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Stable and efficient coalitional networks

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Abstract

We develop a theoretical framework that allows us to study which bilateral links and coalition structures are going to emerge at equilibrium. We define the notion of coalitional network to represent a network and a coalition structure, where the network specifies the nature of the relationship each individual has with her coalition members and with individuals outside her coalition. To predict the coalitional networks that are going to emerge at equilibrium we propose the concepts of strong stability and of contractual stability. Contractual stability imposes that any change made to the coalitional network needs the consent of both the deviating players and their original coalition partners. Requiring the consent of coalition members under the simple majority or unanimity decision rule may help to reconcile stability and efficiency. Moreover, this new framework can provide in- sights that one cannot obtain if coalition formation and network formation are tackled separately and independently.

Keywords: networks, coalition structures, stability, efficiency.

JEL Classification: A14, C70

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1 Introduction

The organization of agents into networks and groups has an important role in the determination of the outcome of many social and economic interactions. For instance, goods can be traded and exchanged through networks, rather than markets, of buyers and sellers. Networks also play important roles in providing mutual insurance especially in developing countries.[1] Partitioning of societies into groups is also important in many contexts, such as the provision of public goods and formation of alliances, cartels and federations. The understanding of how and why such networks and groups form and the precise way in which they affect outcomes of social and economic interactions has been apprehended separately by the coalition theory and the network theory.

One limit of both theories is that it cannot incorporate the existence of bilateral agreements among agents belonging to different coalitions – that is commonly observed in many situations. A first situation has to do with the formation of R&D joint ventures and of bilateral R&D collaborations. On the one hand, Bloch (1995) has analyzed the formation of associations of firms, like R&D joint ventures or groups of firms adopting common standards, in an oligopolistic industry. On the other hand, Goyal and Moraga-González (2001) or Goyal and Joshi (2003) have analyzed the incentives for R&D collaboration between horizontally related firms by considering that collaboration links are bilateral and are embedded within a broader network of similar links with other firms. However, it may happen that firms A and B may decide to form an R&D joint venture while firms B and C sign a bilateral R&D agreement. What is the architecture of the resulting collaboration network and the structure of associations that are likely to emerge?

A second situation has to do with the formation of communication networks (roads, railway tracks or waterways) and the provision of public goods. On the one hand, Jackson and Wolinsky (1996) have studied the incentives for agents to form links, and the strategic stability of communication networks.[2] Bramoullé and Kranton (2007) have studied the incentives to provide goods that are non-excludable along social or geographic links. On the other hand, Ray and Vohra (2001) have studied the provision of (pure) public goods when all agents can form coalitions and write binding agreements regarding their

[1]See Jackson (2008) for a comprehensive introduction to the theory of social and economic networks. Kranton and Minehart (2000) have analyzed the endogenous formation of networks between input suppliers and manufacturers while Mauleon, Sempere and Vannetelbosch (2011) have studied the formation of networks between manufacturers and retailers. Wang and Watts (2006) have examined the formation of buyer-seller networks when sellers can form an association of sellers to pool their customers. Bloch, Genicot and Ray (2008) have studied bilateral insurance schemes across networks of individuals.

[2]Bloch and Dutta (2009) have analyzed the formation of communication networks when agents choose how much to invest in each link. See also Jackson and Rogers (2005) and Johnson and Gilles (2000).

contributions toward the provision of a (pure) public good. However, there are situations where municipalities can form communication links but may belong to different regions, and costs for providing those links or public services are shared at the regional level.[3] What are the incentives for municipalities to form links and coalitions for the provision of a (pure) public good?

There are many other situations where agents are part of a network and belong to groups or coalitions. In labour markets, workers are linked to each other within each firm through a hierarchy – that is, a network – and, at the same time workers may group themselves into unions. Individuals are living their social interactions in clubs or communities as well as through friendship networks. Countries can sign bilateral free trade agreements or multilateral free trade agreements and may belong to customs unions. Connections among different criminal gangs became a major feature of the organized crime during the 1990s. Criminal gangs often develop contract relationships for the provision of certain kinds of services, such as transportation, security, contract killing, and money laundering.[4]

The aim of this paper is to develop a theoretical framework that allows us to study which bilateral links and coalition structures are going to emerge at equilibrium. We define the notion of *coalitional network* to represent a network and a coalition structure, where the network specifies the nature of the relationship each player has with her coalition members and with players outside her coalition. This new framework forces us to redefine key notions of theory of networks, *value functions* and *allocation rules*, and to redefine existing solution concepts, *strong stability* and *contractual stability.*

A strongly stable coalitional network is a coalitional network which is stable against changes in links and coalition structures by any coalition of players. The idea of contractual stability is that adding or deleting a link needs the consent of coalition partners. For instance, in the context of R&D alliances, firms may decide to have a common laboratory with some partners, while developing bilateral R&D agreements with other partners. The

[3]Basque Y is the name given to the Spanish high-speed rail network being built since 2006 between the three cities of the Basque Country Autonomous Community (Bilbao, Vitoria and San Sebastian). Since the Basque Y will connect Spain with the European high-speed network, the decision of linking the three cities and of the Y-shaped layout required the consent of the Basque Parliament and the Spanish authorities. In addition, The Spanish government manages the construction of the stretches in the provinces of Alava and Bizkaia while the Basque government takes care of the stretches in the province Gipuzkoa. See http://www.euskalyvasca.com/en/home.html

[4]Colombian-Sicilian networks brought together Colombian cocaine suppliers with Sicilian groups possessing local knowledge, well-established heroin distribution networks, extensive bribery and corruption networks, and a full-fledged capability for money laundering. Italian and Russian criminal networks have also forged cooperative relationships. See Williams (2001).

signing of a bilateral R&D agreement may need the consent of those partners within the common laboratory or joint venture. Moreover, the formation of new coalition structures may need the consent of original coalition partners.[5] Thus, once a coalition has been formed, the consent of coalitional partners may be required in order to add or delete links that affect some coalition partners, or to modify the existing coalition. As in Drèze and Greenberg (1980) the word "contractual" is used to reflect the notion that coalitions are contracts binding all members and subject to revision only with consent of coalitional partners. Two different decision rules for consent are analyzed: simple majority and unanimity.[6]

Looking at two models of coalitional network formation (a connections model with cost sharing among municipalities and a R&D model where firms form R&D bilateral agreements and belong to alliances), we observe that requiring the consent of coalition members under the simple majority or unanimity decision rule may help to reconcile stability and efficiency.[7] We also show that this new framework provides us results that one cannot obtain if coalition formation and network formation are tackled separately and independently. In general, contractually stable coalitional networks may fail to exist. We show that under the component-wise egalitarian or majoritarian allocation rules, there always exists a contractually stable coalitional network under the simple majority decision rule. However, if the component-wise dictatorial allocation rule is adopted, then a contractually stable coalitional network always exists only under the unanimity decision rule.

Our paper is related to Myerson (1980) who has studied situations in which communication is possible in conferences that can consist of an arbitrary number of players. Hence, Myerson (1980) has modeled the communication possibilities of the players by means of hypergraphs. Each element of an hypergraph is called a conference. Communication between players can only take place within a conference if all players of the conference participate. Since a conference can consist of several players, an hypergraph is a generalization of a network, which has bilateral communication channels only. In our paper, coalitions do not restrict how players can communicate to each other. Each player's payoff depend both on

[5] Rules of exit in alliances (R&D joint ventures, partnerships) are either exit at the will of the larger party subject to forewarning (simple majority rule) or exit without breach via a deadlock implemented by the contractual board where only unanimous decisions are taken (unanimity rule). See Smith (2005).

[6] All individuals who are part of a criminal organization like the Hells Angels are sponsored by an official member and have to gain the approval of 100 percent of members in order to climb the hierarchy. See Morselli (2009). Rules governing entry and exit in labor cooperatives may require the consent of partners. A new partner will enter the cooperative only if (i) she wishes to come in; (ii) her new partners wish to accept her; and (iii) she obtains from her former partners permission to withdraw (only if she was before member of another cooperative). See Drèze and Greenberg (1980).

