Trade, economic geography
and the choice of product quality

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The present paper studies the effect of the choice of product quality on trade and location of firms. We discuss a model where consumers have preferences for the quality of a set of differentiated varieties. Firms do not only develop and sell manufacturing varieties in a monopolistic competitive market but also determine the quality level of their varieties by investing in research and development. We explore the price and quality equilibrium properties when firms are immobile. We then consider a footloose capital model where capital is allocated to the manufacturing firms in the region offering the highest return. We show that the larger region produces varieties of higher quality and that the quality gap increases with larger asymmetries in region sizes and with larger trade costs. Finally, the home market effect is mitigated when firms choose their product quality.

**Keywords:** monopolistic competition, endogenous quality, economic geography
1 Introduction

The present paper studies the effect of the choice of product quality on trade and location of firms. In particular, this paper discusses the role of the size of regions on firms’ choice of location and product quality. It is well-know that firms’ mobility fosters spatial polarization of economic activity (Krugman 1991). It is however less clear how differences in region sizes affect the quality produced in each region. Recently, Picard and Okubo (2012) highlight that firms endowed with higher qualities choose to locate in the larger region. Yet, product quality is not an exogenous factor. Firms invest in research and development to improve their product quality and this investment is likely affect their decisions about plant locations. Such a relationship between quality and location is a topic that has lacked attention. The present paper therefore focuses on the incentives to invest in product quality and the impact of such investments on firms’ location.

In this paper we build a quality-augmented version of Ottaviano et al.’s (2002) model where consumers have preferences for the quality of manufacturing varieties. As in Foster et al. (2008), the linear properties of the demand system of this model are particularly well suited for such an analysis. As is usual in the literature, each firm produces a distinct variety, competes under monopolistic competition and chooses its location in one of two regions. We consider a footloose capital model where capital is allocated to the firms and region offering the highest returns. The main difference with the literature is that firms are also able to determine their quality levels by investing in research and development.

We obtain the following results. We firstly consider the firms’ choice of quality at a given spatial distribution of firms. We show that the larger region attracts the firms that produce varieties of higher quality and that the quality gap between regions increases with larger regional asymmetries and larger trade costs. Hence, the size of the local market is an important determinant of the average product quality and the added value of the goods that are produced in a particular region. In this paper, such a result does not hinge on income effects but rather on a market size and competition effect. On the one hand, firms get higher returns from their investment when they locate in the region where demand is larger. On the other hand, investments in product quality foster competition and make the larger region more competitive. Hence, incentives to invest in quality are
mitigated by a harsher competition in larger regions. Quite interestingly, we show that
the co-agglomeration of firms and consumers in the same locale is good for average quality
and bad for aggregate price index (good for cost of living). Although firms agglomerating
in the larger region face a harsher competition, they benefit from a larger market which
increases their incentives to invest in quality. Therefore, global quality rises. Finally, the
model highlights the existence of complementarity effects between trade costs and returns
to investments in quality improvements. Quality investments reinforce the impact of trade
costs on prices and consumptions.

We secondly consider the location choice of firms that simultaneously choose their
product quality. We assume an investment technology with decreasing returns. We show
that the location equilibrium exists and is unique. In this location and quality equilibrium,
the firms that choose to produce high quality varieties locate in the larger region. As
standard in the economic geography literature, a fall in trade cost entices a larger number
of firms to locate in the larger region. More interestingly, we show that firms invest
more in quality on average and the quality gap decreases as trade costs fall. Removing
trade barriers is always good for quality because firms have access to larger markets and
more easily recoup their investment costs. This market access effect always dominates
the negative effect that quality investments have on competition and profitability. We
also show that market integration reduces regional disparities in terms of product quality.
Better access to consumers increases the economic returns on quality investment. Finally
we provide ambiguous results about the effect of investments in product quality on the
spatial distribution of firms and the home market effect.

Related literature This paper is closely related to several literature strands. First,
quality and location is the focus of a well-known business literature about "sophistication"
and "clustering". Porter (1990, p. 188) reports some qualitative evidence that investment
in product quality turns out to be more important and more successful in regions with
larger demand sizes. A typical example lies in the story of the two German designers of
the rotary press, Koenig and Bauer, who returned from London (U.K.) to Bavaria (Ger-
many) in 1818 to set up their first plant because this region was one amongst the world’s
largest market for printing press. German competitors in the press industry responded
with differentiation strategies based on quality and reliability, which made Germany the country with the highest quality and highest price premium in this market. Similarly, the emergence of a US cluster in patient monitoring equipment after World War II is mainly explained by the fact that the US wealthy private hospitals had higher demands for sophisticated monitoring than any European country with socialized medicine. Finally, the emergence of the Japanese cluster in the robotic industry is also explained by the higher demand for robotics by the Japanese management who had significantly stronger engineering background.

Second, since the recent availability of trade microdata, a recent strand of empirical trade literature is developing around the question of product quality and trade. It is shown that the quality or the value of goods plays an important role in international trade pattern. For instance, using US commodity trade data, Schott (2004) finds that the unit value of trade within one product line is higher for high-wage countries. Hummels and Klenow (2005) find that richer countries export higher value goods. Hallak (2006) finds that rich countries import relatively more from the countries producing high quality goods. Hence, there also exists quantitative evidence of heterogeneity of product quality in the trade patterns. For many authors, trade is better explained by demand or quality heterogeneity than by cost heterogeneity (see Baldwin 2005, Greenaway 1995 and Greenaway et al. 1995; Fukao et al. 2003; Foster et al. 2008). Khandelwal (2010), Baldwin and Harrigan (2011), Manova and Zhang (2012) provide additional support of heterogeneous quality in international trade data. This suggests that the study of the relationship between quality, trade and firms’ location deserves a dedicated attention.

Finally, academic research has produced a theoretical literature about product quality and trade based on vertical differentiation to explain why higher quality products are more likely to be consumed and produced in high wage countries. Murphy and Shleifer (1997) develop a model where high quality products end up being produced in high human capital countries. Feenstra and Romalis (2006) extend the Heckscher Ohlin model

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1Hummels and Klenow (2005) use import data from 76 countries at the six digit level of the Harmonized System and then find that the quality margin is a function of the exporter size. Hallak (2006) analyzes bilateral trade flows among 60 countries.

to product qualities. Recently, Kluger and Verhoogen (2012) theoretically study the issue of endogenous quality in a trade context but focus on the impact of exchange rate devaluations. Eckel et al. (2011) discuss the impact of quality choice of multi-product monopolies and oligopolies serving consumers with linear demands that are similar to ours. However, none of those models study the location of firms and the impact of the co-agglomeration of firms with workers. The relationship between product quality and location choice is recently studied in Picard and Okubo (2012) who show that larger regions attracts better quality firms.\footnote{Okubo et al. (2010) study a similar two-type heterogeneity model.} This paper extends this idea in a model where product quality is a variable chosen by firms.

The paper is structured as it follows. Section 2 presents the model. Section 3 and 4 discuss the choice of quality and location. Section 5 concludes.

## 2 The model

Our model extends the footloose capital model of Ottaviano and Thisse (2004) by allowing for consumer’s preference for product quality. In this section, we present the basic model and characterize the market outcome for any given organizational structure and spatial distribution of firms. In this paper the timing is as follows: first firms simultaneously choose their quality and location, second they set their prices and finally consumers consume the goods produced by the firms. We solve this sequential game by backward induction.

### 2.1 Preferences

Consider a world with two regions, labeled $H$ and $F$. Variables associated with each region will be subscripted accordingly. We assume that there is a mass $L$ of consumers, with a share $\theta_H \leq 1$ located in region $H$. In what follows, we refer to $H$ as the large and to $F$ as the small region.

