biological spillovers amongst plots have an impact on the natural growth rate of the resource. An increase in the number of plots makes biological spillovers less likely to occur, which has in turn a negative effect on the growth rate of the resource. Its growth rate is then the strongest when the spillovers are at their maximum level, which occurs when $D = 1$. 

4
As we will see later, the sensitivity of the natural growth rate \( b(D) \) to the property rights regime depends on the considered resource. There are cases in which splitting the resource into plots is highly detrimental to the growth rate of the resource itself, and other cases in which the enforced property rights regime influences the growth rate of the resource in a milder way.

As in the standard OLG setting, in each period and within each community there are two cohorts living two periods: the young and the old. Since we are not interested in intra-community externalities, we assume that at each time \( t \) and within each community a representative young agent and a representative old agent coexist. Following Mirman and To [22], we make the assumption that only the young can harvest: we denote by \( y^i_t \) the harvest of the \( i \)-th community at time \( t \). Hence, the total harvest is \( Y_t = \sum_{i=1}^{D_t} y^i_t \). The representative young agent of the \( i \)-th community either uses part of the harvest as an input for current consumption, \( z^i_t \), or saves it as capital, \( k^i_{t+1} \). While the representative young extracts and owns the resource, the representative old owns capital, which is used, in conjunction with the resource, to produce the consumption good. As remarked by Mirman and To [22], this structure gives an incentive to the young to save some of the natural resource: by not extracting the whole resource, the young agent ensures that she can consume when old. The following equilibrium constraint must hold

\[
y^i_t - h \left( \frac{X_t}{D_t} \right) = k^i_{t+1} + z^i_t
\]

(2)

where \( X_t / D_t \) is the stock of resource on each plot assigned to each community and \( h \left( X_t / D_t \right) \) measures the maintenance costs (in terms of monitoring costs): at each time \( t \), due to these costs, there is a part of the harvest that cannot be used as input for consumption nor saved as capital. The function \( h() \) satisfies the following properties

**Assumption 2** \( h(X_t / D_t) : R^+ \to R^+ \) is \( C^1 \). \( h'(x_t) > 0, \forall x_t, h \left( \frac{X_t}{D_t} \right) > D_t h \left( \frac{X_t}{D_t} \right) \), \( \forall D_t > 1 \).

Here we can clearly see the second effect that the enforced property rights regime produces. We assume that \( h() \) depends positively on the current stock of resource assigned to each community, \( x_t = X_t / D_t \). Moreover, we assume that when \( D = 1 \) the maintenance costs, that is \( h \left( X_t / 1 \right) \), are larger than the sum of the maintenance costs over plots, that is \( D_t h \left( X_t / D_t \right) \), when \( D > 1 \). We are not claiming here that any maintenance costs satisfy Assumption 2, actually here we are considering a specific class of maintenance costs, like monitoring costs. Indeed, average monitoring costs are increasing in the stock of the resource in the relative plot.

Now that the effects of property rights regime on the natural growth rate and on monitoring costs have been introduced and that the harvesting decision of the representative young agent has been highlighted, it seems worth spending some more words
on what we have previously defined Hardin effect and Meade effect. These two effects encompass the way the evolution of the resource reacts to the enforced property rights regime. There are resources whose evolution is highly affected by the property rights regime defined over it, so that - if the goal is to maximize the stock of the resource at the steady state - splitting the resource turns out to be always detrimental. For this kind of resource the so-called Meade effect always prevails and the peak of the stock of resource is found for \( D = 1 \). However, there are resources whose evolution is only mildly affected by the property rights regime, in a sense that it is possible to maximize the resource stock in the presence of a number of plots larger than one. This is what we call Hardin effect. It is important to bear in mind that in our dynamic general equilibrium approach these two effects characterize the resource in equilibrium, and take into account not only the effect that the enforced property rights regime has on the balance between the natural resource growth rate, \( b \), and the monitoring costs function, \( h \), but also its impact on the agents’ harvesting decisions. Considering all these features together in an equilibrium analysis is the distinct contribution of our paper to the literature on renewable natural resources.

At time \( t \), the \( i \)-th representative young holds \( z_i^t \) and the \( i \)-th representative old holds \( k_i^t \), while they both need capital and resource to produce the homogeneous good. Hence, once harvesting has taken place, a market for \( k \) and \( z \) opens: the young and the old of all communities trade the natural resource, \( z \), and the capital, \( k \), to produce a consumption good, \( c \), using a homogeneous of degree 1 production function \( G(k, z) \).

Assuming that at time \( t \) the number of communities to which plots are assigned is \( D_t \), total supplies for \( z \) and \( k \) are \( Z_t = \sum_{i=1}^{D_t} z_i^t \) and \( K_t = \sum_{i=1}^{D_t} k_i^t \), respectively.

We assume that in each community the maximization problem faced by the young agent consists in maximizing her life-cycle utility function. Utility is defined on youth and old-age consumption, \( c_{ty}^t \) and \( c_{to}^{t+1} \)

\[
u(c_{ty}^t) + u(c_{to}^{t+1})
\]

where \( u \) is increasing and concave.

As in Mirman and To [22], in each period a two-stage game is played. In the first stage the young agents decide how much to extract taking into account the law of motion of the natural resource; while in the second stage production takes place and the young and the old trade on the markets for \( z \) and \( k \). The game is solved by backward induction. In the following section we will analyze the resource’s and agents’ behaviours in a strategic setting, where we assume that representative young and old agents of each community take the decisions of the other \( D - 1 \) communities’ representative agents as given.

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Bréchet and Lambrecht [7] model an OLG economy in which individuals are endowed with a renewable resource that can be exploited at no cost by the young households and provided to production or bequeathed to the next generation. They find that the mere existence of a bequest motive does not guarantee a sustainable outcome. Further, when the resource is preserved in equilibrium, its level does not necessarily coincide with the efficient one. Whether the resource stock is too high or too low the capital stock should be lower than the golden rule level.
3 Strategic equilibrium

3.1 Competitive marketplace

For the sake of simplicity, as in Mirman and To [22], we assume that \( u(c) = \ln(c) \) and \( G(k, z) = k^\beta z^{1-\beta} \). Let \( z \) be the numeraire and \( p \) the price of \( k \) relative to \( z \). At each time \( t \) the representative young and old agents of the \( i-th \) community maximize the consumption when young and old, respectively. That consists in solving the following maximization problem for the community \( i \)'s representative agent, where the index \( j \) stands for young and old age

\[
\max_{\{k_{ij}^t, z_{ij}^t\}} G(k_{ij}^t, z_{ij}^t) \quad j = y, o
\]

s.t.

\[
\begin{align*}
&\text{for } j = o: \quad z_{ij}^t \leq p_t (k_{ij}^t - k_{oj}^t) \\
&\text{for } j = y: \quad p_t k_{ij}^t + z_{ij}^t \leq z_{it}^t
\end{align*}
\]

As we have already claimed, both the young and the old need capital, \( k \), and natural resource, \( z \), in order to produce the consumption good, \( c \). A competitive market for \( k \) and \( z \) opens and agents trade on this market.

Part of the capital held by the old agent, \( k_{it}^o \), is used as an input in the production function, while the remaining part is sold at price \( p_t \) to buy the natural resource. Similarly, the representative young agent holds a part of the natural resource, \( z_{it}^y \), and uses it for production, while the remaining part is sold to buy capital at price \( t \). Since the production function is Cobb-Douglas, from the maximization problems of the representative old and the young agents we get

\[
\begin{align*}
z_{it}^{io} &= (1 - \beta) p_t k_{it}^i \\
k_{it}^{io} &= \beta k_{it}^i \\
z_{it}^{iy} &= (1 - \beta) z_{it}^i \\
k_{it}^{iy} &= \beta \frac{z_{it}^i}{p_t}
\end{align*}
\]

Total demand for \( z \) and \( k \) are given respectively by \( \sum_{i=1}^{D_t} (z_{it}^{iy} + z_{it}^{io}) \) and \( \sum_{i=1}^{D_t} (k_{it}^{iy} + k_{it}^{io}) \).