[7] Notice that strongly stable coalitional networks are not strongly efficient in general.

the network and the coalition structure.[8] In addition, coalitions can represent contracts where each coalition member is entitled to one's say when coalition partners add or delete links to the network.[9]

The paper is organized as follows. In Section 2 we introduce the framework of coalitional networks and we define the concepts of strong stability and of contractual stability. In Section 3 we consider two models to illustrate both the framework of coalitional networks and the concepts of strong stability and of contractual stability. In Section 4 we derive some results about the existence of contractually stable coalitional networks and we look whether efficient coalitional networks are likely to be stable or not. In Section 5 we comment upon some of the features of the framework showing that it is general enough to study the emergence of community structures. Section 6 concludes.

2 Coalitional networks

2.1 Notations and definitions

Let $N = \{1, ..., n\}$ be the finite set of players who are connected in some network relationship and who belong to some coalitions or communities. A coalitional network (g, P) consists of a network $g \in G^N$ and a coalition structure $P \in \mathbb{P}$. A network g is simply a list of which pairs of players are linked to each other with $ij \in g$ indicating that i and j are linked under the network g. Let $G^N = \{g \mid g \subseteq g^N\}$ denote the set of all possible networks on N, where g^N denotes the set of all subsets of N of size 2.[10] A coalition structure $P = \{S_1, S_2, ..., S_m\}$ is a collection of coalitions satisfying: $S_a \cap S_b = \varnothing$ for $a \neq b$, $\cup_{a=1}^{m} S_a = N$ and $S_a \neq \varnothing$ for $a = 1, ..., m$. Let $S(i) \in P$ be the coalition to which player i belongs. Let $\mathbb{P}$ denote the finite set of coalition structures. A sub-coalitional network of (g, P) is (h, Q) with $h \subseteq g$ and Q a sub-collection of coalitions of P (possibly $Q = P$). A sub-coalitional network (h, Q) of (g, P) is nonempty if both h contains at least one link and Q contains at least a coalition. For instance, if $N = \{1, 2, 3, 4, 5, 6, 7, 8\}$, then $(g, P) = (\{12, 23, 45, 56, 78\}, \{\{1\}, \{2, 3, 4, 5\}, \{6, 7, 8\}\})$ is the coalitional network in which there is a link between players 1 and 2, a link between

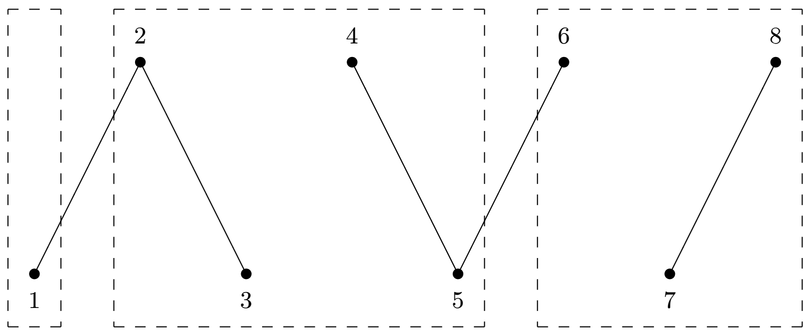
[8] Caulier, Mauleon and Vannetelbosch (2011) have also considered situations where players are part of a network and belong to coalitions. However, each player's payoff only depends on the network, and so, each player's coalition only constrains her ability to add or delete links in the network.

[9] Modeling club structures as bipartite directed networks, Page and Wooders (2010) have formulated the problem of club formation with multiple memberships as a noncooperative game of network formation. See also Bloch and Dutta (2011) for a discussion of some recent literature on the endogenous formation of coalitions and networks.

[10] Throughout the paper we use the notation $\subseteq$ for weak inclusion and $\subsetneq$ for strict inclusion. Finally, # will refer to the notion of cardinality.

players 2 and 3, a link between players 4 and 5, a link between players 5 and 6, and a link between players 7 and 8, and players 2, 3, 4 and 5 are in the same coalition while players 6, 7 and 8 are in another coalition and player 1 is alone. This coalitional network $(g, P) = (\{12, 23, 45, 56, 78\}, \{\{1\}, \{2, 3, 4, 5\}, \{6, 7, 8\}\})$ is depicted in Figure 1.

Figure 1: A coalitional network



For any network g, let $N(g) = \{i \mid \exists j \text{ such that } ij \in g\}$ be the set of players who have at least one link in the network g. For any given sub-collection Q of coalitions of P, $N(Q) = \{i \in S \mid S \in Q\}$ is the set of players that belong to some coalition $S \in Q$. Let $N(g+Q) = N(g) \cup N(Q)$. Finally, let $N(g, P)$ be the set of players who have at least one link in the network g or that belong to a coalition $S \in P$ such that at least one member of S has a link in the network g.

Definition 1. A nonempty sub-coalitional network (h, Q) is connected if for each $i \in N(h+Q)$ and $j \in N(h+Q)$ there exists a sequence of coalitions $S^1, S^2, ..., S^K$ with $i \in S^1 \in Q$ and $j \in S^K \in Q$ $(K > 1)$ such that for any $l \in \{1, ..., K-1\}$, $S^l \in Q$ and there exists $i_l i_{l+1} \in h$ with $i_l \in S^l$ and $i_{l+1} \in S^{l+1}$.[11]

Under this definition of a connected sub-coalitional network, a coalition whose members have no links is not considered as a connected sub-coalitional network.

Definition 2. A component of a coalitional network (g, P) is a nonempty sub-coalitional network (h, Q), with $h \subseteq g$ and Q a sub-collection of coalitions of P, such that

(i) $h = \{ij \in g \mid \exists\ S, S' \in Q \text{ (possibly } S = S') \text{ such that } i \in S \text{ and } j \in S'\}$,

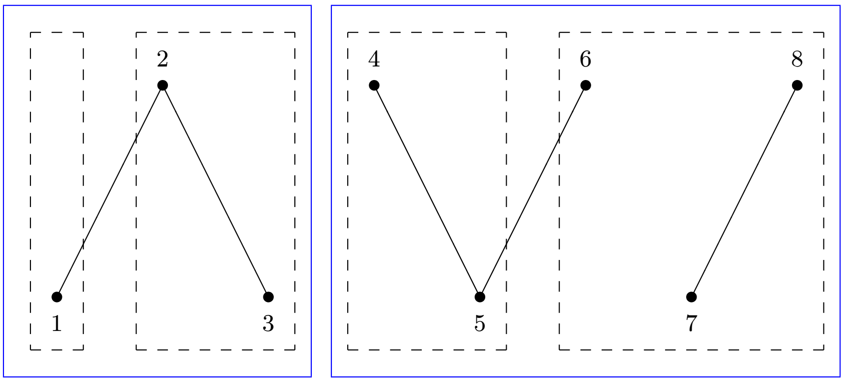
(ii) for all $S, S' \in Q$ there exists a sequence of coalitions $S^1, S^2, ..., S^K$ with $S^1 = S$ and $S^K = S'$ such that for any $l \in \{1, ..., K-1\}$, $S^l \in Q$ and there exists $i_l i_{l+1} \in h$ with $i_l \in S^l$ and $i_{l+1} \in S^{l+1}$,

[11] A nonempty sub-coalitional network consisting of only one coalition is connected since by definition of nonemptyness there is at least one link among players in that coalition.

(iii) $\nexists\, S \in P$, $S \notin Q$ and $ij \in g$ such that $i \in S_l$, $S_l \in Q$ and $j \in S$.

A component (h, Q) of (g, P) consists of a nonempty sub-network h of g and the coalitions in P that contain at least one player with a link in h and that are thus connected through the links in h. The set of components of (g, P) is denoted as $C(g, P)$ and contains the maximal connected sub-coalitional networks of (g, P). Under this definition of a component, a coalition whose members have no links is not considered as a component. Take the coalitional network $(\{12, 23, 45, 56, 78\}, \{\{1\}, \{2, 3\}, \{4, 5\}, \{6, 7, 8\}\})$ depicted in Figure 2. The connected sub-coalitional networks are $(\{12, 23\}, \{\{1\}, \{2, 3\}\})$, $(\{23\}, \{\{2, 3\}\})$, $(\{12\}, \{\{1\}, \{2, 3\}\})$, $(\{45, 56, 78\}, \{\{4, 5\}, \{6, 7, 8\}\})$, $(\{45\}, \{\{4, 5\}\})$, $(\{56\}, \{\{4, 5\}, \{6, 7, 8\}\})$, $(\{78\}, \{\{6, 7, 8\}\})$. The components are the maximal connected sub-coalitional networks, that is $(\{12, 23\}, \{\{1\}, \{2, 3\}\})$ and $(\{45, 56, 78\}, \{\{4, 5\}, \{6, 7, 8\}\})$. These two components are depicted in Figure 2.

