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Social security and economic integration

Lionel Artige, Antoine Dedry and Pierre Pestieau

DISCUSSION PAPER

Center for Operations Research and Econometrics
Voie du Roman Pays, 34
B-1348 Louvain-la-Neuve
Belgium
http://www.uclouvain.be/core
Abstract

The purpose of this letter is to analyze the impact of economic integration when countries differ in their social security systems, more specifically in the degree of funding of their pensions, and in the regulation of the retirement age. Funding and mandatory early retirement are two features which foster capital accumulation relative to pay-as-you-go pensions with flexible retirement. In case of economic integration they both imply some capital outflow and may lead to some utility losses.

Keywords: economic union, tax competition, social security.

JEL Classification: H2, F42, H8

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1 University of Liège, Belgium.
2 University of Liège, Belgium.
3 CREPP, Université catholique de Louvain, CORE, B-1348 Louvain-la-Neuve, Belgium. E-mail: p.pestieau@ulg.ac.be

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1 Introduction

It is well known that economic integration can have unpleasant implications for countries, which are relatively less indebted than others. Whether the debt we have in mind is the traditional sovereign debt or the debt that is implicit to unfunded pension schemes, free capital mobility leads to an outflow from countries with sound public finances to indebted countries. This consideration justified the Maastricht Treaty guidelines of the European Union: a deficit of less than 3% and a debt to GDP ratio not exceeding 60%. It is interesting to observe that the Maastricht Treaty was unable to touch the other less explicit forms of indebtedness.

Besides indebtedness, there are other national characteristics that have the same implications and that have not received the same attention. One of them concerns the more or less flexibility of the retirement decision. There are a wide variety of regulations concerning the age of retirement across OECD countries\(^1\) and this leads to an important range in the effective age of retirement. This has some implications for saving and capital accumulation. The life cycle theory of saving is quite explicit: the later individuals retire, the less they have to save. If someone wants and is allowed to work till the end of his life he will need to save much less than someone who decides or is forced to retire at, let us say age 55, which is frequent in countries such as France or Belgium.

In this paper we are interested in the role of two features of the retirement systems in case of economic integration: whether it is funded or not and whether it comprises flexible or mandatory early retirement age. The impact of funding has been widely studied\(^2\). It is largely equivalent to the impact of public debt in an economic union. In contrast the effect of mandatory versus flexible retirement has received little attention in the literature. Using an overlapping generations model (OLG) in the steady state, we show that both a PAYG pension system and a totally endogenous retirement age imply an inflow of capital from countries with fully funded pensions and mandatory early retirement. In the real world one find all sorts of pension systems even though in the OECD the most frequent one is PAYG systems with mandatory early retirement.\(^3\)

2 The basic model: autarky

We use the standard overlapping generations model. An individual belonging to generation \(t\) lives two periods \(t\) and \(t + 1\). The first one has a unitary length, while

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\(^1\)See Fenge and Pestieau (2005).
\(^2\)See Casarico (2000).
\(^3\)For recent evidence, see EC (2008) and OECD (2011).
the second has a length $\ell \leq 1$, where $\ell$ reflects variable longevity. In the first period, the individual works and earns $w_t$ which is devoted to the first-period consumption, $c_t$, saving $s_t$ and pension contribution $\tau$. In the second period he works an amount of time $z_{t+1} \leq \ell \leq 1$ and earns $z_{t+1}w_{t+1}$. This earning plus the proceeds of saving $R_{t+1}s_t$ and the PAYG pension $p$ finances second period consumption $d_{t+1}$. Working $z_{t+1}$ implies a monetary disutility $v(z_{t+1}, \ell)$ where $\frac{\partial v}{\partial z} < 0$ reflects the idea that an increase in longevity fosters later retirement. Note in this simple model we assume for simplicity that earnings in the second period of life is not taxed; also we assume that a fully funded system is identical to standard saving. Thus the parameter $\tau$ measures the relative size of the unfunded pensions. In other words $\tau = 0$ implies that the whole pension system is funded.