Market clearing conditions imply

\[
\begin{align*}
Z_t &= (1 - \beta) [p_t K_t + Z_t] \\
K_t &= \beta \left[ K_t + \frac{Z_t}{p_t} \right]
\end{align*}
\]

Solving for \( p \) yields the equilibrium price

\[ 7 \]
3.2 Strategic resource harvesting

In this strategic resource harvesting equilibrium, at each time $t$ and within each community, the representative young agent maximizes her own life-cycle utility. We compute the indirect utility function of the representative young agent belonging to the $i$–th community

$$\ln c^{i,y}_t + \ln c^{i,o}_{t+1} = \ln(k^{i,y}_t)^{\beta}(z^{i,y}_t)^{1-\beta} + \ln(k^{i,o}_{t+1})^{\beta}(z^{i,o}_{t+1})^{1-\beta}$$

(10)

By substituting Eqs. (5) – (9) into Eq. (10), we can characterize the young’s maximization problem as follows

$$\max_{\{z^i_t, k^i_{t+1}\}} \ln(1 - \beta) K^\beta T^\beta z^i_t + \ln(1 - \beta) K^{1-\beta}_{t+1} k^{i}_{t+1}$$

(11)

s.t. \[
\begin{align*}
X_{t+1} &= X_t (1 + b(D_t)) - y^i_t - \sum_{j \neq i} y^j_t \\
Z_{t+1} &= Z_{t+1} (X_{t+1})
\end{align*}
\]

For the sake of simplicity, and following Mirman and To [22], we assume that the equilibrium value of $Z_{t+1}$ is a fraction of $X_{t+1}$ and is independent of $K_{t+1}$.

\[7\]

Once the model is solved, these hypothesis are shown to be consistent with the equilibrium solutions, and therefore they are fulfilled.

\[8\]

By solving the maximization problem, one obtains the following

$$X_{t+1} = \left( \frac{\alpha(1 - \beta)}{\alpha(1 - \beta) + (2D_t - 1)} \right)^\alpha$$

(12)

As it can be seen from Eq. (12), the property rights regime defined over the resource shapes the natural resource stock $X_{t+1}$ in a complex way. First of all, the number of plots at $t$ enters the denominator. The higher $D_t$, the larger the denominator and the smaller $X_{t+1}$, other things unchanged. Second, $D_t$ also affects positively $X_{t+1}$ through $h(\cdot)$, and negatively through $b(\cdot)$. So the overall effect is ambiguous.

\[7\] In the general case we would have $Z_{t+1} = Z_{t+1} (K_{t+1}, X_{t+1})$, where $Z_{t+1} (K_{t+1}, X_{t+1})$ is the anticipated equilibrium value of $Z_{t+1}$ and $(K_{t+1}, X_{t+1})$ is the $t + 1$ state.

\[8\] See Appendix A.1.

\[9\] See Appendix A.1 for algebraic details.
Example: functional forms for $b(D)$ and $h\left(\frac{X}{D}\right)$

To go further into the analysis we first choose functional forms for both $b(D)$ and $h\left(\frac{X}{D}\right)$ which satisfy Assumption 1 and Assumption 2. Let us consider the following simple functional forms

$$b(D) = a - cD \quad \text{with } a, c > 0$$

$$h\left(\frac{X}{D}\right) = e \left(\frac{X}{D}\right)^2 \quad \text{with } e > 0$$

We thus assume the monitoring costs to be quadratic in the stock of resource assigned to each plot, where $e$ is a scale parameter. As long as $b(D)$ is concerned, we impose $D \leq a/c$ to satisfy Assumption 1. Natural resources whose growth rate is heavily affected by the enforced property rights regime show high values for the parameter $c$; while for resources whose natural growth rate is less sensitive to the property rights regime the opposite is true. We then set $\alpha = 1/2$. With these functional forms, Eq. (12) provides the steady state resource stock, denoted by $X^*$, as the solution to the following implicit function

$$\Phi(X^*, D) \equiv \left[\frac{1}{2}(1-\beta) + (2D-1)\right] X^{*2} - \frac{1}{2}(1-\beta) \left[ (1+a-cD)X^* - De \left(\frac{X^*}{D}\right)^2 \right] = 0 \quad (13)$$

The solution to Eq. (13), $X^* = \Psi^*(a, c, e, \beta, D)$, defines the stock of the resource in the steady state in the strategic equilibrium as a function of $D$ and parametrized by the coefficients of the model. Equation (13), being a polynomial of degree two, has two distinct roots (two distinct steady states): $X^*_a = 0$ and

$$X^*_b = \frac{D(1+a-cD)(1-\beta)}{D(4D-1-\beta) + e(1-\beta)} \quad (14)$$

In the following we will concentrate our analysis on the non trivial steady state and we will denote it by $X^*$. Before going further, the following proposition establishes uniqueness and local stability of the steady state.

**Proposition 1** In the strategic equilibrium the non trivial steady state is unique.

The steady state is locally stable for $D > \frac{1}{24} \left( 3(1+\beta) + \sqrt{48e(1-\beta) + 9(1+\beta)^2} \right)$ with $e \leq \frac{6(7-\beta)}{1-\beta}$.

**Proof:** See Appendix A.2.

The following Proposition describes the shape of the resource at the steady state in the strategic setting, depending on the values of the parameter $e$. As stated earlier, $e$ is a parameter of scale related to the monitoring costs: the gain from lowering the monitoring costs.
costs by splitting the resource into plots depends on the value of \( e \). When \( e \) is sufficiently high then splitting the resource brings a remarkable reduction in the monitoring costs born by the communities, which makes the Hardin effect likely to appear, other things unchanged.\(^{11}\)

**Proposition 2** Under Proposition 1 in the strategic equilibrium,

1. if \( e \leq \hat{e} = \frac{4(1+a) - c(1+\beta)}{(1-\beta)(1+a-4c)} \) then \( X^* \) is always decreasing in \( D \).

2. if \( \hat{e} < e < \hat{\hat{e}} = \frac{(a/c)2(4(1+a)-c(1+\beta))}{(1-\beta)(1-a)} \) with \((a/c) > 2 \) then \( X^* \) is first increasing and then decreasing in \( D \).

3. if \( e \geq \hat{\hat{e}} \) then \( X^* \) is always increasing in \( D \).

**Proof:** See Appendix A.3.

In Proposition 2 the balance between Hardin and Meade effects is clearly stated: when the condition of Proposition 2.1 is satisfied, the latter prevails and \( X^* \) turns out to be always decreasing in \( D \). On the contrary, in Proposition 2.2 the Hardin effect prevails for small enough values of \( D \), while for large enough values of \( D \) the Meade effect is at work. Finally, in Proposition 2.3 the Hardin effect is always prevalent. However, the Hardin effect is unlikely to always prevail for all \( D \geq 2 \),\(^{12}\) such that the \( X^* \) curve that describes the profile of the resource in the steady state as a function of \( D \) is always increasing (as stated in Proposition 2.3). The reasoning behind this intuition is the following. Let us denote by \( X^*_s \) the steady-state level of the natural resource at \( D = 2 \) and let us assume that the resource initially shows the Hardin effect, such that \( X^*_2 > X^*_s \). At \( D = 3 \) the growth rate of the resource is still high (since, under a Hardin regime, the natural growth rate of the resource is only slightly influenced by the property right regime - especially when the number of plots is low). Nevertheless, at the same time, monitoring costs are high, which represents an incentive for decentralization and increasing the number of plots into which the resource is split. Let us assume then that the maximum stock of the natural resource is reached for a \( D = s > 2 \). After that point, the number of plots will become high enough to have a significant detrimental impact on \( b(D) \). This effect, along with low monitoring costs, will reduce the stock of the resource and lead eventually to over-exploitation, that is \( X^*_{s+1} < X^*_s \). In this case, for small enough values of \( D \) the Hardin effect is at work, and thereafter the Meade effect shall prevail.