All consumers in region $i = H, F$ have identical quasi-linear preferences over a homogeneous good and a continuum of horizontally differentiated varieties, indexed by $v \in V$. As
in Ottaviano et al. (2002) and Ottaviano and Thisse (2004), the utility of a representative agent in region $i$ is given by the following quadratic function:

$$U_i = \int_{\mathcal{V}} \tilde{\alpha}(v)q_i(v)dv - \frac{\beta - \gamma}{2} \int_{\mathcal{V}} [q_i(v)]^2 dv - \frac{\gamma}{2} \left[ \int_{\mathcal{V}} q_i(v)dv \right]^2 + q_i^o;$$

(1)

where $q_i(v)$ denotes the consumption of variety $v$ in region $i$ and $q_i^o$ stands for the consumption of the homogenous good in that same region. The parameter $\gamma$ is a measure of the degree of substitution between varieties whereas $\beta - \gamma (> 0)$ measures the consumer bias toward a more dispersed consumption of varieties.

The new element in this model is the function $\tilde{\alpha}(v): \mathcal{V} \equiv [0, 1] \rightarrow [\alpha, \infty), \alpha \geq \pi > 0$, that reflects the quality of variety $v$. It measures the consumer’s willingness to pay for product variety $v$; that is, the intensity of consumer’s preferences for the differentiated product $v$ with respect to the homogenous good. When $\tilde{\alpha}(v)$ is identical for all varieties, varieties are horizontally and symmetrically differentiated. When $\tilde{\alpha}(v)$ varies, the quality and therefore the willingness to pay varies across varieties so that goods are also vertically differentiated in the sense that consumers have a higher willingness to pay for the variety $v$ than for $v'$ if $\tilde{\alpha}(v) > \tilde{\alpha}(v')$. Note finally that the consumers have identical preferences: there is no a priori ‘regional preferences’. We denote the average quality by $\alpha \equiv \int_{\mathcal{V}} \tilde{\alpha}(v)dv$.

Each consumer in region $i = H, F$ maximizes his/her utility (1) subject to his/her budget constraint:

$$\int_{\mathcal{V}} p_i(v)q_i(v)dv + p_i^o q_i^o \leq w_i + p_i^o \bar{q}^o,$$

(2)

where $p_i(v)$ denotes the consumer price of variety $v$ and $w_i$ stands for each individual’s earning. Following Ottaviano et al. (2002), we assume that consumers own a sufficiently large endowment $\bar{q}^o > 0$ of the numéraire. Consequently, income effects are present in the demand for homogenous goods but are absent in the demand for manufacturing varieties. As will become clear in the sequel, free trade in the homogenous good market leads to price equalization across regions and makes this good a natural choice for the numéraire ($p_i^o = 1, i = H, F$).

We assume that all varieties are consumed. Maximizing the utility (1) subject to the budget constraint (2) yields the following first order condition:

$$\tilde{\alpha}(v) - (\beta - \gamma) q_i(v) - \gamma \int_{\mathcal{V}} q_i(\xi)d\xi - p_i(v) = 0$$

(3)
Integrating the left hand side of this equality yields the average quality

$$\alpha = \beta \int_v q_i(v)dv + \int_v p_i(v)dv$$

(4)

The last two expressions allows us to derive the individual demand for variety $v$ as the following linear formula:

$$q_i(v) = (b + c) [\hat{\alpha}(v) - p_i(v)] + c(\mathbb{P}_i - \alpha)$$

(5)

where

$$\mathbb{P}_i = \int_v p_i(v)dv$$

is the manufacturing price index in region $i$ and where $b$ and $c$ are the following positive coefficients

$$b = \frac{1}{\beta} \quad \text{and} \quad c = \frac{\gamma}{\beta (\beta - \gamma)}$$

(6)

The parameter $b$ measures the price sensitivity of demand and the parameter $c$ the degree of product substitutability. In particular, when $c \to 0$ varieties are perfectly differentiated, whereas they become perfect substitutes when $c \to \infty$.

2.2 Price equilibrium and trade costs

Production takes place in two sectors. In the first sector, the homogenous good is produced under perfect competition using one unit of labor per unit of output. We assume that this good can be costlessly traded between regions, which implies that its price is internationally equalized and equal to wages. Normalizing wages to one we get $p_o^i = w_i = 1$ for $i = H, F$, which justifies our previous choice of this good as the numéraire.

In the second sector, called the manufacturing sector, each firm produces under increasing returns to scale and sells a single differentiated manufacturing variety. Let $\mathcal{V}_i$ and $n_i$ be the set and the mass of manufacturing firms located in region $i$. That is, $n_i = \mu(\mathcal{V}_i) \equiv \int_0^1 d\mu_i(v)$ where $\mu(\mathcal{V}_i)$ is the measure of $\mathcal{V}_i$ and $\mu_i(v)$ is the measure of variety $v$ if it is produced in region $i$ ($\mu_H(v) + \mu_F(v) = 1$ and $\mu_H(v) * \mu_F(v) = 0$). In this section we derive the price equilibrium for a given location structure $(\mathcal{V}_H, \mathcal{V}_F)$ and for a given distribution of product quality $\hat{\alpha}(\cdot)$ across firms. The average quality are therefore
given by
\[
\alpha = \int_0^1 \hat{\alpha}(v) d\mu_H(v) + \int_0^1 \hat{\alpha}(v) d\mu_F(v)
\]

The demand for each variety in each market depends on the set of varieties produced domestically and on the set produced abroad. In accord with empirical evidence (e.g., Head and Mayer, 2000; Haskel and Wolf, 2001), we assume that product markets are segmented. Firms are hence free to set prices specific to each national market they sell their product in. The profit of a manufacturing firm established in region \(i\) is given by

\[
\Pi_i(v) = L \theta_i p_{ii}(v) q_{ii}(v) + L \theta_j (p_{ij}(v) - \tau) q_{ij}(v) - I(\tilde{\alpha}(v)) - r_i(v), \quad v \in \mathcal{V}_i \quad (7)
\]

where \(L\) is the total population, \(\theta_i\) is the share of population in region \(i\), \(I(\tilde{\alpha}(v))\) is the firm’s investment in quality \(\tilde{\alpha}(v)\), \(r_i(v)\) is the remuneration of firm \(v\)’s capital and \(q_{ij}(v)\) and \(p_{ij}(v)\) is the price and demand of variety \(v\) when it is produced in region \(i\) and consumed in region \(j\). In the latter expression we have normalized w.l.o.g. the marginal cost of production to zero and assumed a unit transport cost \(\tau\) paid in numéraire. By (5), the individual demand writes as

\[
q_{ij}(v) = (b + c) [\hat{\alpha}(v) - p_{ij}(v)] + c (P_j - \alpha)
\]

for all \(i, j \in \{H, F\}\). Under monopolistic competition, firms are too small to affect the aggregate variables. So, they set their prices \(p_{ii}(v)\) and \(p_{ij}(v)\) taking the price indices \((P_i, P_j)\) and the distribution of quality \((\tilde{\alpha}(\cdot))\) as givens. The optimal prices are computed as it follows:

\[
p_{ii}(v) = \frac{(b + c) \hat{\alpha}(v) + c (P_i - \alpha)}{2(b + c)} \quad \text{and} \quad p_{ij}(v) = p_{jj}(v) + \frac{\tau}{2} \quad (8)
\]

which depend on the quality of the variety offered. At the equilibrium in the product market, the firm’s prices \((p_{ii}(v), p_{ij}(v))\) are consistent with aggregate prices or price indices.
\((\mathbb{P}_i, \mathbb{P}_j)\). The latter are successively equal to
\[
\mathbb{P}_i = \int_0^1 p_{ii}(v) d\mu_i(v) + \int_0^1 p_{ji}(v) d\mu_j(v)
= \int_0^1 p_{ii}(v) d\mu_i(v) + \int_0^1 \left( p_{ii}(v) + \frac{\tau}{2} \right) d\mu_j(v)
= \int_0^1 p_{ii}(v) dv + \int_0^1 \frac{\tau}{2} d\mu_j(v)
= \frac{\alpha b + c\mathbb{P}_i}{2(b+c)} + \frac{\tau}{2} n_j
\]

Solving for the fixed point yields
\[
\mathbb{P}_i = \frac{\alpha b + (b+c)\tau n_j}{2b+c}
\]