Denoting by $u(\cdot)$ the utility function for consumption $c$ or $d$ and $U$ the lifetime utility, the problem of an individual of generation $t$ is:

$$\max U = u(w_t - \tau - s_t) + \beta \ell u\left(\frac{w_{t+1}z_{t+1} + R_{t+1}s_t + p - v(z_{t+1}, \ell)}{\ell}\right)$$

(1)

where $p = \tau(1 + n)$ and $\beta$ is the time discount factor. $(1 + n)$ is the gross rate of population growth and also the number of children per individual.

The FOC’s are simply:

$$v'_{z_{t+1}}(z_{t+1}, \ell) = w_{t+1}$$

$$-u'(c_t) + \beta R_{t+1}u'(\tilde{d}_{t+1}) = 0$$

where $\tilde{d}_{t+1} = d_{t+1} - v(z_{t+1}, \ell)$

Again for simplicity’s sake, we will use simple forms for $u(\cdot)$ and $v(\cdot)$: $u(x) = \ln x$ and $v(x) = x^2/2\gamma\ell$. One clearly sees that the disutility of working longer is mitigated by an increase in longevity. We can now write the problem of the individual :

$$\mathcal{L} = \ln (w_t - \tau - s_t) + \beta \ell \ln \left(\frac{R_{t+1}s_t + w_{t+1}z_{t+1} - z^2/2\gamma\ell + p}{\ell}\right).$$

(2)

where $p = \tau(1 + n)^4$. The FOC with respect to $z_{t+1}$ and $s_t$ yield

$$z_{t+1} = z^*_{t+1} = \gamma \ell w_{t+1}$$

(3)

$$s_t = \frac{\beta \ell}{1 + \beta \ell} w_t - \frac{\gamma \ell w_{t+1}^2}{2R_{t+1}(1 + \beta \ell)} - \tau \left(\frac{\beta \ell}{1 + \beta \ell} + \frac{1 + n}{1 + \beta \ell} R_{t+1}\right)$$

(4)

\(^{4}\text{We thus assume defined contributions.}\)
In the case there is a mandatory age of retirement $\bar{z}$, we have $z_{t+1}^* \geq \bar{z}$, in other words, workers can be forced to work less or more than what they would choose to do with perfect flexibility. In the case of mandatory retirement age, we rewrite equations (3) and (4) as follows:

\[ z_{t+1} = \bar{z} \]
\[ s_t = \frac{\beta \ell}{1 + \beta \ell} w_t - \frac{\bar{z}}{R_{t+1} (1 + \beta \ell)} (w_{t+1} - \bar{z} / 2 \gamma \ell) - \tau \left( \frac{\beta \ell}{1 + \beta \ell} + \frac{1 + n}{(1 + \beta \ell) R_{t+1}} \right) \]

We now turn to the production side. We use a Cobb-Douglas production function

\[ Y_t = F(K_tL_t) = AK_t^\alpha L_t^{1-\alpha} \]

where the labor force is $L_t = N_t + N_{t-1}z_t = N_{t-1}(1 + n + z_t)$, $K_t$ is the stock of capital and $A$ is a productivity parameter. We distinguish $L_t$ the labor force and $N_t$ the size of generation $t$. We assume that

\[ N_t = N_{t-1}(1 + n) \]

Total population at time $t$ is

\[ N_t + \ell N_{t-1} = N_{t-1}(1 + \ell + n) \]

Denoting $K_t/L_t \equiv k_t$ and $Y_t/L_t \equiv y_t$, we obtain the income per worker (and not per capita):

\[ y_t = f(k_t) = Ak_t^\alpha \]

and the factor prices

\[ R_t = f'(k_t) = A\alpha k_t^{\alpha-1} \]
\[ w_t = f(k_t) - f'(k_t)k_t = (1 - \alpha)Ak_t^\alpha \]