To illustrate how the enforced property rights regime can influence the evolution of the natural resource at the steady state (through its general equilibrium effects on \( b, h \) and agents’ harvesting decisions), we now set numerical values for the parameters. Let us assume that \( a = 0.95 \), \( c = 0.08 \) and \( \beta = 0.52 \). We are then able to study the effect

\(^{11}\)Of course, in alternative and in similar way, in Proposition 2 we could have looked at the behaviour of \( c \), that is the parameter that captures the sensitivity of the resource growth rate to the defined property rights regime.

\(^{12}\)Since we are in a Cournot-Nash equilibrium it is natural to consider the case \( D \geq 2 \).
the parameter $e$ associated to the monitoring costs has on the resource stock level in the steady state. Let us now consider the plot of $X^*$ where $a = 0.95$, $c = 0.08$, $\beta = 0.52$, while $e$ is either 63 (see left panel of Figure 1) or 0.1 (see right panel of Figure 1).

Figure 1: Hardin vs Meade: $X^*$ when $e = 63$ (left panel) and when $e = 0.1$ (right panel). Other parameters: $a = 0.95$, $c = 0.08$ and $\beta = 0.52$.

![Graph showing $X^*$ vs $D$ for different values of $e$](graph.png)

The two panels in Figure 1 illustrate Proposition 2 for two different natural resources. In the former case (left panel) the Hardin and Meade effects are balanced for a $D > 2$, while in the latter case (right panel) the Meade effect dominates for any allocation of property rights.

The left panel of Figure 1 illustrates the case of a resource whose natural evolution is mildly affected by the property rights regime when $D$ is small enough, so that the Hardin effect dominates. When $e = 63$, for values of $D$ small enough, $X^*$ is first increasing in $D$, it attains its maximum ($X_{\text{max}}^* = 0.04$) for $D = 3$, and then it decreases because the Meade effect starts dominating.\(^\text{13}\)

In the right panel of Figure 1 the resource dynamics are always dominated by the Meade effect. It is the case where $e = 0.1$, the values of the other parameters being kept unchanged, and it represents a natural resource characterized by strong biological interactions. In such a case, splitting the resource severely harms its natural dynamics. The effect of $D$ on the monitoring costs prevails for every $D > 2$. The maximum stationary level of the resource stock, $X_{\text{max}}^* = 0.147$, is reached for $D = 2$. In other words, if the objective is to maximize the level of the resource in the steady state, then it can be achieved by splitting the resource in two plots.

Some more hints can be gotten from the comparison of these two cases. First, when the resource is more sensitive to the property rights regime, its maximal stationary level is higher (0.135 vs 0.04). However, in this case the resource stock decreases faster with

\(^{13}\)The stock of the resource at the steady state is actually maximized for a $D = 2.7$ but, since $D$ can assume only integer values, in the remaining of the analysis we will consider the nearest integer value to the maximizing $D$, in this case $D = 3$. 

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$D$, and it reaches a size comparable to the one of a less reactive resource as soon as $D \geq 8$.

In a strategic framework, biological spillovers are not internalized and each community takes the decision of the other $D - 1$ communities as given. This can cause a suboptimal use of the resource and eventually leads to over-exploitation. If biological spillovers are at work and if each plot interacts with all neighbouring parcels, then the boundaries of the community’s resource titles do not coincide with the boundaries of the impact of the resource use, and hence the appropriate geographical scale for decision-making is wider than the community area. The issue is then about the coordination of decisions at the relevant geographical scale. In the following section we will study the case where communities decide to cooperate and analyze how this behaviour shapes the resource dynamics.

4 Cooperative equilibrium

Let us now consider the case where communities cooperate. Cooperation amongst communities characterizes the resource harvesting problem; while as for the competitive marketplace game we use the solution derived in Section 3.1. Here we will also be able to depict the case $D = 1$ as a special case in the cooperative setting.

4.1 The natural resource harvesting problem

Let us consider the number of communities as given. The indirect utility function to be maximized is

$$\ln c_{iy}^t + \ln c_{io}^t = \ln(k_{iy}^t)^{\beta}(z_{iy}^t)^{1-\beta} + \ln(k_{io}^t)^{\beta}(z_{io}^t+1)^{1-\beta}$$  \hspace{1cm} (15)

By substituting Eqs. (5) – (9) into Eq. (15), we characterize the maximization problem as follows

$$\max_{z_i^t,k_{i+1}^t} \ln(1 - \beta) \frac{K_t^\beta}{Z_t^\beta} z_i^t + \ln \beta \frac{Z_{t+1}^{1-\beta}}{K_{t+1}^{1-\beta}} k_{i+1}^t$$

$$\text{s.t.} \left\{ \begin{array}{l}
X_{t+1} = (X_t(1 + b(D_t)) - D_t y_i^t)^{\alpha} \\
Z_{t+1} = Z_{t+1} (X_{t+1})
\end{array} \right.$$

The novelty in this problem is that, while in the Cournot-Nash setting each representative within a community takes the harvesting decisions of the other $D - 1$ communities’ representatives as given, here she is aware that $Y_i = D_t y_i^t$. By solving the maximization

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14Klosowski et al. [19] carry out a conjoint analysis to study the probability that coordinated management programmes would be undertaken. Results show that this probability depends on a number of factors of which the most important are tax incentives, the number of parties and the incentive to defect from the agreement (which is related to the difference between the payoffs under the Nash and cooperative equilibria). See also Gluck [14] for a review on collective property regimes.
problem we end up with Eq. (17), which represents the law of motion of the resource in the cooperative setting\textsuperscript{15}

\[
X_{t+1} = \left( \frac{\alpha(1 - \beta)}{\alpha(1 - \beta) + 1} \left[ \frac{(1 + b(D_t))X_t - D_t h \left( \frac{X_t}{D_t} \right)}{\alpha(1 - \beta)} \right] \right)^{\alpha}
\tag{17}
\]

By comparing Eq. (12) with Eq. (17), one can notice that the effect of $D$ on the denominator disappears in the latter. However, the overall effect of $D$ remains ambiguous, as $D$ enters Eq. (17) through $b()$ and $Dh()$. In other words, in the cooperative setting the externality arising from the strategic behaviour is internalized, but the effects of $D$ on $b()$ and $h()$ are still at work. Even in this framework then changing the property rights regime would change the resource stock.