Hence, the price index depends only on the average quality and the number of firms in each region. It does not depend on the quality chosen by each firm and on the particular location of firms. This property stems from the fact that preferences are linear in the quality parameter. Equilibrium prices are then equal to
\[
p_{ii}^*(v) = \frac{1}{2} \frac{2\alpha b + \tau n_j c}{2b+c} + \frac{\tilde{\alpha}(v) - \alpha}{2} \text{ and } p_{ij}^*(v) = p_{jj}^*(v) + \frac{\tau}{2}
\]

Firms selling higher quality products set higher prices. Indeed, each firm’s prices increase with its idiosyncratic product quality. They however fall with larger average product quality \((\partial p_{ii}^*(v)/\partial \alpha < 0)\). The firm reacts to more attractive competing goods by lowering its prices. Also, note that firms inflate the price of their exports \(p_{ij}(v)\) by half of the transport cost. This means that their exports’ freight in board (f.o.b.) prices \((p_{ij}(v) - \tau)\) are equal to \(p_{jj}(v) - \tau/2\), which includes a discount of half of the transport cost. For a given price index at the destination, larger trade costs result in smaller f.o.b. prices, which is consistent with empirical evidence (e.g. Baldwin and Harrigan, 2011; Manona and Zhang, 2012)

We observe that prices are independent to the precise composition of local production. That is, each firm sets a price \(p_{ii}^*(v)\) that depends only on the quality of its own variety \(\tilde{\alpha}(v)\), on the average quality \(\alpha\) and on the mass of firms \((n_i, n_j)\) in each region. In other words, for any given profile of quality \(\tilde{\alpha}(\cdot)\) that yields the average quality \(\alpha\), the prices \(p_{ii}^*(v)\) and \(p_{ij}^*(v)\) depend only on \(\tilde{\alpha}(v)\) and \((n_i, n_j)\) but not on the sets \((\mathcal{V}_i, \mathcal{V}_j)\). This
independence of prices to the precise composition of local production turns out to be a useful property in the subsequent analysis of spatial selection (see Section 4). Given the above prices, it is easy to show that firms’ optimal output is equal to 
\[ q_{ii}(v) = (b + c)p_{ii}^*(v) \] and 
\[ q_{ij}(v) = (b + c)\left(p_{ij}^*(v) - \tau\right). \] So, firms sizes \((q_{ii}^*(v) + q_{ij}^*(v))\), exports quantities \((L\theta_j q_{ij}^*(v))\) and export values \((L\theta_j (p_{ij}^*(v) - \tau) q_{ij}^*(v))\) fluctuates in the same direction as export prices and f.o.b. prices. This is also consistent with recent empirical literature. The profit of firm \(v\) located in region \(i\) can be written as

\[ \Pi_i(v) = L(b + c)\left[\theta_i(p_{ii}^*(v))^2 + \theta_j\left(p_{ij}^*(v) - \tau\right)^2\right] - I(\alpha(v)) - r_i(v) \quad (11) \]

In this analysis we have assumed that bilateral trade. To guarantee this, we must impose that exports are positive, \(q_{ij}^*(v) > 0\), or equivalently, that the f.o.b. prices \(p_{ij}^*(v) - \tau\) remain positive in equilibrium. Section 3.4 discusses this condition in more detail.

We now analyze the firms’ simultaneous choice of quality and location. For the sake of exposition we make a separate discussion about the choice of quality in Section 3. The analysis of the simultaneous choice follows in Section 4.

### 3 Product quality and trade

In this section we discuss a trade model where the spatial distribution of firms is given and where firms are able to choose their quality investments. We assume decreasing returns to investment in quality where each firm can improve its quality to a quality \(z\) by investing an amount

\[ I(z) = I(z - \alpha)^2 \quad \text{if} \quad z \geq \alpha \]

of numéraire. The investment cost is nil otherwise: \(I(z) = 0\) if \(z < \alpha\). In this expression, the parameter \(\alpha\) is the costless product quality. This is the level of quality that firms can achieve at no cost. Because of decreasing returns in quality, higher product qualities require more than proportional investment levels. For the sake of convenience we define

\[ I \equiv L(b + c)I_o/4 \]

where \(L\) is the total consumer population and where \(I\) measures this return to investment on a per-consumer basis. This formulation is convenient because firms’ investment incentives are proportional to the total size of market (i.e. \(L\) consumers) and because profit...
levels are proportional to the slope of demand functions \((b + c)\). Finally we assume that the quality investment can be made only in the location where the production is located. For instance, quality investments are related to specific management efforts to improve product quality, specific training of local labor force or specific manufacturing immobile equipments.

Under monopolistic competition, the firm is small in the product market and has no influence on other firms’ prices and quality levels indices. Hence, the firm maximizes its profit \(\Pi_i(v)\) with respect to its own quality level \(\hat{\alpha}(v)\) taking \(P_i\) and \(\alpha\) as givens. We get the first order condition

\[
\frac{\partial \Pi_i(v)}{\partial \hat{\alpha}(v)} = L(b + c) \left\{ \left[ \theta_i p_{ii}^*(v) + \theta_j (p_{ij}^*(v) - \tau) \right] - (\hat{\alpha}(v) - \alpha) I_o/2 \right\} = 0 \tag{12}
\]

while the second order condition is satisfied if and only if \(I_o > 1\). The optimal quality is finite only if decreasing returns in quality investment are strong enough, which we assume from now. Indeed, if this condition were not satisfied, investment cost would rise at a smaller pace than the revenue increase associated to quality improvement and the optimal quality would be unbounded.

From expression (12), the optimal increase in quality

\[
\hat{\alpha}^*(v) - \alpha = \frac{2}{I_o} \left[ \theta_i p_{ii}^*(v) + \theta_j (p_{ij}^*(v) - \tau) \right] \tag{13}
\]

is proportional to the average markup on the world population and inversely proportional to the cost of quality investment \(I_o\). The firm balances the marginal cost and the marginal revenues of its quality investment. Under linear demand functions, both outputs and marginal revenues are proportional to markups. The impact of access is clearly apparent: since trade costs reduce markups, marginal revenues and therefore incentives to invest in quality get smaller as the production site is located further away from consumers.

### 3.1 Product quality and aggregate prices

It is instructive to study the optimal quality as a function of the other firms’ prices and the average quality in the economy. It permits to outline how firms react to their competitive environment. The relationships between quality and prices might be used as
a basis for empirical work. Plugging the prices (8) into (12), we compute the following optimal increase in quality:

\[ \alpha_i^* - \alpha \equiv \hat{\alpha}^* (v) - \alpha = \frac{\alpha (b + c) - c \alpha + c (\theta_i p_i + \theta_j p_j) - \tau \theta_j (b + c)}{(I_o - 1) (b + c)}, \quad v \in V_i \quad (14) \]

This expression allows us to make the following observations. First, *ceteris paribus*, firms invest more in quality for smaller average quality and for less substitutable products (smaller \( \alpha \) and \( c \)). Firms get higher returns from improving their own qualities in a world where the industry produces bad qualities and operate under weak competition. Second, the optimal quality increases with the *global average price*, which we define as \( \theta_i p_i + \theta_j p_j \). High aggregate prices allow each firm to set higher prices and give it more incentives to invest in product quality.

Finally, the *quality gap* between regions is given by

\[ \alpha_H^* - \alpha_F^* = \frac{\tau}{I_o - 1} (2 \theta_H - 1) \quad (15) \]

which is simply proportional to the asymmetry in region sizes. As a result, *high quality products are necessarily produced in the larger region*. Furthermore, *higher trade costs increase the quality gap*. This is because the markups on firms’ exports decrease with higher trade costs and reduce the returns to investment in quality. This effect has a stronger impact on the firms that have a larger share of export in their production, which are the firms located in the smaller market. Finally, the above expression shows the complementary effect of trade costs and returns in quality investments. Stronger decreasing returns (smaller \( I_o \)) exacerbate the impact of trade costs on regional asymmetries in product quality.