Figure 2: A coalitional network and its components



Let $\Pi(g, P)$ denote the partition of N induced by (g, P). That is, $S \in \Pi(g, P)$ if and only if (i) there exists $(h, Q) \in C(g, P)$ such that $S = N(h, Q)$, or (ii) $S \in P$ such that for all $i \in S$, $i \notin N(g, P)$. $\Pi(\{12, 23, 45, 56, 78\}, \{\{1\}, \{2, 3\}, \{4, 5\}, \{6, 7, 8\}\}) = \{\{1, 2, 3\}, \{4, 5, 6, 7\}\}$ in the previous example.

2.2 Partition value functions and allocation rules

Different coalitional networks lead to different values of overall production or overall utility to players. These various possible valuations are represented via a partition value function. A *partition value function* is a function $v : G^N \times \mathbb{P} \to \mathbb{R}$. Let $\mathcal{V}$ be the set of all possible partition value functions. A partition value function only keeps track of how the total societal value varies across different coalitional networks. The calculation of partition value is a richer object than a partition function in a partition game and/or a value function in a network game, as it allows the value generated to depend both on the coalition structure and on the network structure. A partition value function v is *component additive* if

$$\sum_{(h,Q)\in C(g,P)} v(h,Q) = v(g,P).$$

Component additivity is a condition that rules out externalities across components but still allows them within components. A coalitional network (g, P) is *strongly efficient* relative to a partition value function v if $v(g, P) \geq v(g', P')$ for all $g' \in G^N$ and all $P' \in \mathbb{P}$.

We also wish to keep track of how that value is allocated or distributed among the players in any coalitional networks. An *allocation rule* is a function $Y : G^N \times \mathbb{P} \times \mathcal{V} \to \mathbb{R}^N$ such that

$$\sum_{i\in N} Y_i(g,P,v) = v(g,P) \text{ for all } v,\ g \text{ and } P.$$

It is important to note that an allocation rule depends on g, P and v. This allows an allocation rule to take full account of a player i's role in the network and in the coalition structure. This includes not only what the network configuration and coalition structure are, but also and how the value generated depends on the overall network and coalition structure. A coalitional network (g, P) is *Pareto efficient* relative to partition value function v and allocation rule Y if no $g' \in G^N$ and no $P' \in \mathbb{P}$ exist such that $Y_i(g', P', v) \geq Y_i(g, P, v)$ for all i with strict inequality for some i.

We propose next three allocation rules that will be helpful for obtaining existence of stable coalitional networks. For any component additive partition value function $v \in \mathcal{V}$, the *component-wise egalitarian* allocation rule Y^{ce} is such that for any $(h, Q) \in C(g, P)$ and each $i \in N(h, Q)$,

$$Y_i^{ce}(g,P,v) = \frac{v(h,Q)}{\#N(h,Q)}.$$

For any partition value function $v \in \mathcal{V}$ that is not component additive, $Y^{ce}(g, P, v)$ splits the value $v(g, P)$ equally among all players. The component-wise egalitarian rule is one in which the value of each component is split equally among the members of the component provided the partition value function is component additive.

Let i^S be the player $i \in S$, $S \subseteq N$, such that $i \leq j$ for all $j \in S$. For any component additive partition value function $v \in \mathcal{V}$, the *component-wise dictatorial* allocation rule Y^{cd} is such that for any $(h, Q) \in C(g, P)$ and each $S \in Q$,

$$Y_i^{cd}(g,P,v) = \begin{cases} v(h,Q)/\#Q & i = i^S, \\ 0 & \forall i \in S,\ i \neq i^S \end{cases}$$

For any partition value function $v \in \mathcal{V}$ that is not component additive, $Y^{cd}(g, P, v)$ splits the value $v(g, P)$ equally among all players. The component-wise dictatorial rule is one in which the value of each component is split equally among one member of each coalition belonging to the component provided the partition value function is component additive.

For any component additive partition value function $v \in \mathcal{V}$, the *component-wise majoritarian* allocation rule Y^{cm} is such that for any $(h, Q) \in C(g, P)$,

$$Y_i^{cm}(g, P, v) = \begin{cases} v(h,Q)/\sum_{S \in Q} \left[\frac{\#S}{2} + \text{mod}[\#S, 2]\right] & \forall i \in S' \subseteq S \\ 0 & \forall i \in S'' \subseteq S \end{cases}$$

with $S' \cap S'' = \varnothing$, $S' \cup S'' = S$, $\#S' \geq \#S'' \geq \frac{\#S}{2} - \text{mod}[\#S, 2]$, and $i^{S''} > j$, $\forall j \in S'$, with $i^{S''}$being the player $i \in S''$, such that $i \leq j$ for all $j \in S''$. For any partition value function $v \in \mathcal{V}$ that is not component additive, $Y^{cm}(g, P, v)$ splits the value $v(g, P)$ equally among all players. The component-wise majoritarian rule is one in which the value of each component is split equally among half members of each coalition belonging to the component provided the partition value function is component additive.

2.3 Notions of stability

A simple way to analyze the coalitional networks that one might expect to emerge in the long run is to examine a sort of equilibrium requirement that no group of players benefits from altering the coalitional network. What about possible deviations?

Definition 3. A coalitional network (g', P') is obtainable from (g, P) via S, $S \subseteq N$, if

(i) $ij \in g'$ and $ij \notin g$ implies $\{i, j\} \subseteq S$, and

(ii) $ij \notin g'$ and $ij \in g$ implies $\{i, j\} \cap S \neq \varnothing$, and

(iii) $\{S' \in P' \mid S' \subseteq N \setminus S\} = \{T \setminus S \mid T \in P,\ T \setminus S \neq \emptyset\}$, and

(iv) $\exists\ \{S'_1, S'_2, ..., S'_m\} \subseteq P'$ such that $\cup_{l=1}^m S'_l = S$.

Condition (i) asks that any new links that are added can only be between players inside S. Condition (ii) requires that there must be at least one player belonging to S for the deletion of a link.[12] Condition (iii) embodies the assumption that no simultaneous deviations are possible. So if players in S deviate leaving their coalition in P, non-deviating players do not move. Condition (iv) allows deviating players in S to form one or several coalitions in the new coalitional structure P'. Non-deviating players do not belong to those new coalitions.

[12] These first two conditions have been introduced by Jackson and van den Nouweland (2005) to define the netwoks obtainable from a given network by a coalition S.

Definition 4. A coalitional network (g, P) is strongly stable with respect to partition value function v and allocation rule Y if for any $S \subseteq N$, (g', P') obtainable from (g, P) via S and $i \in S$ such that $Y_i(g', P', v) > Y_i(g, P, v)$, there exists $j \in S$ such that $Y_j(g', P', v) \leq Y_j(g, P, v)$.

A coalitional network is said to be strongly stable if for any feasible deviation by a coalition S from (g, P) to (g', P') we have that if some player $i \in S$ gains then at least another player $j \in S$ should not gain and block the deviation from (g, P) to (g', P'). This definition of strong stability is due to Dutta and Mutuswami (1997). The definition of strong stability of Dutta and Mutuswami considers a deviation to be valid only if all members of a deviating coalition are strictly better off, while the definition of Jackson and van den Nouweland (2005) is slightly stronger by allowing for a deviation to be valid if some members are strictly better off and others are weakly better off.[13] The weaker definition has sense when transfers among players are not possible.

As in Drèze and Greenberg (1980), we may assume that coalitions are contracts binding all members and that adding or deleting a link or modifying the existing coalition requires the consent of coalition partners. Two different decision rules for consent are analyzed: simple majority and unanimity.

Definition 5. A coalitional network (g, P) is contractually stable under the unanimity decision rule with respect to partition value function v and allocation rule Y if for any $S \subseteq N$, (g', P') obtainable from (g, P) via S and $i \in S$ such that $Y_i(g', P', v) > Y_i(g, P, v)$, there exists $k \in S(j)$ with $S(j) \in P$ and $j \in S$ such that $Y_k(g', P', v) \leq Y_k(g, P, v)$.