As in the Cournot-Nash setting, we assume $\alpha = 1/2$. Plugging the functional forms for $b()$ and $h()$ into Eq. (17) and evaluating it in the steady state provides us with the following implicit function

\[
\Phi^C(D, X^C) \equiv \left[ \frac{1}{2}(1 - \beta) + 1 \right] X^{C2} - \frac{1}{2}(1 - \beta) \left[ (1 + a - cD) X^C - De \left( \frac{X^C}{D} \right)^2 \right] = 0
\tag{18}
\]

The solution to Eq. (18), $X^C = \Psi^C(a, c, e, \beta, D)$, defines the resource stock in the steady state in the cooperative equilibrium as a function of $D$ and the parameters. Eq. (18) is a polynomial of degree two, with two distinct roots. One is trivial, $X^C_a = 0$, while the other, $X^C_2$, is given by

\[
X^C_2 = \frac{D[(1 + a - cD)(1 - \beta)]}{D(3 - \beta) + e(1 - \beta)}
\tag{19}
\]

In the following, we will concentrate on the non trivial steady state and, for the sake of brevity, we will denote it by $X^C$. Let us briefly compare Eq. (14) and Eq. (19). As one can easily see, it turns out that $X^C$ is always larger than $X^*$, since the numerator of the two equations is the same, while the denominator in Eq. (14) is always bigger than in Eq. (19). This implies that in the cooperative framework, for a given $D$, a higher level of the natural resource is reached.

**Proposition 3** In the cooperative equilibrium the non trivial steady state is unique. The steady state is locally stable for $D > \frac{e(1-\beta)}{3(3-\beta)}$ with $e \geq \frac{3(3-\beta)}{1-\beta}$.

**Proof:** See Appendix A.5.
This leads us to our next result.

**Proposition 4** Under Proposition 3 in the cooperative equilibrium,

1. if \( e \leq \tilde{e} = \frac{c(3-\beta)}{(1-\beta)(1+a-2c)} \) then \( X^* \) is always decreasing in \( D \).

2. if \( \tilde{e} < e < \tilde{\tilde{e}} = \frac{c(3-\beta)}{(1-\beta)(1-a)} \) with \( \frac{a}{c} > 2 \) then \( X^* \) is first increasing and then decreasing in \( D \).

3. if \( e \geq \tilde{\tilde{e}} \) then \( X^* \) is always increasing in \( D \).

**Proof:** See Appendix A.6.

Clearly, Proposition 4 parallels 2. Both provide necessary conditions for the *Hardin* or the *Meade* effect to prevail. Although similar in their content, these two propositions differ in the range of values for the parameter \( e \). In particular, it can be easily shown that: \( \hat{e} > \tilde{e} \) always holds;\(^{16}\) meaning that, when communities cooperate the *Hardin effect* is much more likely to appear, even for small values of \( e \), which is a good news for the resource stock and must be seen as an ancillary result to cooperation.

It is useful to display \( X^* \) and \( X^C \) on the same graph, as shown in Figure 2: in each plot the resource stock under strategic and cooperative equilibrium is displayed. The left panel refers to the case where the *Hardin effect* is initially at work; while in the right panel the *Meade effect* always prevails.

*Figure 2:* Strategic vs cooperative behavior: \( X^* \) (solid line) and \( X^C \) (dashed line) with \( e = 63 \) (left panel) and with \( e = 0.1 \) (right panel). Other parameters: \( a = 0.95, c = 0.08 \) and \( \beta = 0.52 \).

The first result that comes out from Fig. 2 is that, whatever the resource dynamics (left or right panel), the resource stock is higher in the cooperative equilibrium than in the

\(^{16}\)The condition ensuring that \( \hat{e} > \tilde{e} \) is \( c < 1 + a \), which is always satisfied under Assumption 1.
strategic equilibrium (in both panels the dashed curve always lies above the solid curve). This is not unexpected, of course, because under cooperation the communities internalize the externality arising from the Cournot-Nash behavior. Nevertheless, it is striking to compare the order of magnitude of the inefficiency due to the strategic interactions and the one due to the biological spillovers. Let us analyze further these results.

As far as the left plot is concerned, we can see that the \( D \) which maximizes the stock of the resource under the Cournot-Nash setting \( (D = 8) \) is different from the \( D \) which maximizes the stock under the cooperative behaviour \( (D = 3) \). Put it differently, when communities cooperate the maximum level of the resource is found for a larger number of plots. As for the evolution of the resource, the gain springing from cooperation is considerable, since the maximum level of \( X^C \) (that is \( X^C_{\text{max}} \)) is about 2.5 times as large as the maximum level of \( X^* \) (that is \( X^*_{\text{max}} \)), and this difference becomes even wider for large enough values of \( D \) (with \( X^C \) being almost six times as large as \( X^* \) when \( D = 10 \)).

Given the values of the parameters the cooperative steady state turns out to be stable for \( D \geq 4 \).

Let us now analyze the case of a resource which is very sensitive to the enforced property rights regime also for small values of \( D \). This is the case where the Meade effect always prevails: interactions between plots are strong and splitting the resource is highly detrimental. As a consequence, the maximum level of the resource is reached when \( D = 1 \) \( (X^C_{\text{max}} = 0.364) \). As in the Cornot-Nash framework, also in the cooperative equilibrium the maximum level reached by a very sensitive resource is higher than a less sensitive resource’s \( (0.364 \text{ vs } 0.101) \). Due to this high sensitivity to the enforced property rights regime, however, the resource is depleted fast and severely as \( D \) increases. Finally, the proportional gain of cooperating is comparable to the one obtained in the case of a less sensitive resource.

In both cases, it is worth stressing that the gain from cooperation (that is, jumping vertically from the solid curve to the dashed one for a given \( D \)) is larger than the one that would be obtained by simply tuning \( D \) in the strategic setting, so as to maximize the stock of the resource (that is, moving along the solid curve).

It is also worth drawing attention to the case where the parameter \( e \) takes intermediate values, let us say \( e = 15 \). The result is displayed in Figure 3. This figure shows the case where Hardin meets Meade. In the previous cases (see Figure 2) the same arbitrage between Hardin and Meade held both in the Nash-Cournot equilibrium and in the cooperative equilibrium: \( X^C \) and \( X^* \) were bell-shaped functions of \( D \) in the case of a poorly sensitive resource, and monotonically decreasing functions of \( D \) in the case of a more sensitive resource. However, we see here that this balance is not a pure technological or biological effect, merely reflecting a feature of the natural resource, but that it also depends on the market outcome.

\[\text{It goes without saying that the magnitude of the difference between } X^C \text{ and } X^* \text{ depends on the values of the parameters. Some sensitivity analyses show that the qualitative results hold. Our analysis should be valued more from a qualitative than a quantitative point of view, meaning that cooperation entails a remarkable gain in terms of stationary resource stock.}\]
When $e = 15$, in the non-cooperative setting the negative impact of $D$ on the natural growth rate always prevails, so that the resource reaches its maximum when $D = 2$, and then monotonically decreases. Hence, it looks like a highly sensitive resource, as discussed above. Yet, under cooperation the resource behaves like a low sensitive resource as it first increases in $D$, reaches a maximum at $D = 6$, and then monotonically decreases in $D$. This case clearly shows how the Hardin and Meade effects not only encompass the balance between the impacts of the number of plots on the natural growth rate of the resource and on the maintenance costs, but also the agents’ harvesting decisions in equilibrium. In other words, it is shown how the same natural resource can behave differently and give birth to the Hardin or Meade effects depending on the decisions taken by the agents in equilibrium.

More generally, from these three cases (Figure 1 and Figure 2) we have learnt that having a good understanding of the resource’s characteristics is key to manage it efficiently. However, the assignment of property rights and its implications in terms of strategic interactions may be even more important for sustainability. Furthermore, it has also been shown that no property rights regime can be as efficient as cooperation. This result questions the idea discussed at the beginning of the article, that the resource stock could be maximized by choosing the appropriate property rights regime. If correcting for the strategic interactions is much more efficient than simply tuning the number of allocated property rights in a strategic setting, then an additional instrument is required.

