Self-commitment of combined cycle units under electricity price uncertainty

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Abstract

Day-ahead energy market clearing relies on a deterministic equivalent model with a limited time horizon, which may lead to inefficient scheduling of generating units from the point of view of generators. For this reason, generators may wish to forgo the profit hedging offered by day-ahead electricity markets and assume the risk of self-committing their units with the hope of securing greater profits. This phenomenon may undermine the depth of the day-ahead market, especially in conditions of high price volatility due to deep renewable energy integration. In this paper we investigate the influence of risk aversion and price volatility on the decision of generators to self-commit units. We present a stochastic programming model for self-committing combined cycle units under price uncertainty with a conditional value at risk criterion. We use Bender’s decomposition to solve the problem and present results on a case study to draw conclusions.

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1 Introduction

The clearing of day-ahead markets relies on a deterministic equivalent model where uncertainty is represented through its expected value. In addition, the horizon of day-ahead market models is often too short to account for operating constraints that couple the operations of units from one day to the next. These two factors may lead to the inefficient commitment of conventional units. This is especially true for combined cycle units due to the increased complexity of their technical constraints and cost characteristics.

When generators participate in the day-ahead market, they are required to place market bids that consist of startup cost, minimum load cost, a multi-segment heat rate curve as well as minimum and maximum capacity limits, minimum up and down times and ramping rates. In this case, the system operator commits these resources according to the optimal solution of a simplified deterministic market model. As in any forward market, the benefit of bidding in the day-ahead electricity market is the insurance of generators against real-time price uncertainty. Motivated by the aforementioned inefficiency in day-ahead dispatch, generators may instead opt to self-schedule or self-commit resources. Self-commitment refers to the situation where a generator fixes its day-ahead unit commitment schedule and determines power output based on market price. In the case of self-scheduling, the generator also fixes its production level in advance. Self-scheduling usually takes place based on contract terms and will not be investigated further in this paper. Resources that are self-scheduled may be use-limited and are not able to follow the system operator automatic control. The drawback of self-scheduling or self-committing resources is that the generator assumes the full risk of real-time price uncertainty. The advantage of this approach is that a generator can better optimize the commitment and dispatch of its assets by solving a more detailed optimization with a detailed representation of uncertainty and a longer time horizon.

The self-commitment of units can reduce the depth and undermine the very purpose of the day-ahead market. The large-scale integration of renewable resources with the resulting increase of real-time price uncertainty and the increased complexity of combined cycle units exacerbates the weakness of a deterministic day-ahead market model. On the other hand, the risk aversion of generators is the primary reason for the existence of a day-ahead market.

1The time horizon of the California ISO Integrated Forward Market is 24 hours.

2In the California ISO market, there will be a high demand for flexible resources as renewable generation is integrated increasingly in the market. Self-commitment and more so self-scheduling undermine this purpose by rendering combined cycle units as inflexible resources from the point of view of the system operator, although these resources are inherently flexible. An initiative is underway from the California ISO and the California Public Utilities Commission to enforce flexible capacity (flexible resource adequacy) in addition to the current generic resource adequacy requirement on California utilities.

3An additional reason for self-commitment is the fact that certain cost components of combined cycle units are not accurately represented in the ISO model due to the bid cap rules. For instance, in the California ISO the fixed startup cost is capped by twice the cost of startup fuel which is not sufficient to recover the true actual startup cost of combined cycle units, that exceeds cost factors not related to fuel.
In this paper we investigate how these two factors influence the preference of generators to bid in the market versus self-committing combined cycle units.

The use of combined cycle units in short-term balancing is becoming increasingly important due to the better controllability of these units, their modularity, and their flexibility in terms of the fuel they consume [11]. The proliferation of these units has increased the need to represent their operations and costs accurately in day-ahead commitment models. This need was underscored in a recent presentation by Ott [15] for the Pennsylvania Jersey Maryland (PJM) system. A detailed survey of literature on combined cycle unit modeling is presented by Anders et al. [1].