Under the unanimity decision rule, the move from a coalitional network (g, P) to any obtainable coalitional network (g', P') needs the consent of every deviating player as well as the consent of every member of the initial coalitions of the deviating players. Then, a coalitional network is contractually stable if any deviating player or any member of the former coalitions of the deviating players is not better off from the deviation to any obtainable coalitional network (g', P').

Definition 6. A coalitional network (g, P) is contractually stable under the simple majority decision rule with respect to partition value function v and allocation rule Y if for any

[13] Notice that Jackson and van den Nouweland's (2005) version of strongly stability implies pairwise stability from Jackson and Wolinsky (1996). A network is pairwise stable if no player benefits from severing one of her links and no two players benefit from adding a link between them, with one benefiting strictly and the other at least weakly. However, Dutta and Mutuswami's (1997) version of strongly stability only implies the strict version of pairwise stability when no two players strictly benefit from adding a link between them.

$S \subseteq N$, (g', P') obtainable from (g, P) via S and $i \in S$ such that $Y_i(g', P', v) > Y_i(g, P, v)$, there exists

(*i*) $l \in S$ such that $Y_l(g', P', v) \leq Y_l(g, P, v)$, or

(*ii*) $\widehat{S} \subseteq S(j)$ with $S(j) \in P$ and $j \in S$ such that $Y_k(g', P', v) \leq Y_k(g, P, v)$ for all $k \in \widehat{S}$ and $\#\widehat{S} \geq \#S(j)/2$.

Under the simple majority decision rule, the move from a coalitional network (g, P) to any obtainable coalitional network (g', P') needs the consent of every deviating player as well as the consent of more than half members of each initial coalition of the deviating players. Then, a coalitional network is contractually stable if any deviating player or half members of some former coalition of the deviating players are not better off from the deviation to any obtainable coalitional network (g', P'). Obviously, a coalitional network that is strongly stable is contractually stable under the simple majority decision rule, and a coalitional network that is contractually stable under the simple majority decision rule is contractually stable under the unanimity decision rule. In fact each decision rule requires the consent of coalitional partners above some proportion for a deviation not to be blocked. Let q denote the proportion of coalition partners whose consent is needed for a deviation not to be blocked, $0 \leq q \leq 1$. For instance, the simple majority decision rule reverts to a proportion $q > \frac{1}{2}$ while the unanimity decision rule reverts to a quota $q = 1$.[14]

3 Two models of coalitional networks

3.1 The connections model with cost sharing

To illustrate both the framework of coalitional networks and the concepts of contractual stability we consider an alternative version of Jackson and Wolinsky (1996) symmetric connections model. Municipalities form communication links (roads, railway tracks or waterways) with each other in order to be connected and form coalitions in order to share communication costs. If municipality i is "connected" to municipality j, by a path of t links, then municipality i receives a payoff of δ^t from her indirect connection with municipality j. It is assumed that $0 < \delta < 1$, and so the payoff δ^t decreases as the path connecting municipalities i and j increases; thus travelling a long distance is less valuable.

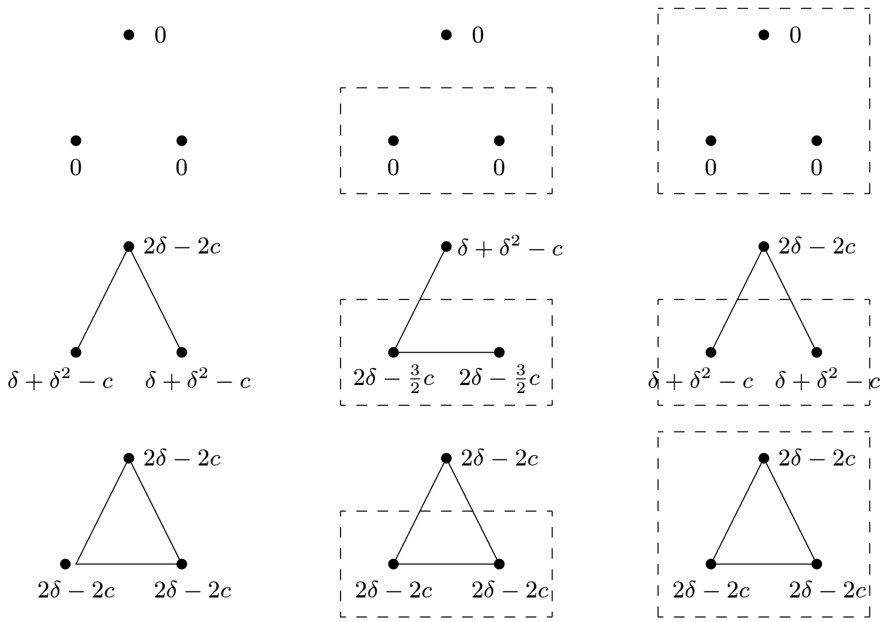
[14] The relationship between contractual stability under any decision rule embodied by a proportion q is obvious: a proportion $q' < q$ refines stability. That is, the set of contractually stable coalitional networks under q' is (weakly) included in the set of contractually stable coalitional networks under q. Indeed, the probability to block a deviation is greater the higher the proportion q. When the proportion approaches zero ($q \to 0$), coalitional membership has no matter in terms of consent.

Each direct link ij results in a cost c to both i and j. This cost can be interpreted as the cost a municipality must spend for building and maintaining a direct link with another municipality. The communication costs are shared equally within coalitions. Municipality i's payoff from a network g in a coalition $S(i) \in P$ is given by

$$Y_i(g,P) = \sum_{j \neq i} \delta^{t(ij)} - \frac{1}{\#S(i)} \sum_{j \in S(i)} \left(\sum_{k: jk \in g} c \right),$$

where $t(ij)$ is the number of links in the shortest path between i and j (setting $t(ij) = \infty$ if there is no path between i and j). Inside each coalition, the consent of members may be needed in order to modify the network and/or the coalition structure. The contractually stable coalitional networks in case of three municipalities under the simple majority decision rule are depicted in Figure 3.

Figure 3: Stable coalitional networks in the connections model with costs shared within groups



The contractually stable coalitional networks in case of three municipalities under the simple majority decision rule are $(\varnothing, \{\{1\},\{2\},\{3\}\})$ if and only if $c > \max\{3(\delta+\delta^2)/4, \delta\}$; $(\{12,13,23\}, \{\{1\},\{2\},\{3\}\})$ if and only if $c < \delta - \delta^2$; $(\varnothing, \{\{1,2,3\}\})$ if and only if $c > 3\delta/2$; $(\{12,13,23\}, \{\{1,2,3\}\})$ if and only if $c < \min\{\delta - \delta^2, 3\delta/4\}$; $(\varnothing, \{\{i,j\},\{k\}\})$ if and only if $c > \max\{3(\delta+\delta^2)/4, \delta\}$; $(\{ij,ik\}, \{\{i\},\{j\},\{k\}\})$ if and only if $\delta - \delta^2 < c < \delta$; $(\{ij,ik\}, \{\{i\},\{j,k\}\})$ if and only if $\delta - \delta^2 < c < \delta$; $(\{ij,ik\}, \{\{i,j\},\{k\}\})$ if and only if $c < \min\{\delta + \delta^2, 4\delta/3\}$; $(\{ij,ik,kj\}, \{\{i,j\},\{k\}\})$ if and only if $c < \delta - \delta^2$. While the allocation rule depends on the coalitions in P, the partition value function does not depend on the coalitions in P. Hence, the strongly efficient coalitional networks are like the strongly efficient networks of the original symmetric connections model where each municipality only bears her own costs. The strongly efficient coalitional networks are $(\{ij, ik, kj\}, \{\{i\}, \{j\}, \{k\}\})$, $(\{ij, ik, kj\}, \{\{i, j\}, \{k\}\})$ and $(\{ij, ik, kj\}, \{\{i, j, k\}\})$ if $c < \delta - \delta^2$; $(\{ij, ik\}, \{\{i\}, \{j\}, \{k\}\})$, $(\{ij, ik\}, \{\{i, j\}, \{k\}\})$, $(\{ij, ik\}, \{\{i\}, \{j, k\}\})$ and $(\{ij, ik\}, \{\{i, j, k\}\})$ if $\delta - \delta^2 < c < \delta + (\delta^2)/2$; and, $(\varnothing, \{\{i\}, \{j\}, \{k\}\})$, $(\varnothing, \{\{i, j\}, \{k\}\})$ and $(\varnothing, \{\{i, j, k\}\})$ if $\delta + (\delta^2)/2 < c$. We have that, for any parameter values, there is always a strongly efficient coalitional network which is contractually stable under the simple majority decision rule.