5 Fiscal policy in support of property rights

In the previous section we have shown that the steady state resource stock cannot be higher in the strategic equilibrium than in the cooperative one, whatever the assignment of property rights. This suggests that changing the property rights regime defined on the resource has some effects on its size, but that it cannot be sufficient to maximize the resource stock. To fill the inefficiency gap between non-cooperative and cooperative outcomes, an additional policy instrument is required. Finding such an instrument is the
purpose of this section.

Let us assume that, besides the assignment of property rights, the government is also able to levy a lump-sum transfer $\omega$ (which can be positive or negative). In such a case Eq. (3) becomes

$$y_t - h\left(\frac{X_t}{D_t}\right) = k_{t+1} + z_t^i + \omega \quad (20)$$

The following government’s budget constraint must be met

$$D_t \omega = \theta p_t K_t \quad (21)$$

This budget constraint means that, if $\omega > 0$, then the tax revenue is distributed through a subsidy on capital. Otherwise, the positive transfer $\omega > 0$ is financed through a tax on capital. By solving the agent maximization problem\(^{18}\) we end up with the following equation, which defines the steady-state level of the resource, $X^\omega$, in the presence of a lump-sum tax $\omega$ as a function of $D$

$$\Phi^\omega(D, X^\omega) \equiv X^{\omega^2}\left[(2D - 1) + \frac{1}{2}(1 - \beta)\right] - \frac{1}{2}(1 + b(D))(1 - \beta)X^\omega +$$

$$+ \frac{1}{2}(1 - \beta)D \left(h\left(\frac{X^\omega}{D}\right) + \omega\right) = 0 \quad (22)$$

The solution to Eq. (22), $X^\omega = \Psi^\omega(a, c, e, \beta, D)$, defines the resource stock in the steady state in the cooperative equilibrium as a function of $D$ and the parameters. Eq. (22) is a polynomial of degree two which, for every $\omega \neq 0$, has two distinct, non trivial roots: $X^\omega_a$ and $X^\omega_b$.\(^{19}\)

The $\omega$ which decentralizes the cooperative outcome in a strategic setting with fiscal policy is found by imposing

$$X^\omega_{a,b} = X^C$$

which leads to the following proposition

**Proposition 5** There exists a $e > 0$ such that, for every $D \geq 2$, the unique positive and stable steady state can replicate the cooperative equilibrium with a lump-sum transfer $\omega^*(D)$

$$\omega(D)^* = -\frac{4D(D - 1)(1 + a - cD)^2(1 - \beta)}{(D(3 - \beta) + e(1 - \beta))^2} < 0 \quad (23)$$

**Proof:** See Appendix A.8

\(^{18}\)Computations are given in Appendix A.7.

\(^{19}\)See Appendix A.8 for the values of $X^\omega_a$ and $X^\omega_b$. 

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The level of $\omega^*$ depends on the defined property rights regime and, according to Proposition 5, turns out to be a subsidy financed through a tax on capital. Actually, young agents should receive a subsidy from the government in order to restrain themselves from harvesting too much so as to reach the resource stock level that would be attained under full cooperation. Such a finding is consistent with many empirical evidence, e.g. Chomitz et al. [9], Pagiola [24] or Sanchez-Azofeifa et al. [26]. For example, in Costa Rica in the late '90s the government launched a program of payments for environmental services. The payments were given to communities and private landowners for reforestation and sustainable forest management (see Lambin and Meyfroidt [20]).

The subsidy level $\omega^*$ is such that, for every $D \geq 2$, the resource stock in the strategic equilibrium coincides with that in the cooperative framework. Interestingly, we can go even further in the analysis. Let us consider that for some reason, that can be found in the political economy literature, changing the property rights regime is uneasy for the policy maker. Then, the issue is to find the level of $\omega$ (if it exists) such that, for every given $D \geq 2$, the resource stock in the strategic equilibrium coincides with the maximum resource stock reached in the cooperative equilibrium, denoted by $X_{C_{\text{max}}}$. The following corollary shows that such a fiscal policy exists.

**Corollary 1.** Let $D$ be the number of communities for which the maximum resource stock level is reached in the cooperative equilibrium. Under Proposition 5, for every $D \geq 2$, there exists $\omega(D) < 0$, such that $X_{C_{\text{max}}}$ can be decentralized in the strategic equilibrium.

**Proof:** See Appendix A.9

### 6 Conclusions

Hardin’s [16] theory, by depicting a set of herders inexorably trapped in the overuse of their common pasture, was thought for many years to be typical for common-pool resources not owned privately or by a government. Hardin has advocated two solutions to prevent future tragedies: state control or individual ownership. His theory has led national governments to put restrictions on the local systems of resource management and/or to assign property rights to individual owners or local communities. However, empirical evidence has showed that often decentralization and privatization led to over-exploitation of the resources. To understand successes and failures in the field of resource management we have introduced a new ingredient: namely, the existence of positive externalities across owners, as depicted by Meade [21]. The existence of spatial or biological spillovers amongst landowners has been studied by several theoretical works and recently confirmed by empirical research.

In this article we have then taken advantage of both Hardin’s theory on property rights and the existence of spatial spillovers amongst private owners, so that the boundaries of private titles and the boundaries of the impacts of resource use may not coincide.

We have assumed that defining property rights and dividing the natural resource in plots plays an important role in influencing the evolution of the resource itself: on
the one hand it reduces the overall maintenance costs, but on the other hand it makes
biological spillovers among plots less likely to occur, thus having a negative impact on
the growth rate of the resource. The central issue became finding the optimal level of
plots that maximizes the stock of the resource: in this analysis, the sensitivity of the
resource to the enforced property rights regime played a crucial element in explaining
the final outcome, since some resources are more reactive than others.

We have tackled this issue in an OLG framework: we have assumed that at each time
t the natural resource is divided into $D_t$ plots. Each plot is assigned to a community that
owns property rights on it and must manage it. Within each community and at each time
t a representative young and a representative old agent exist: the young agent harvests
the resource, while the old agent owns the capital. Both capital and the natural resource
are used for production. We have first analyzed the problem of over-exploitation in a
Cournot-Nash framework, where each community takes as given the harvesting decisions
of the other $D - 1$ communities. We have then compared the result obtained in the
strategic setting to that arising in a cooperative framework. It has been shown that
the gain from cooperation is always remarkable. A fiscal policy able to decentralize
the cooperative outcome has then been studied and some policy implications have been
stressed. It has also been shown that cooperation alone is not enough to maximize the
stock of the resource in the steady state. However, assuming that the number of property
rights is fixed, by means of a subsidy and for every given $D$, it is possible to replicate in
the strategic setup the maximum level of the natural resource reached in the cooperative
framework.

A Appendix

A.1 Strategic resource harvesting in the Cournot-Nash set-up

Equation (11) can be rewritten as follows

$$\ln(1 - \beta) + \beta \ln K_t - \beta \ln Z_t + \ln z_i + \ln \beta + (1 - \beta) \ln Z_{t+1} - (1 - \beta) \ln K_{t+1} + \ln k_{t+1}$$

The maximization problem is solved by substituting $k_{t+1} = y_i - h\left(\frac{X_t}{D_t}\right) - z_i$ and $Z_{t+1} = \eta_{t+1}X_{t+1}$ into Eq. (11).