The equilibrium conditions in the labor and capital markets are respectively

\[ L_t = N_{t-1}(1 + n + z_t) \]
\[ K_{t+1} = N_ts_t \]

We can now write the dynamic equation with perfect foresight

\[ (1 + n + z_{t+1})k_{t+1} = s_t \]
\[(1 + n)k_{t+1} + \gamma \ell A (1 - \alpha) k_{t+1}^{\alpha+1} = \frac{\beta \ell}{1 + \beta \ell} A(1 - \alpha)k_t^{\alpha} - \frac{\gamma \ell k_{t+1}^{1-\alpha} A^2 (1 - \alpha)^2}{2(1 + \beta \ell) A\alpha} - \tau \left( \frac{\beta \ell}{1 + \beta \ell} + \frac{(1 + n) k_{t+1}^{\alpha-1}}{A\alpha (1 + \beta \ell)} \right) \]  

(9)

when \(z\) is flexible, or

\[(1 + n + \bar{z})k_{t+1} = \frac{\beta \ell}{1 + \beta \ell} A(1 - \alpha)k_t^{\alpha} - \frac{\bar{z}k_{t+1}^{\alpha-1}}{(1 + \beta \ell) A\alpha} (A(1 - \alpha)k_{t+1}^{\alpha} - \bar{z}/2\gamma \ell) - \tau \left( \frac{\beta \ell}{1 + \beta \ell} + \frac{(1 + n) k_{t+1}^{\alpha-1}}{A\alpha (1 + \beta \ell)} \right) \]  

if \(z\) is constrained and mandatory. Differentiating totally these two equations taken in the steady state and assuming both stability and unicity of \(k^*\), namely \(0 < \frac{dk_{t+1}}{dk_t} < 1\), we can show:

\[
\frac{dk^*}{d\tau^*} < 0 , \quad \frac{dk^*}{d\gamma} < 0 , \quad \frac{dk^*}{dn} < 0.
\]

These three inequalities are standard. It is indeed well-known that a PAYG pension \((\tau)\) depresses capital accumulation, that working longer \((\gamma)\) has a negative impact on saving and that a lower fertility rate \((n)\) increases the steady-state capital stock\(^5\). However, the effect of an increase in longevity or in mandatory retirement on capital accumulation is ambiguous:

\[
\frac{dk^*}{d\ell^*} \approx -2A\ell^{-2}(1 + n)k^* - 2\tau \ell^{-2}(1 + n)k^* + 2A^2 \alpha \beta \gamma (1 - \alpha)k^{\alpha+1} \geq 0 \quad (11)
\]

\[
\frac{dk^*}{d\bar{z}} \approx 1 - \frac{1}{(1 + \beta \ell) \alpha} (\alpha \beta \ell - \bar{z}/A \gamma \ell k) \geq 0 \quad (12)
\]

The ambiguity of \(\frac{dk^*}{d\ell}\) depends on the presence of a flexible age of retirement along with a PAYG system. Without pension and flexible retirement, increasing longevity unambiguously fosters capital accumulation. As to \(\frac{dk^*}{d\bar{z}}\), its sign is expected to be negative: as people work later, they need less saving. However, when they retire

\(^{5}\)At least with defined contributions pensions. See Artige et al. (2013) where the defined contributions and defined benefits systems are compared
very late, near the end of life, the disutility becomes so high that they have to be compensated for by increasing saving.

It is important to note at this point that some of these results, particularly the unambiguous comparative statics, comes from our particular specification of preferences and technology. As shown by de la Croix and Michel (2002), as soon as we depart from the Cobb-Douglas specification, one faces problems of unicity and stability.