But what happens for more than three municipalities? For $\delta < c < \delta + ((n-2)/2)\delta^2$, the strongly efficient coalitional networks consist of a star network associated to any coalition structure but is never strongly stable. A coalitional network consisting of a star network associated to a coalition structure where (i) the central municipality is a singleton (she is alone in a coalition) is never contractually stable under the simple majority decision rule because this central municipality has incentives to cut links, (ii) the central municipality belongs to a coalition consisting of at least three municipalities is never contractually stable under the simple majority rule because the partners of the central municipality have incentives to break the coalition to become singletons. The last case to be considered is the star network associated to a coalition structure where the central municipality forms a coalition with a single partner. If $\delta + ((n-2)/n)\delta < c < \delta + ((n-2)/2)\delta^2$ then both the central municipality and her partner have incentives to cut all their links. However, if $\delta < c < \min\{\delta + ((n-2)/n)\delta, \delta + ((n-2)/2)\delta^2\}$, then the central municipality's partner does not want to cut the link she has with the central municipality. We conclude that, for $\delta + ((n-2)/n)\delta < c < \delta + ((n-2)/2)\delta^2$, no strongly efficient coalitional network is contractually stable under the simple majority decision rule; for $\delta < c < \min\{\delta + ((n-2)/n)\delta, \delta + ((n-2)/2)\delta^2\}$, the coalitional network consisting of the star network associated to a coalition structure where the central municipality forms a coalition with a single partner is strongly efficient and contractually stable under the simple majority decision rule. For $c < \delta(1-\delta)$, it is straightforward that the strongly efficient coalitional network consisting of the complete network and the coalition structure where all municipalities are singletons is contractually stable under the simple majority rule.

Proposition 1. *Take the symmetric connections model with communication costs shared within groups. For $c < \delta(1-\delta)$, (g, P) is contractually stable under the simple majority rule and strongly efficient if g is the complete network and $\#S(i) = 1$ $\forall i \in N$. For $\delta < c < \min\{\delta + ((n-2)/n)\delta, \delta + ((n-2)/2)\delta^2\}$, (g, P) is contractually stable under the simple majority rule and strongly efficient if g is a star network encompassing all municipalities and $\#S(i^*) = 2$, $S(i^*) \in P$ with i^* being the center of the star network.*

For $\delta + ((n-2)/n)\delta < c < \delta + ((n-2)/2)\delta^2$, no strongly efficient (g, P) is contractually stable under the simple majority rule. For $\delta + ((n-2)/2)\delta^2 < c$, (g, P) is contractually stable under the simple majority rule and strongly efficient if g is the empty network and $\#S(i) = 1 \; \forall i \in N$.

3.2 R&D coalitional networks

We develop a three-stage game in a setting with n competing firms that produce some homogeneous good. Let q_i denote the quantity of the good produced by firm $i \in N$. In the first stage, firms decide the bilateral R&D collaborations (or links) they are going to establish and the alliances (or coalitions) they want to form in order to maximize their respective profits. The collection of pairwise links between the firms and the alliances define a R&D coalitional network. In the second stage, firms can undertake R&D to look for cost reducing innovations. The cost function for technology is given by $\frac{\gamma}{2}(x_i)^2$, where x_i is the research output undertaken by firm i, $i \in N$. Firms belonging to the same alliance (or coalition) decide the amount of research output that each of them has to undertake in order to maximize the joint profits of the alliance. Given a network g and the collection of research outputs $\{x_1, ..., x_n\}$, the marginal cost of production of firm i is given by

$$c_i(g, P) = c - x_i(g, P) - \sum_{\substack{j:ij \in g \\ \text{or } j \in S(i)}} x_j(g, P) - \sum_{\substack{j:ij \notin g \\ \text{and } j \notin S(i)}} \mu \cdot x_j(g, P)$$

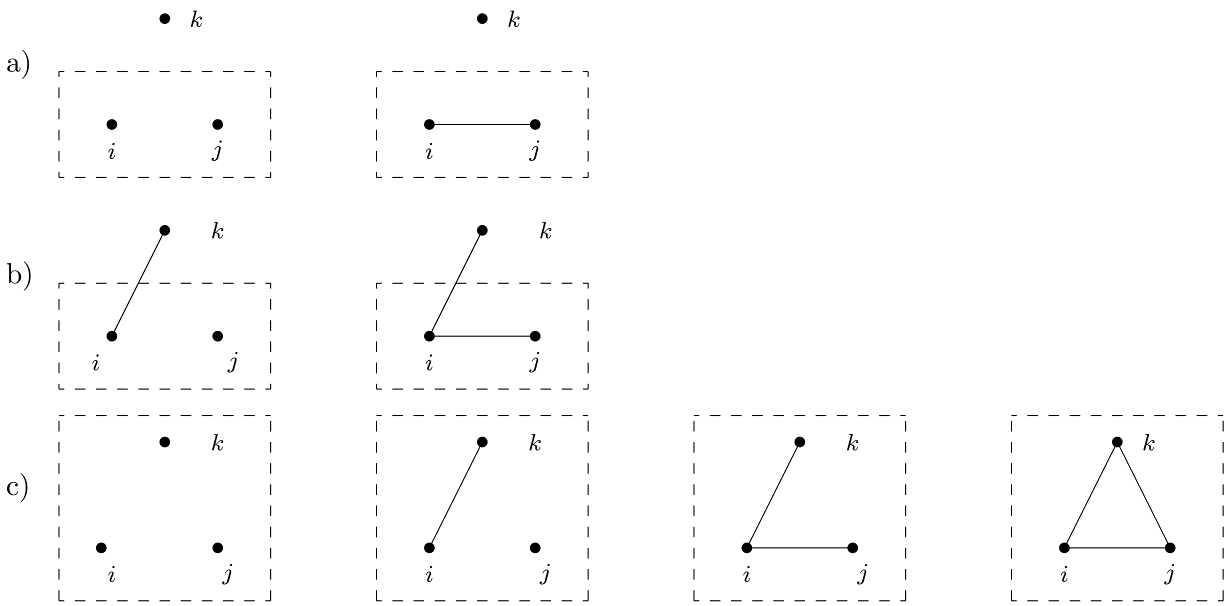
where the parameter $\mu \in (0, 1)$ measures the public knowledge spillovers obtained from indirectly connected partners and unconnected firms that are not in the same alliance. Notice that the transmission of knowledge among linked firms and among firms in the same alliance is fully appropriated. In the third stage, firms compete in quantities in the oligopolistic market, taking as given the costs of production. Let $p(q) = a - q$ be the market-clearing price when aggregate quantity on the market is $q \equiv \sum_{i \in N} q_i$. More precisely, $p(q) = a - q$ for $q < a$, and $p(q) = 0$ otherwise, with $a > 0$. Given a R&D coalitional network (g, P), the profits of firm $i \in N$ are given by

$$Y_i(g, P) = \left(a - q_i(g, P) - \sum_{j \neq i} q_j(g, P) - c_i(g, P) \right) \cdot q_i(g, P) - \frac{\gamma}{2} (x_i(g, P))^2 .$$

This three-stage game is solved backwards. We first look for subgame perfect equilibria of the multi-stage game made up of stage two to stage three. Then, stage one is solved using the concept of contractual stability.