As in the symmetric Nash equilibrium $y_i^j = y_j^i$ and $z_i^j = z_j^i \forall i, j = 1...D$, we obtain the
following first-order conditions with respect to $z_i^j$ and $y_i^j$

$$\frac{1}{z_i^j} - \frac{\beta}{Z_t} + \frac{1 - \beta}{Y_t - D_t h\left(\frac{X_t}{D_t}\right) - Z_t} - \frac{1}{y_i^j - h\left(\frac{X_t}{D_t}\right) - z_i^j} = 0$$ (A.1)

$$- \frac{\alpha (1 - \beta)}{X_t(1 + b(D_t)) - Y_t} - \frac{(1 - \beta)}{Y_t - D_t h\left(\frac{X_t}{D_t}\right) - Z_t} + \frac{1}{y_i^j - h\left(\frac{X_t}{D_t}\right) - z_i^j} = 0$$ (A.2)
By solving Eq. (A.1) for $Z_t$ we get

$$Z_t = \frac{(D_t - \beta) \left( Y_t - D_t h \left( \frac{X_t}{D_t} \right) \right)}{2D_t - 1} \quad (A.3)$$

By using the relationship $Y_t - D_t h \left( \frac{X_t}{D_t} \right) = Z_t + K_{t+1}$, Eq. (A.3) can also be rewritten as

$$K_{t+1} = \frac{(D_t - 1 + \beta) \left( Y_t - D_t h \left( \frac{X_t}{D_t} \right) \right)}{2D_t - 1} \quad (A.4)$$

By substituting $K_{t+1}$ into Eq. (A.2) and solving for $Y_t$ we get

$$Y_t = \frac{(2D_t - 1)(1 + b(D_t))}{\alpha(1 - \beta) + (2D_t - 1)} X_t + \frac{\alpha(1 - \beta)}{\alpha(1 - \beta) + (2D_t - 1)} D_t h \left( \frac{X_t}{D_t} \right) \quad (A.5)$$

As we can see from Eq. (A.5), the total harvesting, $Y_t$, is influenced by both $b(D_t)$ and $D_t h(D_t, X_t)$. By using Eq. (A.5) and substituting it into Eq. (A.3) and Eq. (A.4) we get respectively\(^{20}\)

$$Z_t = \frac{(D_t - \beta) \left[ X_t(1 + b(D_t)) - D_t h \left( \frac{X_t}{D_t} \right) \right]}{\alpha(1 - \beta) + (2D_t - 1)} \quad (A.6)$$

$$K_{t+1} = \frac{(D_t + \beta - 1) \left[ X_t(1 + b(D_t)) - D_t h \left( \frac{X_t}{D_t} \right) \right]}{\alpha(1 - \beta) + (2D_t - 1)} \quad (A.7)$$

By substituting Eq. (A.5) into the law of motion, $X_{t+1} = [X_t(1 + b(D_t)) - Y_t]^\alpha$ we obtain

$$X_{t+1} = \left( \frac{\alpha(1 - \beta) \left[ (1 + b(D_t)) X_t - D_t h \left( \frac{X_t}{D_t} \right) \right]}{\alpha(1 - \beta) + (2D_t - 1)} \right)^\alpha \quad (A.8)$$

finally, by assuming $\alpha = 1/2$ and by evaluating at the steady state we get Eq. (13).

### A.2 Proof of Proposition 1

The non trivial steady state $X^*_2$ is unique and, under Assumption 1, can only takes positive values.

Moreover, the steady state implicitly defined by Eq. (12) is locally stable if

$$\left| \frac{\partial}{\partial X^*} \left( \frac{1}{2}(1 - \beta) \left[ (1 + a - cD)X^* - De \left( \frac{X^*}{D} \right)^2 \right] \right)^{\frac{1}{2}} \right| < 1$$

\(^{20}\)As we can see from Eq. (A.7), the equilibrium level of $Z_t$ is a fraction of $X_t$, which satisfies the assumption made in Section 3.2.
where we have set $\alpha = \frac{1}{2}$ and plugged the functional forms for $b(D)$ and $h(X/D)$. Which means

$$\frac{(1 + a - cD)(1 - \beta) \left( 4D^2 - e(1 - \beta) - D(1 + \beta) \right)}{2(4D - 1 - \beta) \left( D(4D - 1) + e - (D + e)\beta \right)} \sqrt{\frac{D^2(1+a-cD)^2(\beta-1)^2}{(D(4D-1)+e-(D+e)\beta)^2}} < 1$$

evaluated at (14).

The above inequality admits two solutions

$$\frac{1}{8} \left( 1 + \beta - \sqrt{1 - 16e + 2\beta + 16e\beta + \beta^2} \right) < D < \frac{1}{8} \left( 1 + \beta + \sqrt{1 - 16e + 2\beta + 16e\beta + \beta^2} \right)$$

and

$$D < \frac{1}{24} \left( 3 + 3\beta - \sqrt{3\sqrt{3 + 16e + 6\beta - 16e\beta + 3\beta^2}} \right) \lor \quad D > \frac{1}{24} \left( 3 + 3\beta - \sqrt{3\sqrt{3 + 16e + 6\beta - 16e\beta + 3\beta^2}} \right)$$

Since we are in a strategic setting, we impose $D \geq 2$. The first solution satisfies this condition only for negative values of the parameter $e$ and hence cannot be accepted. As for the second solution, the first inequality yields negative values of $D$ and must hence be rejected, while the second inequality is compatible with $D \geq 2$ for $e \leq \frac{6(1-\beta)}{1-\beta}$.

$$D > \frac{1}{24} \left( 3(1 + \beta) + \sqrt{48e(1 - \beta) + 9(1 + \beta)^2} \right) \quad \text{(A.9)}$$

### A.3 Proof of Proposition 2

We look at the derivative of Eq. (14) with respect to $D$

$$\frac{\partial X^*}{\partial D} = \frac{[(1+a-2cD)(1-\beta)] \left[ (D-4D^2+\beta D-e(1-\beta))-(8D-1-\beta) \right] (1-\beta)(D+aD-cD^2)}{\left[(D-4D^2+\beta D-e(1-\beta))^2 \right]}$$

Since the denominator is always positive, the derivative can be either positive ($X^*$ increasing in $D$) or negative ($X^*$ decreasing in $D$) depending on the sign of the numerator, which can be simplified as follows

$$(1 - \beta) \left[ e(1 + a)(1 - \beta) - 2ceD(1 - \beta) + D^2 (c(1 + \beta) - 4(1 + a)) \right]$$

The term $(1 - \beta)$ is always positive (since $0 < \beta < 1$), hence we concentrate our analysis on the term in the square brackets. We obtain that the derivative equals zero (local
maximum) when
\[
e = \frac{D^2 [4(1 + a) - c(1 + \beta)]}{(1 - \beta)(1 + a - 2cD)} \quad \text{with} \quad a > [c(1 + \beta) - 1]/4 \tag{A.10}
\]

Let us characterize the value of \(e\) such that the maximizing \(D\) is equal to 2 and to \(a/c\) (with \((a/c) > 2\)), respectively. We obtain
\[
\hat{e} = 4 \frac{[4(1 + a) - c(1 + \beta)]}{(1 - \beta)(1 + a - 4c)} \tag{A.11}
\]
\[
\hat{\hat{e}} = \frac{(a/c)^2 [4(1 + a) - c(1 + \beta)]}{(1 - \beta)(1 - a)} \quad \text{with} \quad a < 1 \tag{A.12}
\]

Since Eq. (A.10) is monotonically increasing in \(D\), when \(e \leq \hat{e}\) the maximum stock of the resource is found for \(D = 2\), while for every \(\hat{e} < e < \hat{\hat{e}}\) it will be found for an integer \(D > 2\) and, finally, for \(D = a/c\) when \(e = \hat{\hat{e}}\).