Note that the quality gap is independent from the location of firms. We have seen above that the quality of a firm depends just on the global average prices \( \theta_i p_i + \theta_j p_j \) but is not directly related to the distribution of firms \( (n_i, n_j) \). So, the relocation of firms does not alter on the quality gap.

### 3.2 Product quality and location

We now study the optimal quality and prices as a function of firms’ spatial distribution. In particular, we wish to highlight the impact of the co-agglomeration of firms with
consumers in the larger market.

We first compute the average quality $\alpha^* = n_H \alpha_H^* + n_F \alpha_F^*$. Using (14) and (10) we get the following relationship:

$$\alpha^* - \alpha = 2b \frac{\alpha - \tau (\theta_H n_F + \theta_F n_H)}{I_o (2b + c) - 2b}$$

(16)

where the denominator is positive because $I_o > 1$. We note the following points. First, the average quality increases with higher costless quality $\alpha$. This is because a larger $\alpha$ makes it cheaper to achieve any quality level. Second, the average quality decreases with larger trade cost $\tau$. As explained above, higher trade costs reduce markups on exports and diminish the incentives to invest in quality. Finally, let us define the co-agglomeration of firms with consumers as follows:

**Definition 1** Firms co-agglomerate with consumers in larger market if the number of firms producing in this market increases. That is, $n_H$ rises while $\theta_H > 1/2$.

Remembering that $n_F = 1 - n_H$ and $\theta_F = 1 - \theta_F$, one can readily check that the term $(\theta_H n_F + \theta_F n_H)$ decreases in $n_H$ for any $\theta_H > 1/2$. Hence, average quality falls when more firms locate in region $H$ (higher $n_H$). As a result, co-agglomeration of firms with consumers in the large market is good for average quality. This result depends on two opposite effects. First, markups are larger when firms locate in the larger market and therefore spend less on trade costs, which entices them to invest more in quality. Second, product market competition increases when firms collocate in the larger market, which decreases markups and incentives to invest in quality. In this model, the first effect dominates.

We can perform a similar analysis on the following measure of aggregate cost of living

$$\theta_i \mathbb{P}_i + \theta_j \mathbb{P}_j = \alpha - 4 \frac{[(b + c)I_o - 2b] [\alpha - \tau (\theta_H n_F + \theta_F n_H)]}{I_o (2b + c) - 2b}$$

One can show that it increases with higher costless quality $\alpha$ and that it increases with higher trade cost $\tau$ iff $I_o > 2b/(b + c)$. In the aggregate, prices rise if quality improvement starts from a larger costless quality level and if market access gets better. A better market access fosters quality and increase prices. Also, because $(\theta_H n_F + \theta_F n_H)$ decreases in $n_H$ for any $\theta_H > 1/2$, the aggregate cost of living decreases when more firms locate
in the larger region. Although co-agglomeration of firms with consumers in the larger market improves average quality, it increases competition so that prices tend to fall. The competition effect is therefore stronger than the quality improvement effect!

The optimal quality is given by the following formula:

\[
\alpha^*_i - \alpha = \frac{2 \alpha b}{I_o (2b + c) - 2b} + \frac{ct I_o (\theta_F n_F + \theta_H n_H)}{(I_o - 1) (I_o (2b + c) - 2b)} - \frac{\tau \theta_j}{I_o - 1}
\]  

(17)

It can be shown that the optimal quality increases with larger costless quality \((d\alpha^*_i / d\alpha)\), it falls with co-agglomeration of firms with consumers in the larger region \((\alpha^*_i \text{ increases with } \theta_F n_F + \theta_H n_H \text{ which decreases in } n_H)\) and it increases with larger local market (smaller \(\theta_j\)). It reflects the trade-off discussed earlier. It can be shown that \(\alpha^*_H\) increases with trade cost \(\tau\) whereas \(\alpha^*_F\) decreases with it.

We summarize our results in the following proposition.

**Proposition 2**  
(i) The larger region produces varieties of higher quality. (ii) The quality gap increases with larger asymmetries in region sizes and with larger trade costs. (iii) The aggregate cost of living falls as the larger region hosts more firms. (iv) Co-agglomeration of firms with consumers in the larger market is bad for individual quality, good for average quality and bad for aggregate prices (thus good for cost of living).

This proposition brings a contrasting perspective on global quality and firms’ spatial distribution. Indeed, on the one hand, the asymmetry in consumers’ spatial distribution is a cause spatial disparities in product quality. On the other hand, disparities in firms’ spatial distribution lead to a rise in the average quality. This result is novel in the literature.

We are now equipped to discuss the relationship between prices and location.

### 3.3 Prices and location

As mentioned above, because \(q^*_{ii} = (b + c)p^*_{ii}\) and \(q^*_{ij} = (b + c) (p^*_{ij} - \tau)\), local sales and export sales move in the same direction as local prices, export prices and export f.o.b. prices. Given (10), a firm located in region \(i\) and choosing its optimal quality sets the following domestic and export prices:

\[
p^*_{ii} = \frac{1}{2} \frac{2 \alpha^* b + \tau n_j c}{2b + c} + \frac{\alpha^*_i - \alpha^*}{2} \quad \text{and} \quad p^*_{ij} = \frac{1}{2} \frac{2 \alpha^* b + \tau n_i c}{2b + c} + \frac{\alpha^*_i - \alpha^* + \tau}{2}
\]

(18)
The two (first) terms of each expression reflect two location forces underlying the choice of quality. The first term represents the optimal price for a hypothetical ‘average quality good’ ($\alpha^*_i = \alpha^*$). Prices therefore increase with average quality $\alpha^*$. In Ottaviano and Thisse’s (2004) model, a higher (homogenous) quality increases the willingness to pay for each product and reduces the impact of trade costs on firms’ competition and therefore on prices. Consequently, trade barriers cannot offer as much protection against competitors and entice firms to agglomerate. Trade costs therefore exert a smaller dispersion force and agglomeration is more likely to take place. The same process takes place here with the average quality $\alpha^*$, which, however, depends on the location of firms and consumers. Hence, by (16), the co-agglomeration of firms with consumers in the larger market increases average quality and therefore inflates prices and outputs upwards. This effect strengthens when the decreasing returns of investment in product quality $I_o$ weaken.

The second term represents a markup for quality (resp. a discount if $\alpha^*_i < \alpha^*$) for the firm’s quality advantage (resp. disadvantage). In this model without income effects, quality affects the consumer’s individual demand in the same way in each region. As a result, the firm sets the same markup (resp. discount) for quality in both markets. The markup for quality of firms located in the larger region $H$ can be expressed as

$$\frac{\alpha^*_H - \alpha^*}{2} = \frac{\tau}{2(I_o - 1)} (\theta_H - \theta_F) (1 - n_H) > 0$$

which increases with region size asymmetries (larger $\theta_H$) but decreases with the co-agglomeration of firms in the domestic market (larger $n_H$). Markups for quality create a repulsion force for firms locating in the larger market because those firms see the individual benefit of their quality advantage reduced as more firms locate there. Firms located in the smaller region get a quality discount

$$\frac{\alpha^*_F - \alpha^*}{2} = -\frac{\tau}{2(I_o - 1)} (\theta_H - \theta_F) n_H < 0$$

whose absolute value increases with stronger region size asymmetries (larger $\theta_H$) and with further co-agglomeration of firms in the larger market (larger $n_H$). As more firms co-agglomerate in the larger region $H$, the average quality rises and aggravates the quality discount of the firms producing in the smaller market. However, the overall effect is that the quality markups and discounts exert a dispersion force on firms, which counters
too much agglomeration of firms in the larger locale. This effect also strengthens when decreasing returns of quality investments $I_o$ weaken, in particular when $I_o$ is close to one.

To sum up, the choice of quality creates two conflicting forces: an agglomeration force because average price-trade cost ratio rises and a dispersion forces because the net benefit of quality advantage diminishes as more firms agglomerate in the larger region. Those forces are exacerbated by smaller decreasing returns of quality investments.