Suppose that $n = 3$ and $\gamma = 4$ (minimum value for γ that ensures that all equilibrium outputs are positive for any $\mu \in (0, 1)$). The third stage equilibrium can be easily computed as a function of the different firms' marginal costs for any given coalitional network (g, P). The equilibrium quantity and the profits of firm i in any coalitional network (g, P) are: $q_i(c_1(g, P), c_2(g, P), c_3(g, P)) = \frac{a-4c_i+\sum_j c_j}{4}$ and $Y_i(g, P) = (q_i(c_1(g, P), c_2(g, P), c_3(g, P))^2 - \frac{\gamma}{2}(x_i(g, P))^2$ with $i \in N = \{1, 2, 3\}$. Next equilibrium research output levels are computed according to the R&D structure defined by any given coalitional network (g, P). Finally, the contractually stable coalitional networks under the unanimity decision rule are depicted in Figure 4.[15]

Figure 4: Contractually Stable coalitional networks in the RD model



Proposition 2. *The contractually stable coalitional networks under the unanimity decision rule are:*

a) $(\varnothing, \{\{i, j\}, \{k\}\})$ *and* $(\{ij\}, \{\{i, j\}, \{k\}\})$*, one coalition of two firms (linked or not) and an isolated singleton firm if* $\mu < \frac{1}{2}$*.*

b) $(\{ik\}, \{\{i, j\}, \{k\}\})$ *and* $(\{ij, ik\}, \{\{i, j\}, \{k\}\})$*, one coalition of two firms (linked or not) with one of the two firms linked to the singleton firm if* $\mu < 0.737$*.*

c) $(g, \{\{N\}\})$*, the grand coalition of firms with any possible network* g*,* $\forall g \in G^3$*.*

Note that the set of contractually stable coalitional networks under the unanimity decision rule includes three different types of coalitional networks. It is interesting to note that the network structure inside a coalition of a contractually stable coalitional network does not affect the stability of the coalitional network since the transmission of information can take place through the link or through the coalition. All firms get the same profits in $(g, \{\{N\}\})$ regardless of the particular g. This is a general fact in this example. In case of

[15] All equilibrium expressions and proofs are available from the authors upon request.

a coalition with two firms, the existence or not of a link between them does not affect the level of profits they obtain. In case of no public spillovers, i.e. $\mu = 0$, all the coalitional networks in a), b) and c) are contractually stable. However, as the level of public spillovers increases the set of contractually stable coalitional networks under the unanimity decision rule shrinks. The asymmetric coalitional networks become unstable since the three firms gain moving to the grand coalition due to the fact that a larger μ reduces the strategic use of R&D output levels.

One interesting question is to investigate whether the efficient coalitional networks are included in the set of contractually stable coalitional networks. Note that, for this example, efficiency is attained when the grand coalition forms since joint industry profits are maximized when the equilibrium research outputs are set jointly. Moreover, since links are not costly, any network inside the grand coalition give rises the same level of profits. Hence, the efficient coalitional networks are $(g, \{\{N\}\})$ for all $g \in G^3$. Thus, the efficient coalitional networks are contractually stable coalitional networks under the unanimity decision rule for any μ.

Proposition 3. *The contractually stable coalitional networks under the simple majority decision rule are:*
a) $(\varnothing, \{\{i,j\},\{k\}\})$ *and* $(\{ij\}, \{\{i,j\},\{k\}\})$*, one coalition of two firms (linked or not) and an isolated singleton firm if* $\mu < \frac{1}{2}$*.*
b) $(\{ik\}, \{\{i,j\},\{k\}\})$ *and* $(\{ij, ik\}, \{\{i,j\},\{k\}\})$*, one coalition of two firms (linked or not) with one of the two firms linked to the singleton firm if* $\mu < 0.737$*.*
c) $(\{ij, ik, jk\}, \{\{N\}\})$ *and* $(\{ij, ik\}, \{\{N\}\})$*, the grand coalition of firms with the complete or the star networks for all* μ*.*
d) $(\varnothing, \{\{N\}\})$ *and* $(\{ij\}, \{\{N\}\})$*, the grand coalition of firms with the empty or the partially connected networks for all* $\mu > \frac{1}{2}$*.*

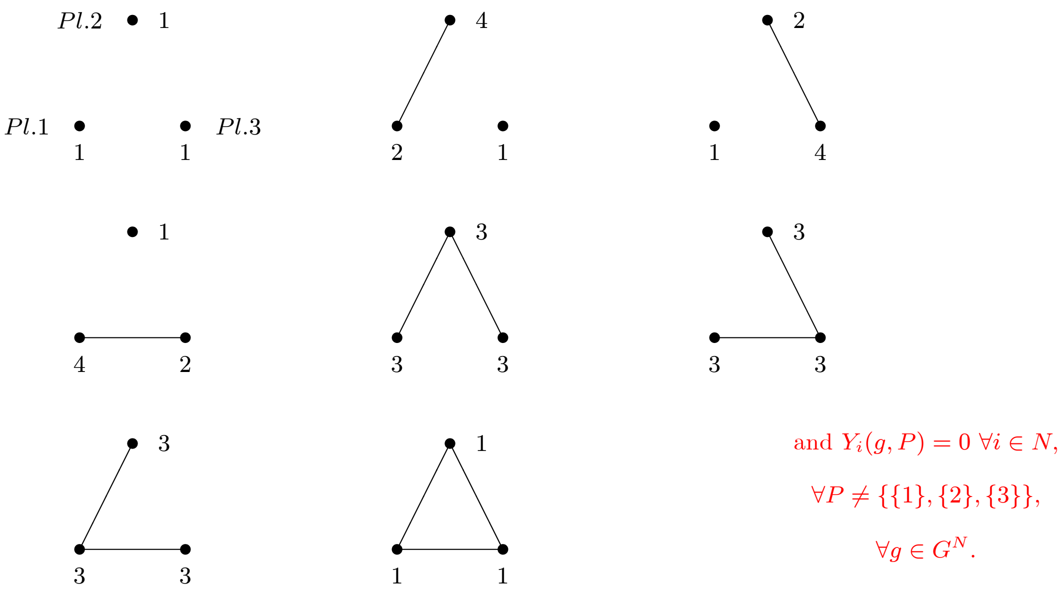
The change in the decision rule affects the set of contractually stable coalitional networks for small public spillovers. The coalitional networks with the empty or partially connected networks and the grand coalition (part d) in Proposition 3) are unstable against deviations of two firms (a simple majority of firms) who form a coalition leaving behind the other firm. By doing so, the deviating firms obtain a significant strategic advantage over the firm left alone that implies higher profits than the ones obtained under the grand coalition. Therefore, when $\mu = 0$, only parts a), b) and c) of Proposition 3 apply. However, coalitional networks with the complete or star networks and the grand coalition are contractually stable for any value of μ. This is an interesting illustration of the claim that coalition formation and network formation cannot be tackled independently. In this particular example, any change in the network structure inside a coalition has no effect on firms profits but it has consequences on the stability of coalitional networks. In contrast with the unanimity decision rule case, not all the efficient coalitional networks are contractually stable coalitional networks under the simple majority decision rule for any μ.

Finally, we investigate whether the efficient coalitional networks are contractually stable under the unanimity and the simple majority decision rules for an arbitrary number of firms, $n \geq 3$. First of all, note that the research outputs chosen under the grand coalition maximize the aggregate profits of the industry. Therefore, the efficient coalitional networks are contractually stable under the unanimity decision rule since any potential improvement of a group of deviating firms reduces the profits of the non-deviating firms. Hence, any deviation from the grand coalition will be blocked. Take now the case of no public spillovers, $\mu = 0$. Then, the efficient coalitional network $(\varnothing, \{\{N\}\})$ is not contractually stable under the simple majority decision rule. The deviation of a coalition consisting of $n-1$ firms to $(\varnothing, \{\{N-i\}, \{i\}\})$ is profitable for the deviating firms and they have the majority inside the grand coalition. However, if $\mu = 1$, the efficient coalitional network $(\varnothing, \{\{N\}\})$ is stable against the same type of deviation. In fact, it is stable against any deviation that splits the grand coalition into two coalitions. Indeed, the coalition that benefits by deviating it is always the smaller one. Then, the non-deviating firms can always block the deviation of the smaller coalition.

4 Stability and Pareto efficiency

There can be many contractually stable coalitional networks in the connections model or in the R&D model. However, it is easy to find an example where a contractually stable coalitional network fails to exist. Take $N = \{1, 2, 3\}$ and $\overline{P} = \{\{1\}, \{2\}, \{3\}\}$. Payoffs are $Y_i(\varnothing, \overline{P}) = 1$, $Y_1(\{23\}, \overline{P}) = 1$, $Y_2(\{23\}, \overline{P}) = 2$, $Y_3(\{23\}, \overline{P}) = 4$, $Y_1(\{13\}, \overline{P}) = 4$, $Y_2(\{13\}, \overline{P}) = 1$, $Y_3(\{13\}, \overline{P}) = 2$, $Y_1(\{12\}, \overline{P}) = 2$, $Y_2(\{12\}, \overline{P}) = 4$, $Y_3(\{12\}, \overline{P}) = 1$, $Y_i(\{13, 23\}, \overline{P}) = 3$, $Y_i(\{12, 13\}, \overline{P}) = 3$, $Y_i(\{12, 23\}, \overline{P}) = 3$, $Y_i(\{12, 13, 23\}, \overline{P}) = 1$, and $Y_i(g, P) = 0$ $\forall i \in N$, $\forall P \neq \overline{P}$, $\forall g \in G^N$. The coalitional networks with positive payoffs are depicted in Figure 4.