### A.4 Strategic resource harvesting in the cooperative framework

Equation 16 can be rewritten as follows
\[
\ln(1 - \beta) + \beta \ln K_t - \beta \ln Z_t + \ln z^i_t + \ln \beta + (1 - \beta) \ln Z_{t+1} - (1 - \beta) \ln K_{t+1} + \ln k^i_{t+1}
\]

Once again, the maximization problem is solved by substituting \(k^i_{t+1} = y^i_t - h \left(\frac{X_t}{D_t}\right) - z^i_t\) and \(Z_{t+1} = \eta_{t+1} X_{t+1}\) into Eq. (16). Since communities cooperate, \(Y_t = D_t y_t\) and \(Z_t = D_t z_t\), and we obtain the following first-order conditions with respect to \(z^i_t\) and \(y^i_t\)

\[
\frac{1}{z^i_t} \left(1 - \beta\right) \frac{D_t}{Z_t} + \frac{(1 - \beta)D_t}{Y_t - D_t h \left(\frac{X_t}{D_t}\right) - Z_t} - \frac{1}{y^i_t - h \left(\frac{X_t}{D_t}\right) - z^i_t} = 0 \tag{A.13}
\]
\[
- \frac{\alpha (1 - \beta) D_t}{X_t (1 + b(D_t)) - Y_t} - \frac{(1 - \beta) D_t}{Y_t - D_t \left(\frac{X_t}{D_t}\right) - Z_t} + \frac{1}{y^i_t - h \left(\frac{X_t}{D_t}\right) - z^i_t} = 0 \tag{A.14}
\]

By solving Eq. (A.13) for \(Z_t\) we get
\[
Z_t = (1 - \beta) \left[ Y_t - D_t h \left(\frac{X_t}{D_t}\right) \right] \tag{A.15}
\]

By using the relationship \(Y_t - D_t h \left(\frac{X_t}{D_t}\right) = Z_t + K_{t+1}\), Eq. (A.15) can also be rewritten as
\[
K_{t+1} = \beta \left[ Y_t - D_t h \left(\frac{X_t}{D_t}\right) \right] \tag{A.16}
\]
By substituting $K_{t+1}$ into Eq. (A.14) and solving for $Y_t$ we get

$$Y_t = \frac{X_t (1 + b(D_t)) + D_t \alpha (1 - \beta) h \left(\frac{X_t}{D_t}\right)}{\alpha(1 - \beta) + 1}$$  \hspace{1cm} (A.17)$$

$Y_t$ is influenced by both $b(D_t)$ and $D_t h(D_t, X_t)$. By using Eq. (A.17) and substituting it into Eq. (A.15) and Eq. (A.16) we get respectively:\footnote{As we can see from Eq. (A.18), the equilibrium level of $Z_t$ is a fraction of $X_t$, which satisfies the assumption made in Section 4.1.}

$$Z_t = (1 - \beta) \left( \frac{X_t(1 + b(D_t)) - D_t h \left(\frac{X_t}{D_t}\right)}{\alpha(1 - \beta) + 1} \right)$$  \hspace{1cm} (A.18)$$

$$K_{t+1} = \beta \left( \frac{X_t (1 + b(D_t)) - D_t h \left(\frac{X_t}{D_t}\right)}{\alpha(1 - \beta) + 1} \right)$$  \hspace{1cm} (A.19)$$

By substituting Eq. (A.17) into the law of motion, $X_{t+1} = [X_t (1 + b(D_t)) - Y_t]^\alpha$ we obtain

$$X_{t+1} = \left( \frac{\alpha(1 - \beta) \left[ (1 + b(D_t)) X_t - D_t h \left(\frac{X_t}{D_t}\right) \right]}{\alpha(1 - \beta) + 1} \right)^\alpha$$  \hspace{1cm} (A.20)$$

finally, by assuming $\alpha = 1/2$ and by evaluating at the steady state we get Eq. (18).

**A.5 Proof of Proposition 3**

In the cooperative setting, the non trivial steady state $X_C^C$ is unique and, under Assumption 1, can take positive values only. Moreover, the steady state implicitly defined by Eq. (17) is locally stable if

$$\left| \frac{\partial}{\partial X_C} \left( \frac{1}{2}(1 - \beta) \left[ (1 + a - cD) X_C^C - De \left(\frac{X_C^C}{D}\right)^2 \right] \right)^{1/2} \right| < 1$$

where we have set $\alpha = \frac{1}{2}$ and plugged the functional forms for $b(D)$ and $h(X/D)$. Which means

$$\left| \frac{(1 + a - cD)(\beta - 1)(e + D(\beta - 3) - c\beta)}{2(D(\beta - 3) + e(\beta - 1))(\beta - 3)\sqrt{\frac{D^2[1+a-cD][\beta-1]^2}{(D(\beta-3)+e(\beta-1))^2}}} \right| < 1$$

evaluated at (19).

The above inequality admits two distinct solutions
\[ D < \frac{e(1-\beta)}{\beta - 3} \]

and

\[ D > \frac{e(1-\beta)}{3(3-\beta)} \]

The first solution cannot be accepted as it gives negative values of \( D \). As for the second solution, since we are in a cooperative framework, we have to impose that \( D \geq 1 \), which gives us in turn the following condition on \( e \): \( e \geq \frac{3(3-\beta)}{1-\beta} \).

### A.6 Proof of Proposition 4

We look at the derivative of Eq. (19) with respect to \( D \)

\[
\frac{\partial X^C}{\partial D} = \frac{(3-\beta) [D(1 + a - c D)(1-\beta)] - [(1 + a - c D)(1-\beta)] [D(3-\beta) + e(1-\beta)]}{[D(3-\beta) + e(1-\beta)]^2}
\]

Since the denominator is always positive, we concentrate our analysis on the numerator

\[
(1-\beta) [e(1+a)(1-\beta) - c D (D(3-\beta) + 2 e(1-\beta))]
\]

(A.21)

The term \((1-\beta)\) is positive (since \(0 < \beta < 1\)), hence we obtain that the derivative equals zero (local maximum) when

\[
e = \frac{e D^2 (3-\beta)}{(1-\beta)(1 + a - 2 c D)} \quad \text{with} \quad a > 2 c D - 1
\]

(A.22)

Let us characterize the value of \( e \) such that the maximizing \( D \) is equal to 1 and to \( a/c \) (with \((a/c) > 1\)), respectively. We obtain

\[
\tilde{e} = \frac{c(3-\beta)}{(1-\beta)(1+a-2c)}
\]

(A.23)

\[
\tilde{\tilde{e}} = \frac{e(3-\beta)}{(1-\beta)(1-a)} \quad \text{with} \quad a < 1
\]

(A.24)

Since Eq. (A.10) is monotonically increasing in \( D \), when \( e \leq \tilde{e} \) the maximum stock of the resource is found for \( D = 2 \), while for every \( \tilde{e} < e < \tilde{\tilde{e}} \) it will be found for an integer \( D > 2 \) and, finally, for \( D = a/c \) when \( e = \tilde{\tilde{e}} \).