### 3.3.1 Impact of trade costs

The present analysis also allows us to disentangle the various effects of trade costs on export prices and quantities. Differentiating the above price, we get the following changes in the markup

$$\frac{d}{d\tau} (p_{ij}^* - \tau) = -\frac{1}{2} \left( -\frac{n_i c}{2b + c} + \frac{b}{2b + c} \frac{d\alpha^*}{d\tau} + \frac{1}{2} \frac{d(\alpha_i^* - \alpha^*)}{d\tau} \right)$$

$$+ \left[ \frac{1}{2} \frac{\tau c}{2b + c} + \frac{b}{2b + c} \frac{d\alpha^*}{dn_i} + \frac{1}{2} \frac{d(\alpha_i^* - \alpha^*)}{dn_i} \right] \frac{dn_i}{d\tau}$$

which is also proportional to export and f.o.b. price changes. A fall in trade cost has various effects on markups and exports. It firstly has the standard effects that we find in homogenous quality models. One the one hand, it raises the markups and exports because of the presence of an imperfect pass-through (first term $-1/2$). Firms indeed pass through a half of trade costs to their foreign consumers and subsidize the other half. Lower trade costs reduce such subsidy incentives and raise markups and export output. On the other hand, the fall in trade costs pushes consumer prices down in the foreign market and intensifies the price competition there, which pushes markups and exports down (second term). It can readily been shown that the former effect dominates the latter.

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4Since changes in trade costs are equivalent to bilateral changes in tariffs and changes in distances between trade partners, the following analysis applies for the study of pass-through effects of tariffs and transport costs.

5For instance, the existence of an imperfect pass-through is reported in De Loecker et al. (2012).
A fall in trade cost also has an effect through the direct changes in product quality. It firstly improves market access so that firms invest more in product quality so that the average product quality rises and firms set higher export prices (third negative term). This is the main effect of endogenous quality when regions have similar sizes \((\theta_H \simeq \theta_F)\). However when countries have different sizes, a fall in trade cost affects each country’s exports in a different way (fourth term). Because it reduces the market access disadvantage of the smaller region \((i = F)\), the firms producing there have additional incentives to raise their quality level, markup and export price and output. By contrast, the firms in the larger region \((i = H)\) benefit from a lower market access advantage and cannot raise their quality, export price and output as much. So, a fall in trade cost improves the smaller region’s position on the quality ladder more than the larger one.

Finally, a fall in trade costs has an impact through the relocation of firms (see the three terms in squared bracket). Suppose that the fall in trade costs leads to the co-agglomeration of firms with consumers in the larger market. Accordingly, the number of firms increases in the larger region and decreases in the other \((dn_H/d\tau < 0 < dn_F/d\tau)\). This will be shown to be the case in the footloose capital equilibrium discussed in Section 4. Consider the firms producing in the larger region \((i = H)\). First, their export market is served by fewer local firms so that the weaker competition entices all firms to set higher prices there (first positive term). Second, the coagglomeration of firms with consumers in the larger market improves the average product quality and increases export prices and output (second positive term). Those two effects on product quality therefore entice firms to raise their export prices and output when trade costs fall. As above, this is the main effect of firms relocation when regions have similar sizes \((\theta_H \simeq \theta_F)\). By contrast, when countries have different sizes, the larger country offers a market access advantage. This advantage however diminishes because local entry of firms induces stronger competition, which restrains or reverts the rise of export prices and output from the larger region. The overall effect depends on the economic parameters.\(^6\) So, the choice of product quality and the co-agglomeration of firms in the larger market attenuate and may reverse the negative

\(^6\)For instance, the overall effect is positive for the parameters \(\alpha = 1, \beta = 1, \gamma = .9, I_o = 5, \theta_H = 0.55\) and the endogenous value \(n_H = 0.718\) satisfying \((21)\) and \((22)\).
impact of trade costs on f.o.b. prices and exports from the higher quality region.\textsuperscript{7}

We now turn to the discussion of the existence of bilateral trade.

### 3.4 Bilateral trade

Bilateral trade takes place when firms export from all regions. It may not be feasible for all values of economic parameters.\textsuperscript{8} In particular, firms located in the smaller region have lower quality and sell at a competitive disadvantage. Those firms are the first that stop exporting. The export quantities $q_{FH}^*$ are proportional to the markups $p_{FH}^* - t$. Using (16) and (17), it can be shown that those quantities fall with the number of firms in the larger market $n_H$ as competition intensifies there.\textsuperscript{9} So, bilateral trade occurs if and only if $q_{FH}^* > 0$, or equivalently, if $n_H$ remains lower than the threshold

$$\bar{n}_H (I_o) \equiv \frac{2b}{\tau} \frac{\alpha - \tau + \frac{2b}{I_o(2b+c) - 2b} (\alpha - \tau \theta_H)}{c + (\theta_H - \theta_F) \left( \frac{2b+c}{I_o-1} - \frac{4b^2}{I_o(2b+c) - 2b} \right)}$$

(21)

where the second term in the denominator is positive because $I_o > 1$. It is apparent that the threshold $\bar{n}_H (I_o)$ falls with $\theta_H$ and $\tau$. Therefore, bilateral trade is more likely to be supported for smaller region size asymmetries and trade costs. The role of investments in product quality is however ambiguous.

Consider firstly the case where decreasing returns to quality investments are small enough (e.g. $I_o \to 1$). Then we get $\bar{n}_H (1) = 0$ and we can conclude that bilateral trade is never feasible. In this case, firms have more incentives to invest in quality in the larger region because they benefit from a better access to the larger market and because the cost of quality investment does not increase that much as quality rises. Regional quality and markups diverge dramatically and create large price discrepancies and large export differences. As a consequence, exports from the smaller region fall to zero for any trade cost and any regional size asymmetries.

\textsuperscript{7}This argument about endogenous quality may be consistent with Baldwin and Harrigan’s (2011) empirical evidence according which average U.S. export prices fall with proximity.

\textsuperscript{8}Behrens (2005) and Okubo et al (2010) provide a study of bilateral and unilateral trade flows in a homogenous quality model.

\textsuperscript{9}Indeed, $\frac{dq_{FH}^*}{dn_H} = -\frac{1}{2} \frac{c \tau}{2b+c} [1 + \frac{I_o(4b+c) - 2b}{(I_o-1)(I_o(2b+c) - 2b)} (\theta_H - \theta_F)] < 0$
Consider secondly the case where decreasing returns to quality investment are very large (e.g. \( I_o \rightarrow \infty \)). In this case, firms choose a quality level close to the costless quality \( \alpha \) and the markups for quality \( (\alpha_i^* - \alpha^*) / 2 \) tend to zero. We therefore return to a model with a homogenous quality \( \alpha \) where, as explained in the previous sub-section, a higher quality increases the willingness to pay for each product and reduces the impact of trade costs on firms’ competition and therefore on prices. Firms’ location will then have a smaller impact on the existence of exports from region \( F \) if trade barriers are low. Using the previous formula, we get that bilateral trade is feasible iff \( n_H < \pi_H (\infty) \equiv 2b (\alpha - \tau) / (\tau c) \). One can check that bilateral trade is always feasible if \( \tau < \tau^\infty \equiv 2b \alpha / (2b + c) \) because \( \pi_H (\infty) \) lies above one, while it is never feasible if \( \tau > \alpha \) because \( \pi_H (\infty) \) is smaller than zero. For trade costs between \( \tau^\infty \) and \( \alpha \), bilateral trade is feasible only if the larger region does not host too many manufacturing competitors.