3 Economic union

Let us assume that we have \( m \) \((i = 1, ..., m)\) countries that are identical in all respects but in the flexibility of retirement choice (measured by the presence or not of \( z \)) and the degree of unfunding of their pension system (\( \tau \)). The utility of country \( i \) in the steady state is equal to

\[
U_{i,t} = \ln(w_{i,t} - \tau_i - s_{i,t}) + \beta \ell \ln\left(\frac{w_{i,t+1}z_{i,t+1} + R_{i,t+1}s_{i,t} - (z_{i,t+1})^2/2\gamma\ell + \tau_i(1+n)}{\ell}\right)
\]

(13)

where \( z_{i,t+1} \) is equal to \( \bar{z} \) or to \( \gamma \ell w_{i,t+1} \) depending on whether in this country the age of retirement is mandatory or flexible. In autarky and in the steady state, \( R_i \) and \( w_i \) depend on \( s_i \). For the sake of presentation we distinguish among six types of countries with subscript \( F \) for funded and \( P \) for unfunded, and another subscript \( E, O \) and \( L \) for mandatory early retirement, optimal retirement and mandatory late retirement. Consider first a set of 4 countries: PE, PO, FE, FO. In autarky we expect the following ranking in terms of capital and utility:

\[
k_{FE} > k_{PE} \geq k_{FO} > k_{PO} \\
U_{FO} \leq U_{FE} ; U_{PO} \leq U_{PE} ; U_{FO} > U_{PO} ; U_{FE} > U_{PE}
\]

Whereas the extreme cases for \( k \) are unambiguous, the intermediate cases are ambiguous; their ranking will depend on the size of \( \tau \) and of the gap between \( \bar{z} \) and \( z^* \).

If the gap between the optimal and the mandatory early retirement is small and if the PAYG pension is large, one expects to have \( k_{PE} < k_{FO} \). In terms of utility, the comparison is not immediate. Throughout the paper we assume dynamic efficiency \((r > n)\). This implies that mandatory early retirement can yield more steady state welfare than optimal retirement for either a funded or an unfunded pension if the 'static' inefficiency it entails is small relative to the boost to capital accumulation it
gives. This boost brings the economy closer to the Golden Rule \( r = n \). Consider now the set of 4 countries: PL, PO, FL, FO. In autarky we expect the following ranking in terms of capital and utility:

\[
\begin{align*}
  k_{FO} &> k_{PO} \gtrsim k_{FL} > k_{PL} \\
  \quad \quad &\star \star U_{FO} > U_{FL} \lesssim U_{PO} > U_{PL}
\end{align*}
\]

In this sample, the comparison of the four levels of \( k \) is like the one above: the extreme cases are unambiguous and the intermediate cases are not. In terms of utility, the ideal is a funded system with flexible retirement; the worst case is that of unfunded pensions with mandatory late retirement. The intermediate cases are ambiguous; the ranking will depend on the size of \( \tau \) and of the gap between \( \bar{z} \) and \( z^* \).

With capital mobility, we have a uniform value of \( k \) with an outflow from the high saving countries to the low saving ones. The overall utility and the global capital stock do increase. Some countries can experience a loss in utility. To go further, we use a numerical example.

4 Numerical examples.

To better grasp the sensitivity of the solutions to changes in policy parameters \( \tau \) and \( z \), we resort to numerical simulations. In these simulations, we use the same specification as above with: \( y_t = Ak_t^\alpha \) where \( A = 50 \) and \( \alpha = 1/3 \). As to preferences, \( \beta = 1 \) and \( \gamma = 0.005 \). The demographic parameter values are given by \( n = 0.05 \) and \( \ell = 0.9 \). Finally the policy instruments are \( \tau = 10 \), \( \bar{z} = 0.2 \) or 0.7.

Table 1 gives the capital stock and welfare that prevail in autarky and in the steady state. The values obtained correspond to the theoretical expectations: early retirement and fully funded pensions imply the highest capital stock and late retirement with PAYG the lowest capital stock. Flexible retirement along with fully funded pensions yield the highest welfare and late retirement with PAYG the lowest. Naturally the cases that were theoretically ambiguous can now be ranked.