### A.7 Strategic resource harvesting with a lump-sum transfer

We characterize the community maximization problem

\[
\max_{z_i,k_i+1} \ln \left( (1-\beta) \frac{K_i^3}{Z_i^{-1}} z_i^i \right) + \ln \left[ \beta \frac{Z_{i+1}^{1-\beta}}{K_{i+1}^{1-\beta}} k_{i+1}^i \right]
\]

(A.25)
\begin{align*}
\text{s.t.} \quad & X_{t+1} = \left( X_t(1 + b(D_t)) - \sum_{i=1}^{D_t} y_t^i \right)^\alpha \\
& Z_{t+1} = Z_{t+1}(X_{t+1})
\end{align*}

Equation (A.25) can be rewritten as follows

\[
\ln(1 - \beta) + \beta \ln K_t - \beta \ln Z_t + \ln z_t^i + \ln \beta + (1 - \beta) \ln Z_{t+1} - (1 - \beta) \ln K_{t+1} + \ln k_{t+1}^i
\]

The community maximization problem is solved by substituting \( k_{t+1}^i = y_t^i - h \left( \frac{X_t}{Y_t} \right) - z_t^i - \omega \) and \( Z_{t+1} = \eta X_{t+1} \) into Eq. (A.25). As in the symmetric Nash equilibrium \( y_t^i = y_t^j \) and \( z_t^i = z_t^j \) \( \forall i, j = 1...D \), we obtain the following first order conditions with respect to \( z_t^i \) and \( y_t^i \):

\[
\frac{1 - \beta}{Y_t - D_t \left( h \left( \frac{X_t}{Y_t} \right) + \omega \right) - Z_t} = \frac{1}{(1 - \tau)y_t^i - h \left( \frac{X_t}{Y_t} \right)} - \frac{1}{z_t^i - \omega} - \frac{1 + \beta}{Z_t}
\]

(A.26)

\[
\frac{1 - \beta}{Y_t - D_t \left( h \left( \frac{X_t}{Y_t} \right) + \omega \right) - Z_t} = \frac{1}{(1 - \tau)y_t^i - h \left( \frac{X_t}{Y_t} \right)} - \frac{\alpha(1 - \beta)}{X_t(1 + b(D_t)) - Y_t}
\]

(A.27)

By solving Eq. (A.26) for \( Z_t \) we get

\[
Z_t = \frac{(D_t - \beta) \left[ Y_t - D_t \left( h \left( \frac{X_t}{Y_t} \right) + \omega \right) \right]}{2D_t - 1}
\]

(A.28)

By using the relationship \( Y_t - D_t h ((D_t, X_t) + \omega) = Z_t + K_{t+1} \), Eq. (A.28) can also be rewritten as

\[
K_{t+1} = \frac{(D_t - 1 + \beta) \left[ Y_t - D_t \left( h \left( \frac{X_t}{Y_t} \right) + \omega \right) \right]}{2D_t - 1}
\]

(A.29)

By substituting \( K_{t+1} \) into Eq. (A.27) and solving for \( Y_t \) we get

\[
Y_t = \frac{(2D_t - 1)(1 + b(D_t))X_t + \alpha(1 - \beta)D_t \left( h \left( \frac{X_t}{Y_t} \right) + \omega \right)}{\alpha(1 - \beta) + (2D_t - 1)}
\]

(A.30)

By substituting Eq. (A.30) into the law of motion, \( X_{t+1} = [X_t(1 + b(D_t)) - Y_t]^\alpha \), we get

\[
X_{t+1} = \frac{\alpha(1 - \beta)[X_t(1 + b(D_t)) - D_t \left( h \left( \frac{X_t}{Y_t} \right) + \omega \right)]}{\alpha(1 - \beta) + (2D_t - 1)}
\]

(A.31)

by assuming \( \alpha = 1/2 \) and by evaluating at the steady state we obtain Eq. (22).
A.8 Proof of Proposition 5

For any $\omega \neq 0$, Eq. (22) has two distinct roots, $X_\omega^a$ and $X_\omega^b$

$$X_\omega^a = \frac{D(1+a-cD)(1-\beta) - \sqrt{\Delta}}{2[D(4D-1)+e-\beta(D+e)]} \tag{A.32}$$

$$X_\omega^b = \frac{D(1+a-cD)(1-\beta) + \sqrt{\Delta}}{2[D(4D-1)+e-\beta(D+e)]} \tag{A.33}$$

where $\Delta = D^2(\beta-1) \left[ (1+a-cD)^2(\beta-1) + 4\omega(D(4D-1)+e-\beta(D+e)) \right]$.

In order to find the optimal $\omega^*$ (that is the $\omega$ that allows to decentralize the cooperative outcome) we impose

$$X^*_{\omega} = X_C$$

After some algebra we get

$$\omega^*(D) = -\frac{4D(D-1)(1+a-cD)^2(1-\beta)}{(D(3-\beta)+e(1-\beta))^2} \tag{A.34}$$

which is exactly Eq. (23). Because both the numerator and the denominator of Eq. (A.34) are always positive for $D \geq 2$, $\omega^*$ turns out to be a positive transfer: in order to discourage the young from harvesting, instead of levying a tax on the harvest, the government gives them a transfer, which is financed through a tax on capital.

Moreover, since by plugging Eq. (23) into Eq. (A.33), $X_\omega^a$ turns out to be always negative, we can assert that the positive steady state $X^\omega$ is unique.

We are finally interested in studying the stability of the positive steady state. In order to do that, we have to identify for which conditions the absolute value of the derivative of $X^\omega_{t+1}$ w.r.t. $X^\omega_t$ evaluated in Eq. (A.33) and Eq. (23) is lower than 1. Due to the complexity of the derivative, we are not able to find explicit conditions on $D$ and the parameter $e$ which ensures that the absolute value of the derivative is lower than 1, as done in Proposition 4.1 and Proposition 4.3. However, it is possible to study its limit behaviour. In particular, considering the parameter $e$ it is possible to prove that

$$\lim_{e \to 0^+} \left| \frac{\partial X_{t+1}^\omega}{\partial X_t^\omega} \right| = \left| \frac{(1+a-cD)(1-\beta)}{2(4D-1-\beta)R} \right| < 1$$

where $R = \frac{(1+a-cD)(1-\beta)+\sqrt{(D(1+a-cD)^2(1-\beta)^2+e(3-\beta)^2)^2}}{4D-1-\beta}$ with $D \leq \frac{1+a}{c}$.

Since the denominator is always bigger than the nominator, the condition is satisfied.

Similarly, we compute
\[
\lim_{\epsilon \to +\infty} \left| \frac{\partial X_t^{\omega+1}}{\partial X_t^\omega} \right| = \\
= \left| \frac{(1-\beta)M (1+D-\beta+\epsilon(D-\beta)) + 2(DN+N(1+\epsilon(1-\beta)))+\epsilon (M(D(1-\beta)+D-\beta))}{2\sqrt{2N(1-\beta)}(M(M(D-\beta)+1))} \right| > 1
\]

which is always bigger than 1 for \( \epsilon \to +\infty \) and where \( M = 1+a-cD \) and \( N = 4D-1-\beta \)
with \( D \leq \frac{1+a}{c} \).

Since the derivative is continuous, it means that there exist a \( \epsilon > 0 \) for which the condition on the stability of the steady state is satisfied.

### A.9 Proof of Corollary 1

Let \( \bar{D} \) be the number of communities for which \( X^C_{\text{max}} \) is reached in the cooperative framework. We impose

\[
X^{\omega}(D) = X^C_{\text{max}}
\]

and we obtain

\[
\bar{\omega} = -\frac{\bar{D}(1+a-c\bar{D})(1-\beta)}{D^2(\bar{D}(3-\beta)+e(1-\beta))^2} \Theta < 0 \tag{A.35}
\]

where

\[
\Theta = \bar{D} \left( 4(1+a)D-D^2 \left( 4(1+a) + c(3-\beta) \right) - e(1+a)(1-\beta) + De(1+a-cD)(1-\beta) + \right. \\
\left. + cD^2 (D(4D-1)+e-\beta(D+e)) \right)
\]
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