For intermediate values of returns \( I_o \), the impact of firms’ location on exports from region \( F \) depends on how average quality and quality differences move. One can check that the function \( \pi_H (I_o) \) is a bell-shaped function of \( I_o \) over the interval \([0, 1]\) if \( \theta_H < \bar{\theta}_H \) and \( \tau < \alpha \) where

\[
\bar{\theta}_H \equiv \frac{1}{2} + \frac{b (2\alpha - \tau)}{2\alpha (4b + c) - 2\tau (3b + c)} < 1
\]

Otherwise, \( \pi_H (I_o) \) is an increasing function of \( I_o \) over this interval. As a result, on the one hand, when returns to quality investment are strong (small \( I_o \)) or region sizes differ a lot (high \( \theta_H \)), \( \pi_H (I_o) \) is an increasing function. A rise in the number of firms in market \( H \) mainly accentuates product quality differences between regions, intensifies competition and impedes firms to export to this market. Weaker returns to quality investment (larger \( I_o \)) then diminish quality differences and competition so that they entice foreigners to export in this market. On the other hand, when returns to quality investment are weak (high \( I_o \)) and region sizes do not differ too much (low \( \theta_H \)), \( \pi_H (I_o) \) becomes a decreasing function. In this case, regional product quality differences do not constitute a dominant channel. Rather, weaker returns to quality investment (larger \( I_o \)) diminish the average quality, consumers’ willingness to pay and product prices. As trade costs become a larger component of product prices, exporting becomes more difficult, in particular from the

\[10\] If \( \tau < \alpha \), this function has an increasing and a decreasing section on the interval \([0, 1]\). If \( \tau > \alpha \), the decreasing section lies above the interval \([0, 1]\).
smaller market $F$.

We summarize this discussion in the following proposition:

**Proposition 3** Bilateral trade is more likely to be supported for smaller trade costs and/or lower dispersion of consumers and/or smaller number of firms in the larger market. Bilateral trade is more likely to be supported for weaker returns to quality investments if and only if those returns are weak or region sizes are sufficiently different.

### 3.5 Optimality of investments in product quality

We here finally study whether the equilibrium implies too large or too low quality levels. The issue of efficient product quality is discussed by Spence (1975) for the case of monopoly. In this seminal paper, the efficiency of equilibrium quality depends on how the firm internalizes the benefit of a quality increase on inframarginal consumers. This subsection extends this seminal analysis to the case of monopolistic competition and trade. It emphasizes the role of the externality of each firm’s quality choice on consumers and on other firms. For simplicity, we assume that bilateral trade is possible at the costless quality $\alpha$ so that $\tau < \alpha$.

Note first that firms individually choose a higher quality than the quality that maximizes industry profits $\Pi^{tot} \equiv \sum_i \int_{V_i} \Pi_i(v) dv$. Over-investment in quality takes place because firms do not internalize the negative effect of their quality increases on their competitors’ sales. Indeed, an increase in average quality reduces the profit of any firm that does not simultaneously raise its quality level. This easiest way to show this is to study the simultaneous rise of each quality $\hat{\alpha}^*(v)$ to $\hat{\alpha}^*(v) + \varepsilon$ where $\varepsilon$ is a sufficiently small positive real number. Then, we get

$$\frac{d \Pi^{tot}}{d \varepsilon} = -\frac{L}{2b+c} \frac{I_o}{2} (\alpha^* - \alpha) < 0$$

(see Appendix). So, firms would prefer lower quality and there exists over-investment in quality from the producers’ perspective. Such over-investments in quality increase with stronger product substitutability. Indeed, the above expression is nil when products are independent ($c = 0$) and rises with stronger product substitution (larger $c$). As each firm tries to raise its product quality to steal the business of other firms, firms end up reaching
quality levels that they would not implement if they were alone in the market. There is an undesired curse for quality from the perspective of producers. Investors and industry lobbies should call for lower quality levels.

The role of trade and consumer location can be analyzed as follows. Because lower trade costs increase the average quality $\alpha^*$, they increase the extent of over-investment in quality. This is because firms lose the market protection of trade barriers and endure a tougher competition and curse for quality. Similarly, because co-agglomeration of firms with consumers in the larger market raises the average quality, it exacerbates the effect of over-investment in quality. A growing number of firms in the larger market obviously intensifies competition and the curse for quality there.

Although the equilibrium level of quality is too high from the perspective of firms, it still remains lower than what consumers prefer. Firms are indeed not able to collect the whole consumer surplus from an increase of their quality so that they do not internalize the whole benefit of a better quality to infra-marginal consumers. More formally, using condition (3) one can write the consumer surplus of an individual located in region $i$ as

$$U_i^* = \frac{1}{2} \int_v \left[ \tilde{\alpha}^*(v) - p^*(v) \right] q(v)dv + \bar{\eta}_i^0$$

If we raise again each quality $\tilde{\alpha}^*(v)$ to $\tilde{\alpha}^*(v) + \varepsilon$, we get (see Appendix)

$$\frac{dU_i}{d\varepsilon} = \frac{b(b+c)^2}{(2b+c)^2} \left( \alpha^* - \tau n_j \right)$$

which is positive because $\alpha^* \geq \alpha > \tau$. Hence, consumers would prefer higher quality goods even though those goods would be priced higher. There is under-investment in product quality from the consumers’ perspective. This is true for consumers residing in all regions. Yet, those residing in the region hosting the largest number of firms benefit from a better access to products and would gain more from an increase in product quality ($dU_i/d\varepsilon > dU_j/d\varepsilon \iff n_i > n_j$). Finally, the impact of trade and consumer location is naturally the opposite of the one on firms: from the perspective of the average consumer, under-investment in product quality is aggravated by lower trade costs and co-agglomeration of firms with consumers in the larger market. Indeed, some lines of algebra readily show that the aggregate utility $L\theta_iU_i + L\theta_jU_j$ is a decreasing function of $\tau (\theta_H n_F + \theta_F n_H)$ where $$(\theta_H n_F + \theta_F n_H)$$ falls with co-agglomeration of firms with consumers in the larger market.
Since a global quality increase affects industries and consumers in opposite ways, it is natural to ask whether such an increase would raise aggregate welfare. In the Appendix, we compute that, at the equilibrium average quality $\alpha^*$ given by (16),
\[
\frac{d}{d\varepsilon} (L\theta_iU_i + L\theta_jU_j + \Pi_{tot}) = 2b^2I_o(2b + c) \frac{\alpha - \tau (\theta_Hn_F + \theta_Fn_H)}{I_o(2b + c) - 2b}
\]
which is positive because $\theta_Hn_F + \theta_Fn_H < 1$ and $\alpha \geq \tau$. So, global welfare would be increased by a global increase of quality. There is thus under-investment in quality from a welfare viewpoint. Because the latter expression is a decreasing function of $\tau (\theta_Hn_F + \theta_Fn_H)$, we can make the same conclusion as for consumers: from a welfare perspective, under-investment in product quality is aggravated by lower trade costs and co-agglomeration of firms with consumers.

We summarize this discussion in the following proposition:

**Proposition 4** The equilibrium average quality is set too high for producers and too low for both consumers’ and welfare’s viewpoint. Those effects are exacerbated by lower trade costs and by the co-agglomeration of firms with consumers in the larger market.

We now turn to the discussion of the firms’ location choice.

## 4 Product quality and economic geography

In this section we study how firms’ choices of location and quality shape the economic geography. It is well-know that firm or capital mobility fosters spatial polarization of economic activity. It is however less clear how differences in region sizes affect the quality produced in each region and the number of firms locating in each region. We here show that, compared to the case with exogenous quality levels, the home market effect can be stronger or weaker under endogenous quality.

We present a footloose capital model in which unit mass of capital is inelastically supplied by a set of immobile capital owners. Because of the immobility of the capital owners and because of the absence of income effect in the demand for manufacturing goods, the residence place of those agents has no importance on product demands, profits and location of firms. For the sake of exposition, we assume that the capital market is perfectly
competitive and that each firm requires one unit of capital in order to operate. As a result, the mass of varieties is equal to unity and the previous analysis holds.

The equilibrium in the capital market is obtained as follows. Capital owners allocate their capital to the firms that offer the highest return across regions. To obtain a unit of capital, each firm chooses the location that maximizes its profit and bids for capital up to the value that cancels its profit. As a result, we get two possible configurations. On the one hand, the whole capital flows in region $H$ (resp. $F$) so that $n_H = 1$ (resp. $n_F = 1$) because firms producing in this region always offers a better return: $r_H > r_F$ (resp. $r_F > r_H$). On the other hand, the capital spreads across regions so that $n_H \in [0, 1]$ because firms offer the same return in both regions: $r_H = r_F$. Therefore, the location of firms is given by the rent differential $\Delta r^* (n_H) = r_H - r_F$.