Figure 4: Non-existence of contractually stable networks



We now study the existence of stable coalitional networks. Let g^S be the set of all subsets of $S \subseteq N$ of size 2. Let

$$(h, Q)[S] = \underset{\substack{h \subseteq g^S, Q \subseteq P \\ \text{s.t. } (h,Q) \text{ is connected}}}{\operatorname{argmax}} \frac{v(h, Q)}{\# N(h, Q)}$$

be the connected sub-coalitional network with the highest per capita value out of those that can be formed by players in $S \subseteq N$. Given a component additive partition value function v, find a coalitional network $(g, P)^{v,ce}$ through the following algorithm. Pick some $(g_1, Q_1) \in (h, Q)[N]$. Next, pick some $(g_2, Q_2) \in (h, Q)[N \setminus N(g_1, Q_1)]$. At stage k pick some $(g_k, Q_k) \in (h, Q)[N \setminus \cup_{i \leq k-1} N(g_i, Q_i)]$. Since N is finite this process stops after a finite number K of stages. The union of the components picked in this way defines a coalitional network $(g, P)^{v,ce}$ which is Pareto efficient.[16]

Proposition 4. *Under a component additive partition value function v, a coalitional network $(g, P)^{v,ce}$ defined by the preceding algorithm is strongly stable under the component-wise egalitarian allocation rule Y^{ce}.*

Proof. Given the algorithm and the component-wise egalitarian allocation rule Y^{ce}, the players in $N(g_1, Q_1)$ obtain the highest possible payoff they can get. So, no player in

[16] Jackson (2005) has proposed a similar algorithm for finding a network that is pairwise stable and Pareto efficient under the classic component-wise egalitarian rule.

$N(g_1, Q_1)$ will deviate from $(g, P)^{v,ce}$. Players in any $N(g_k, Q_k)$, $k = 2, ..., K$, obtain the highest possible payoff they can get among the players in $N \setminus \cup_{i\leq k-1} N(g_i, Q_i)$. However, their payoff is smaller than the payoff of players in $N(g_j, Q_j)$ with $j = 1, ..., k-1$. Although players in $N(g_k, Q_k)$ would like to be in any $N(g_j, Q_j)$ with $j = 1, ..., k-1$, no player in that components would like to change its position in $N(g_j, Q_j)$, $j = 1, ..., k-1$, with the position of any player in $N(g_k, Q_k)$. □

Let

$$(h, Q)[S] = \underset{\substack{h \subseteq g^S, Q \subseteq P \text{ s.t.} \\ (h,Q) \text{ is connected}}}{\text{argmax}} \frac{v(h, Q)}{\left(\sum_{S' \in Q} \left(\frac{\#S'}{2} + \text{mod}[\#S', 2] \right) \right)}.$$

be the connected sub-coalitional network out of those that can be formed by players in $S \subseteq N$ with the highest per capita value for a majority of players in each S', $S' \in Q$. Given a component additive partition value function v, a similar algorithm as before provides us a coalitional network $(g, P)^{v,cm}$.

Proposition 5. *Under a component additive partition value function v, a coalitional network $(g, P)^{v,cm}$ defined by the preceding algorithm is contractually stable under the simple majority decision rule and the component-wise majoritarian allocation rule Y^{cm}.*

Proof. Given the algorithm and the component-wise majoritarian allocation rule Y^{cm}, a majority of players in each coalition S', $S' \in Q_1$, in the component $(g_1, Q_1) \in (h, Q)[N]$ obtain the highest possible payoff they can get. So, any (g', P') obtainable from $(g, P)^{v,cm}$ via some coalition S containing some members of $N(g_1, Q_1)$ would be blocked by the majority of players in each coalition S', $S' \in Q_1$. Moreover, a majority of players in each coalition S', $S' \in Q_k$, $k = 2, ..., K$, in the component $(g_k, Q_k) \in (h, Q)[N \setminus \cup_{i\leq k-1} N(g_i, Q_i)]$ obtain the highest possible payoff they can get among the players in $N \setminus \cup_{i\leq k-1} N(g_i, Q_i)$. So, any (g', P') obtainable from $(g, P)^{v,cm}$ via some coalition $S \subset N \setminus \cup_{i\leq k-1} N(g_i, Q_i)$ containing some members of $N(g_k, Q_k)$ would be blocked by the majority of players in each coalition S', $S' \in Q_k$. However, the majority of players in each S', $S' \in Q_k$, in the component (g_k, Q_k) receive a smaller payoff than a majority of players in each S', $S' \in Q_j$, in each component (g_j, Q_j), for $j = 1, ..., k-1$. But any (g', P') obtainable from $(g, P)^{v,cm}$ via S, involving some players in some (g_j, Q_j), $j = 1, ..., k-1$, would be blocked by a majority of players in each coalition S', $S' \in Q_j$. □

Let

$$(h, Q)[S] = \underset{\substack{h \subseteq g^S, Q \subseteq P \\ \text{s.t. } (h,Q) \text{ is connected}}}{\text{argmax}} \frac{v(h, Q)}{\#Q}.$$

be the connected sub-coalitional network out of those that can be formed by players in $S \subseteq N$ with the highest per capita value for a single player in each S', $S' \subseteq Q$. Given a component additive partition value function v, a similar algorithm as before provides us a coalitional network $(g, P)^{v,cd}$.

Proposition 6. *Under a component additive partition value function v, a coalitional network $(g, P)^{v,cd}$ defined by the preceding algorithm is contractually stable under the unanimity decision rule and the component-wise dictatorial allocation rule Y^{cd}.*

Proof. Given the algorithm and the component-wise dictatorial allocation rule Y^{cd}, a single player in each coalition S', $S' \in Q_1$, in the component $(g_1, Q_1) \in (h, Q)[N]$ obtains the highest possible payoff she can get. So, any (g', P') obtainable from $(g, P)^{v,cm}$ via some coalition S containing some members of $N(g_1, Q_1)$ would be blocked by the player that obtains the highest possible payoff in each coalition S', $S' \in Q_1$. Moreover, a single player in each coalition S', $S' \in Q_k$, in the component $(g_k, Q_k) \in (h, Q)[N \setminus \cup_{i \leq k-1} N(g_i, Q_i)]$ obtains the highest possible payoff she can get among the players in $N \setminus \cup_{i \leq k-1} N(g_i, Q_i))$. So, any (g', P') obtainable from $(g, P)^{v,cd}$ via some coalition $S \subset N \setminus \cup_{i \leq k-1} N(g_i, Q_i)$ containing some members of $N(g_k, Q_k)$ would be blocked by the player obtaining the highest payoff in each coalition S', $S' \in Q_k$. Finally, any (g', P') obtainable from $(g, P)^{v,cd}$ via S, involving some players in some (g_j, Q_j), $j = 1, ..., k-1$, would be blocked by the player receiving the highest payoff in each coalition S', $S' \in Q_j$. □

5 Community structures

Many real world social and economic networks are composed of many communities of nodes, where the nodes of the same community are highly connected, while there are few links between the nodes of different communities.[17] The framework of coalitional networks is general enough to study the emergence of "community structures" where links between individuals belonging to different communities are infeasible. Suppose that two players can be linked to each other only if they belong to the same coalition. Then, the set of feasible coalitional networks becomes

$$\{(g, P) \in G^N \times \mathbb{P} \mid ij \in g \text{ only if } S(i) = S(j)\}.$$

This situation may be interpreted as a limit case of community structures.

Proposition 7. *Suppose that two players can be linked to each other only if they belong to the same coalition. Then, under a component additive partition value function v, strongly*

[17] See for instance Jackson (2008) or Wasserman and Faust (1994). Research on community structures mainly deals with the detection of these communities in network data.

efficient community structures are always contractually stable under the unanimity decision rule.