Tables 2-4 present the key results when capital is allowed to move freely. Two important findings: the overall welfare \( \Delta U \) increases while overall capital accumulation decreases. Individually, countries which experiences an outflow of capital do also have a loss in welfare. Also in Tables 2 and 3 we observe some symmetry in outflows and inflows of capital. For example in Table 2 the inflow of capital from FE is equal to the inflow in NO; this is due to the linear structure of the saving function. To summarize, we observe that in aggregate terms countries benefit from
capital mobility but countries that export capital lose and countries that import capital benefit from economic integration. In other words, virtuous countries are penalized and indebted countries rewarded.

5 Conclusion

In this paper we have tried to evaluate the economic implications of different social security systems on the welfare of member states of an economic union relative to autarky. We have chosen a simple setting in which countries differ in the structure of their pension system with a focus on two key dimensions: is it funded or not and is the age of retirement mandatory or not? The first dimension has been widely discussed in the literature with the idea that countries with a PAYG system would benefit from joining an Economic Union more than countries having a social security system that is fully funded. To the contrary the second dimension has been relatively neglected; in other words it was not acknowledged that mandatory early retirement could induce more capital accumulation than flexible retirement and thus compensate for the depressive effects of PAYG pensions on saving. It should be noted that these findings are only relevant for the steady-state and were obtained within a Cobb-Douglas setting. The first feature is more important than the second. Results are likely to be different in the short run dynamics. It is not clear that results would be different with more general utility and production functions.
References


Table 1: Autarky

<table>
<thead>
<tr>
<th>FE</th>
<th>FO</th>
<th>FL</th>
<th>PE</th>
<th>PO</th>
<th>PL</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>37.14</td>
<td>27.34</td>
<td>24.11</td>
<td>28.59</td>
<td>21.76</td>
</tr>
<tr>
<td>U</td>
<td>8.28</td>
<td>8.30</td>
<td>8.20</td>
<td>8.17</td>
<td>8.15</td>
</tr>
</tbody>
</table>

Table 2: Open Economy. Early Retirement

<table>
<thead>
<tr>
<th>FE</th>
<th>FO</th>
<th>PE</th>
<th>PO</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>8.24</td>
<td>8.31</td>
<td>8.16</td>
<td>8.23</td>
</tr>
<tr>
<td>ΔU</td>
<td>-0.04</td>
<td>0.01</td>
<td>-0.01</td>
<td>0.08</td>
</tr>
<tr>
<td>(1 + n + z)k - s</td>
<td>-8.48</td>
<td>0.69</td>
<td>-0.69</td>
<td>8.48</td>
</tr>
</tbody>
</table>

\[ \bar{s} = 38.34; \bar{s}_{\text{aut.}} = 39.34 \]

Table 3: Open Economy. Late Retirement

<table>
<thead>
<tr>
<th>FL</th>
<th>FO</th>
<th>PL</th>
<th>PO</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>8.19</td>
<td>8.26</td>
<td>8.09</td>
<td>8.17</td>
</tr>
<tr>
<td>ΔU</td>
<td>-0.01</td>
<td>-0.04</td>
<td>0.07</td>
<td>0.02</td>
</tr>
<tr>
<td>(1 + n + z)k - s</td>
<td>-1.62</td>
<td>-5.79</td>
<td>5.79</td>
<td>1.62</td>
</tr>
</tbody>
</table>

\[ \bar{s} = 37.08; \bar{s}_{\text{aut.}} = 37.46 \]

Table 4: Open Economy. Late and Early Retirement

<table>
<thead>
<tr>
<th>FO</th>
<th>FL</th>
<th>PE</th>
<th>PO</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>8.28</td>
<td>8.21</td>
<td>8.14</td>
<td>8.20</td>
</tr>
<tr>
<td>ΔU</td>
<td>-0.02</td>
<td>0.01</td>
<td>-0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>(1 + n + z)k - s</td>
<td>-2.98</td>
<td>1.52</td>
<td>-3.14</td>
<td>4.60</td>
</tr>
</tbody>
</table>

\[ \bar{s} = 37.59; \bar{s}_{\text{aut.}} = 38.28 \]
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