For the sake of simplicity, we focus on the case of bilateral trade. We study the equilibrium where firms simultaneously choose their location and quality investment. This means that firms see the two decisions with the same degree of irreversibility. From the previous section we know that quality investments depend on firms’ locations. The rent differential writes as

$$\Delta r^* (n_H) = L \left\{ \theta_H \left[ (p_{HH}^*)^2 - (p_{FH}^* - \tau)^2 \right] - \theta_F \left[ (p_{FF}^*)^2 - (p_{HF}^* - \tau)^2 \right] \right\}$$

$$- \frac{1}{I_o} \left\{ [\theta_H p_{HH}^* + \theta_F (p_{HF}^* - \tau)]^2 - [\theta_F p_{FF}^* + \theta_H (p_{HF}^* - \tau)]^2 \right\}$$

where optimal prices are functions of the individual and average quality ($\alpha_H^*, \alpha_F^*$) that depend on the location of firms. Bilateral trade imposes that the spatial equilibrium $n_H$ satisfies condition (21).

After some simplifications, we get that the rent differential $\Delta r^* (n_H)$ is proportional to the function $\Delta(\theta_H, I_o) - (n_H - 1/2)$ where

$$\Delta(\theta_H, I_o) = \frac{2b(2\alpha - \tau)}{c_T} (\theta_H - 1/2) \ G(\theta_H - 1/2, I_o) \quad (22)$$

11Note first that the present model also applies to the sequential model where firms choose their locations before their quality investments. Second, Picard and Okubo (2012) study a same model quality is exogenous. Extending the latter analysis to a sequential model with a quality decision before the location choice adds up a coordination problem between quality and location decision time periods. Finally, the present model can be loosely interpreted as a dynamic model where, in each period, some firms die because their varieties become obsolete and are replaced by new firms that choose their location and quality.
and
\[ G(x, I_o) = \frac{(2b + c) I_o (I_o - 1)}{(I_o - 1) ((2b + c) I_o - 2b) + ((4b + c) I_o - 2b) 4x^2} \]

The numerator and denominator of the function \( G \) are positive because \( I_o > 1 \). Because \( \Delta r^* \) decreases in \( n_H \), the location equilibrium exists and is unique. Therefore the location equilibrium is given by
\[ n_H^* = \min\left[\frac{1}{2} + \Delta(\theta_H, I_o), 1\right] \]

We first highlight the effects of trade costs. It is trivial to check that \( \Delta(\theta_H) \) decreases with larger \( \tau \). Furthermore, using (16) and (15), it can readily be shown that
\[ \frac{d(\alpha_H^* - \alpha_H^*)}{d\tau} = \frac{2\theta_H - 1}{I_o - 1} > 0 \quad \text{and} \quad \frac{d\alpha^*}{d\tau} \propto \left(\theta_H n_F + \theta_F n_H\right) + \frac{\tau}{2} \left(2\theta_H - 1\right) \frac{dn_H^*}{d\tau} < 0 \]

Therefore, whereas the quality gap falls when trade cost falls, average quality rises. The directions of those effects are the same as in the model with exogenous firm locations that we discussed in the previous section.

The following proposition summarizes those results.

**Proposition 5** Under bilateral trade, the location equilibrium exists and is unique. In this equilibrium, high quality varieties are produced in the larger region. As trade costs fall, more firms locate in the larger region, the average quality rises and the quality gap between regions decreases.

We can now discuss how the spatial distribution of firms changes as region size asymmetries rise. In particular we study the home market effect (HME) according to which the market equilibrium may involve a more than proportionate share of industry in the region with the larger population. That is, we measure the home market effect as
\[ HME \equiv \frac{n_H^* - 1/2}{\theta_H - 1/2} > 1 \]

In our model with endogenous regional product qualities \((\alpha_H^*, \alpha_F^*)\), we get
\[ HME(\alpha_H^*, \alpha_F^*) = \frac{2b (2\alpha^* - \tau)}{c\tau} G(\theta_H - 1/2, I_o) \]

The function \( G(x, I_o) \) decreases in \( x \) for all \( x \in [0, 1/2] \): it is larger than one if and only if
\[ x < \bar{x} \equiv \sqrt{b(I_o - 1)/[2(4b + c)I_o - 4b]} \]. Hence the home market effect falls with stronger
region size asymmetries. It is interesting to compare this home market effect with the case of homogenous product quality. We focus on two cases where the product quality is set to either the costless quality $\alpha$ or the average quality $\alpha^*$.

Firms implement the costless quality for too prohibitive quality investments. Since $\lim_{x \to \infty} G(x, \infty) = 1$, we can use (23) to obtain

$$HME(\alpha, \alpha) = \frac{2b(2\alpha - \tau)}{cT}$$

which is smaller than $HME(\alpha^*_H, \alpha^*_F)$ if $\theta_H$ smaller than $1/2 + \bar{x}$. In this situation, the home market effect is independent of region sizes and lower than in the case of endogenous product quality provided that regions are not too dissimilar. Indeed, under endogenous product quality, product quality and consumers’ willingness to pay increase as the larger region hosts more consumers. As a result, firms set higher prices and trade costs offer a smaller protection against competition. This weakens the dispersion forces and strengthens the home market effect. This mechanism reflects the impact of quality investments on ‘average quality’.

Applying the last formula to the case where the homogenous quality is equal to $\alpha^*$ we get

$$HME(\alpha^*, \alpha^*) = \frac{2b(2\alpha^* - \tau)}{cT}$$

In the absence of region size asymmetries ($\theta_H = 1/2$), product quality, we have that $\alpha^*_H = \alpha^*_F = \alpha^*$ so that $HME(\alpha^*, \alpha^*) = HME(\alpha^*_H, \alpha^*_F)$. Therefore since $HME(\alpha^*_H, \alpha^*_F)$ falls with stronger size asymmetry (higher $\theta_H$), we have $HME(\alpha^*_H, \alpha^*_F) < HME(\alpha^*, \alpha^*)$ if $\theta_H > 1/2$. As a consequence, the home market effect is smaller when firms set their own product quality than when they are forced to produce at the average quality. This is because the co-agglomeration of firms with consumers in the larger market is bad for the product quality of each individual firm producing in that market. This reduces profits there and refrains the incentives to locate in the larger market. This mechanism reflects the pro-competitive effects of product quality differences that quality investments generate.

This yields the following proposition.

**Proposition 6** Under bilateral trade, the home market effect decreases with stronger region size asymmetries. It is weaker than the home market effect existing under a homoge-
nous ‘average quality’ while it is stronger than the home market effect prevailing under a homogenous ‘costless quality’ if regions are not too dissimilar.

We finally study the impact of a fall in the decreasing returns of quality investment $I_o$ on the location of firms. One can show that $G(x, 1) = 0$ and $G(x, \infty) = 1$ and that $G(x, I_o)$ increases in $I_o$ for any

$$I_o < T_o(x) \equiv \begin{cases} 1 + \frac{2x^2(2b+c)+\sqrt{2b(2b+c)(1-4x^2)x^2}}{b-(8b+2c)x^2} & \text{if} \quad x^2 < b/[2(4b + c)] \\ \infty & \text{if} \quad x^2 \geq b/[2(4b + c)] \end{cases}$$

A fall in the decreasing returns of quality investment $I_o$ has a non monotone effect on firms location asymmetries $n_H$ and home market effect if region size asymmetries are small enough ($(\theta_H - 1/2)^2 < b/[2(4b + c)]$). As $I_o$ falls to one, location asymmetries $n_H$ and home market effect first increase and then decrease. Otherwise, if region size asymmetries are large enough, location asymmetries and home market effect always diminish as $I_o$ falls.