If there are no externalities among coalitions (which coincide with components since players cannot be linked to players belonging to other coalitions), then it is possible to stabilize the strongly efficient community structures thanks to the unanimity decision rule, and this, whatever the allocation rule. However, once only the consent of more than half members of the initial coalitions of the deviating players is required, then we need to impose a specific allocation rule to stabilize the strongly efficient community structures.

Proposition 8. *Suppose that two players can be linked to each other only if they belong to the same coalition. Under a component additive partition value function v, strongly efficient community structures are contractually stable under the simple majority decision rule and the component-wise majoritarian allocation rule.*

Proof. Let $\{(g, P) \in G^N \times \mathbb{P} \mid ij \in g \text{ only if } S(i) = S(j)\}$ be the set of feasible coalitional networks. Then, for any component additive partition value function v, the component-wise majoritarian allocation rule Y^{cm} is such that for any $(h, S) \in C(g, P)$, $Y_i^{cm}(g, P, v) = v(h, S)[\frac{\#S}{2} + \text{mod}[\#S, 2]]^{-1}\ \forall i \in S' \subseteq S$ and $Y_i^{cm}(g, P, v) = 0\ \forall i \in S'' \subseteq S$, with $S' \cap S'' = \varnothing$, $S' \cup S'' = S$, $\#S' \geq \#S'' \geq \frac{\#S}{2} - \text{mod}[\#S, 2]$, and $i^{S''} > j\ \forall j \in S'$. Let $(g, P)^*$ be an efficient coalitional network with $P = \{S_1^*, S_2^*, ..., S_m^*\}$. First, any deviation from $(g, P)^*$ to any (g', P) by a coalition $S \subseteq S_j^*$ will be blocked because $(g, P)^*$ is efficient and hence in (g', P) players in $S' \subseteq S_j^*$ are worse off than in $(g, P)^*$ and players in $S'' \subseteq S_j^*$ are equal off. Second, any deviation from $(g, P)^*$ to any (g', P') by a coalition $S = S_1^* \cup S_2^* \cup ...$ with $P' = P \backslash \{S_1^*, S_2^*, ...\} \cup \{S_1^* \cup S_2^* \cup ...\}$ will be blocked by all the deviating players in $S_1^* \cup S_2^* \cup ...$ that now obtain a payoff of zero (and a positive payoff in $(g, P)^*$). Third, any deviation from $(g, P)^*$ to any (g', P') by a coalition $S \subseteq S_j^*$ with $P' = P \setminus \{S_j^*\} \cup \{S_j' \cup S_j'' \cup ...\}$ and $S_j^* = S_j' \cup S_j'' \cup ...$ will be blocked by all the deviating players that now obtain a payoff of zero in every $S_j', S_j'', ...$, with $S_j^* = S_j' \cup S_j'' \cup ...$ Fourth, any deviation from $(g, P)^*$ to any (g', P') by a coalition S with $P' = P \setminus \{S_1^*, S_2^*\} \cup \{S\} \cup \{S_1^* \setminus (S_1^* \cap S)\} \cup \{S_2^* \setminus (S_2^* \cap S)\}$ will be blocked by all the deviating players that now obtain a payoff of zero in (g', P'). □

6 Conclusion

We have developed a theoretical framework that allows us to study which bilateral links and coalition structures are going to emerge at equilibrium. We have introduced the notion of coalitional network to represent a network and a coalition structure, where the network specifies the nature of the relationship each individual has with her coalition members and with individuals outside her coalition. To predict the coalitional networks that are going to emerge at equilibrium we have used the concepts of strong stability and of contractual stability. Contractual stability requires that any change made to the coalitional network needs the consent of both the deviating players and their original coalition partners. We have shown that there always exists a contractually stable coalitional network under the simple majority decision rule and the component-wise egalitarian or majoritarian allocation rules. However, once we adopt the component-wise dictatorial allocation rule, a contractually stable coalitional network always exists only under the unanimity decision rule. Hence, requiring the unanimity for consent may be too strong since it can help to support extreme allocations. Finally, we have shown that requiring the consent of group members under the simple majority or the unanimity decision rule may help to reconcile stability and efficiency of coalitional networks or community structures.

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References

[1] Bloch, F., “Endogenous Structures of Association in Oligopolies,” *Rand Journal of Economics* 26, 537-556 (1995).

[2] Bloch, F. and B. Dutta, “Communication Networks with Endogenous Link Strength,” *Games and Economic Behavior* 66, 39-56 (2009).

[3] Bloch, F. and B. Dutta, “Formation of Networks and Coalitions,” in *Handbook of Social Economics*, edited by J. Benhabib, A. Bisin and M.O. Jackson, North Holland: Amsterdam (2011).

[4] Bloch, F., G. Genicot and D. Ray, “Informal Insurance in Social Networks,” *Journal of Economic Theory* 143, 36-58 (2008).

[5] Bramoullé, Y. and R. Kranton, “Public Goods in Networks,” *Journal of Economic Theory* 135, 478-494 (2007).

[6] Caulier, J.F., A. Mauleon and V. Vannetelbosch, “Contractually Stable Networks,” mimeo (2011).

[7] Drèze, J.H. and J. Greenberg, “Hedonic Coalitions: Optimality and Stability,” *Econometrica* 48, 987-1003 (1980).

[8] Dutta, B. and S. Mutuswami, “Stable Networks,” *Journal of Economic Theory* 76, 322-344 (1997).

[9] Goyal, S. and S. Joshi, “Networks of Collaboration in Oligopoly,” *Games and Economic Behavior* 43, 57-85 (2003).

[10] Goyal, S. and J.L. Moraga-González, “R&D Networks,” *Rand Journal of Economics* 32, 686-707 (2001).

[11] Jackson, M.O., “A Survey of Models of Network Formation: Stability and Efficiency,” in *Group Formation in Economics: Networks, Clubs and Coalitions*, edited by G. Demange and M. Wooders, Cambridge University Press: Cambridge (2005).

[12] Jackson. M.O., *Social and Economic Networks*, Princeton University Press: Princeton (2008).

[13] Jackson, M.O. and B.W. Rogers, “The Economics of Small Worlds,” *Journal of the European Economic Association* 3, 617-627 (2005).

[14] Jackson, M.O. and A. van den Nouweland, “Strongly Stable Networks,” *Games and Economic Behavior* 51, 420-444 (2005).

[15] Jackson, M.O. and A. Wolinsky, “A Strategic Model of Social and Economic Networks,” *Journal of Economic Theory* 71, 44-74 (1996).

[16] Johnson, C. and R.P. Gilles, “Spatial Social Networks,” *Review of Economic Design* 5, 273-300 (2000).

[17] Kranton, R. and D. Minehart, “Networks versus Vertical Integration,” *Rand Journal of Economics* 31, 570-601 (2000).

[18] Mauleon, A., J.J. Sempere-Monerris and V. Vannetelbosch, “Networks of Manufacturers and Retailers,” *Journal of Economic Behavior and Organization* 77(3), 351-367 (2011).

[19] Morselli, C., *Inside Criminal Networks*, Springer: New York (2009).

[20] Myerson, R.B., “Conference Structures and Fair Allocation Rules,” *International Journal of Game Theory* 9(3), 169-182 (1980).

[21] Page Jr., F.H. and M. Wooders, “Club Networks with Multiple Memberships and Noncooperative Stability,” *Games Economic Behavior* 70, 12-20 (2010).

[22] Ray, D. and R. Vohra, “Coalitional Power and Public Goods,” *Journal of Political Economy* 109, 1355-1384 (2001).

[23] Smith, D.G., “The Exit Structure of Strategic Alliances,” *University of Illinois Law Review* 1, 303-318 (2005)

[24] Wang, P. and A. Watts, “Formation of Buyer-Seller Trade Networks in a Quality-Differentiated Product Market,” *Canadian Journal of Economics* 39, 971-1004 (2006).

[25] Wasserman, S. and K. Fraust, *Social Network Analysis: Methods and Applications*, Cambridge University Press: Cambridge (1994).

[26] Williams, P., “Transnational Criminal Networks,” in *Networks and Netwars: The Future of Terror, Crime, and Militancy*, edited by J. Arquilla and D. Ronfeldt, RAND Corporation: Santa Monica (2001).

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