This result must be related to Proposition 3 and stems from the agglomeration and dispersion forces of the choice of quality on firms’ location. For large $I_o$, the agglomeration effect dominates as a fall in $I_o$ increases ‘average quality’ more than it decreases the quality markups. For small enough $I_o$, the dispersion effect dominates as a fall in $I_o$ affects more negatively the quality markups than it affects ‘average quality’. Hence, firms tend to agglomerate more for large $I_o$ and disperse more for smaller $I_o$. To sum up, weaker decreasing returns to quality investment monotonically increase the number of firms in the larger region only if region size asymmetries are strong enough. Otherwise, they can have non-monotone effects. This is summarized in our last proposition.

**Proposition 7** Under bilateral trade, changes in the technology of investment in product quality may raise or diminish countries’ inequalities.

5 Conclusion

The present paper studies the effect of the choice of product quality on trade and location of firms. In this model consumers have preferences for the quality of a set of manufacturing
varieties. Firms do not only develop and sell manufacturing varieties in a monopolistic competitive market but also determine the quality level of their varieties by investing in research and development. We show that the larger region produces varieties of higher quality and that the quality gap increases with larger asymmetries in region sizes and with larger trade costs. In a footloose capital model we find that the home market effect is qualified.

This paper sets the stage for further investigations. A traditional research direction is the study of workers’ mobility in a core-periphery model. As discussed in our analysis, we expect that the investments in product quality exacerbate the agglomeration forces and give more prevalance to the central places. This might fit the difference in product quality between rural and urban areas and between large and small cities. It would interesting to highlight the effect of investments on the average quality and quality markups and therefore to outline the pro-competitive effects resulting from quality choices. A more challenging study would be to extend the model to income heterogeneity, as the patterns of trade depend on product quality and therefore on the subtle interplay of the income distribution and non-homothetic preferences. Also, because they depend on exogenous regional differences and quality markups, income distribution might reinforce the tendency of firms producing high quality to agglomerate in the high income region. Such a study should discuss a model with non-homothetic preferences and income effects on the consumption of the differentiated varieties.

References


Appendix

Let us raise each quality $\hat{\alpha}(v)$ to $\hat{\alpha}(v) + \varepsilon$ where $\varepsilon$ is a sufficiently small positive real number. We then get

$$\frac{d\Pi^{tot}}{d\varepsilon} = \sum_i \int_{\nu_i} \left( \frac{\partial \Pi_i^*(v)}{\partial \hat{\alpha}(v)} + \frac{\partial \Pi_i^*(v)}{\partial \hat{\alpha}(v)} \right) \, dv$$

$$= -L \frac{c(b+c)}{2b+c} \sum_i \int_{\nu_i} \left[ \theta_i p_{ii}^*(v) + \theta_j (p_{ij}^*(v) - \tau) \right] \, dv$$

$$= -L \frac{c(b+c)}{2b+c} \sum_i \int_{\nu_i} \left[ \theta_i p_{ii}^*(v) + \theta_j (p_{ij}^*(v) - \tau) \right] \, dv$$

where we apply the envelop theorem in the first line ($\partial \Pi_i^*(v)/\partial \hat{\alpha}(v) = 0$ by (12)), we use $\partial p_{ij}^*(v)/\partial \alpha = -c/ (2b + c) < 0$ in the second line and we use (13) and integrate over varieties in the third line.

The consumer surplus is given by

$$U_i^* = \frac{1}{2} \int_{\nu} [\hat{\alpha}(v) - p^*(v)] q(v) \, dv + \pi_i^p$$

which can be broken down by country as

$$U_i^* = \frac{(b+c)}{2} \int_{\nu_i} [\hat{\alpha}(v) - p_{ii}^*(v)] p_{ii}^*(v) \, dv + \frac{(b+c)}{2} \int_{\nu_j} [\hat{\alpha}(v) - p_{ji}^*(v)] [p_{ji}^*(v) - \tau/2] \, dv + \pi_i^p$$

If we raise again each quality $\hat{\alpha}(v)$ to $\hat{\alpha}(v) + \varepsilon$, we successively get
\[
\frac{dU_i}{d\varepsilon} = \frac{1}{2} \int_{V_i} \frac{d}{d\varepsilon} \left[ (\tilde{\alpha}^*(v) - p_{ii}^*(v)) q_{ii}(v) \right] dv + \frac{1}{2} \int_{V_j} \frac{d}{d\varepsilon} \left[ (\tilde{\alpha}^*(v) - p_{ji}^*(v)) q_{ji}(v) \right] dv
\]

Also,

\[
= \frac{b + c}{2} \int_{V_i} \frac{d}{d\varepsilon} \left[ (\tilde{\alpha}^*(v) - p_{ii}^*(v)) p_{ii}(v) \right] dv + \frac{b + c}{2} \int_{V_j} \frac{d}{d\varepsilon} \left[ (\tilde{\alpha}^*(v) - p_{ji}^*(v)) (p_{ji}^*(v) - \tau) \right] dv
\]

\[
= \frac{b + c}{2 (2b + c)} \left[ \int_{V_i} [\tilde{\alpha}^*(v) b + cp_{ii}(v)] dv + \int_{V_j} [\tilde{\alpha}^*(v) b + cp_{ji}^*(v) - \tau (b + c)] dv \right]
\]

where we use \( dp_{ji}^*(v)/d\varepsilon = b/ (2b + c) < \frac{1}{2} \). Furthermore, we can firstly integrate \( \tilde{\alpha}^*(v) \) over the sets \( V_i \) and \( V_j \), secondly substitutes for the values of prices and then simplify to get:

\[
\frac{dU_i}{d\varepsilon} = \frac{(b + c)}{2 (2b + c)} \left[ ab + \int_{V_i} cp_{ii}^*(v) dv + \int_{V_j} [cp_{ji}^*(v) - (b + c) \tau] dv \right]
\]

\[
= \frac{(b + c)}{2 (2b + c)} \left[ \int_{V_i} \left[ ab + c \frac{1}{2} \frac{2ab + \tau n_j c}{2b + c} + c \tilde{\alpha}(v) - \frac{\alpha}{2} \right] dv \right]
\]

\[
= \frac{(b + c)}{2 (2b + c)} \left[ \int_{V_j} \left[ c \frac{1}{2} \frac{2ab + \tau n_j c}{2b + c} + c \tilde{\alpha}(v) - \frac{\alpha}{2} - (2b + c) \frac{\tau}{2} \right] dv \right]
\]

\[
= \frac{(b + c)}{2 (2b + c)} \left[ \alpha b + c \frac{1}{2} \frac{2ab + \tau n_j c}{2b + c} - (2b + c) \frac{\tau}{2} n_j \right]
\]

\[
= \frac{(b + c)}{2 (2b + c)} \left[ 2b \alpha \frac{b + c}{2b + c} - 2b \tau n_j \frac{b + c}{2b + c} \right]
\]

\[
= \frac{2b(b + c)^2}{2 (2b + c)^2} (\alpha - n_j) > 0
\]

which is positive because \( \alpha^* \geq \alpha > \tau \).

Finally,

\[
\frac{d}{d\varepsilon} \left( L\theta_i U_i + L\theta_j U_j + \Pi^{tot} \right) = L \frac{2b(b + c)^2}{2 (2b + c)^2} (\alpha^* - \tau (\theta_i n_j + \theta_j n_i)) - L \frac{c(b + c) I_o}{2b + c} \frac{\alpha^* - \alpha}{2}
\]

\[
= L \frac{(b + c)}{2 (2b + c)^2} \left[ 2b (b + c) \alpha - 2\tau b(b + c) (\theta_i n_j + \theta_j n_i) \right]
\]

\[
+ (2b(b + c) - c(2b + c) I_o) (\alpha^* - \alpha)
\]

Using (16) we have

\[
\frac{d}{d\varepsilon} \left( L\theta_i U_i + L\theta_j U_j + \Pi^{tot} \right) = 2b^2 I_o (2b + c) \frac{\alpha - \tau (\theta_H n_F + \theta_F n_H)}{I_o (2b + c) - 2b} > 0
\]

because \( \alpha > \tau \geq \tau (\theta_H n_F + \theta_F n_H) \).
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