The difficulty of modeling combined cycle units stems from the fact that they consist of multiple components, each of which has its own independent technical and cost characteristics, as well as dependencies with the other components of the unit. Combined cycle units typically consist of multiple combustion turbines. The waste heat from the combustion turbines can be used for fueling steam turbines. The operation of these units can be represented either by a bottom-up modeling of the components (combustion and steam turbines) of the units, or a reduced modeling of the modes (the combination of combustion and steam turbines that are operational). Liu et al. [10] present a detailed comparison of a component model and a mode model for combined cycle units. Both approaches result in a mixed integer linear program which is substantially more complex than simplified models of conventional units.

Early work on the modeling of combined cycle units is presented by Cohen and Ostrowski [6]. The authors use Lagrangian relaxation to decompose the problem by component. The authors solve single-unit problems by dynamic programming and augment the state space of their model in order to account for how long a unit has been operating a given component. Subsequent deterministic models of combined cycle units include the work of Lu and Shahidehpour, [11], [12]. In [11] the authors present a model for combined cycle units with combustion turbines and steam turbines that they incorporate in a security constrained economic dispatch model. The model is solved via Lagrangian relaxation and Benders decomposition. In [12] the authors extend their model to account for the operation of flexible generating units of three types, mixed fuel units, combined cycle units and dual fuel units. Li and Shahidehpour [9] compare the solution of unit commitment models with combined cycle units based on Lagrange relaxation and branch and bound methods. Simoglou et al. [19] present a detailed model for self-scheduling combined cycle units, however they also do not account for uncertainty.

The modeling of uncertainty in self-scheduling has been addressed by various authors. Cerisola et al. [5] present a stochastic unit commitment model for deciding on the day-ahead and balancing market trades and production quantities of an owner of a hydrothermal portfolio. The horizon of the model is one week with hourly resolution. The source of uncertainty is electricity prices. Tseng and Zhu [21] also present a model for self-scheduling generators subject to uncertainty in electricity prices. Garces and Conejo [7] present a stochastic programming model for a generator that decides on self-scheduling units, for-
ward contracting and offering bids in the pool. The authors also account for risk aversion in the model through the use of the conditional value at risk (CVaR), using the theory provided by Rockafellar and Uryasev [18]. Although this work focuses on self-scheduling, none of the aforementioned papers account for combined cycle unit operations and uncertainty simultaneously. In addition, with the exception of Garces and Conejo [7], none of the papers account for risk aversion.

The incorporation of price uncertainty in stochastic unit commitment models was first introduced by Takriti et al. [20]. The authors in [20] account for price uncertainty in medium-term (monthly) fuel price fluctuations and use Benders decomposition. Fuel price uncertainty in medium term scheduling models was concurrently addressed by Pereira et al. [16] and Gjelsvik et al. [8]. The authors combine stochastic dynamic programming techniques with Benders decomposition in order to address the fact that price uncertainty results in non-convexity of the value function. These models, however, are not focused on self-commitment and do not account for the operating details of combined cycle units.

The purpose of this paper is to investigate the influence of risk aversion and price volatility on the decision of generators to self-commit combined cycle units versus bidding them in the day-ahead market. This requires the integration of uncertainty and risk aversion in the self-commitment model. The previously cited literature either addresses self-commitment under uncertain price conditions without accounting for the complexity of combined cycle unit operations, or addresses the complex operations of combined cycle units without accounting for uncertainty and risk aversion. In addition to introducing a model that accounts for these features simultaneously, in this paper we develop a methodology that exploits this model in order to analyze the influence of risk aversion and price volatility on the willingness of generators to participate or not in the day-ahead market.

In Sect. 2 we present a motivating example that highlights the difference between self-commitment and market bidding. In Sect. 3 we describe our methodology and the models used in our analysis. A solution algorithm for the risk-averse self-commitment model is presented in Sect. 4. We use the case study of Sect. 5 in order to draw conclusions, which are presented in Sect. 6.

2 A Motivating Example

In order to clarify the difference between self-commitment and bidding into the market, we examine the example of Fig. 1. This system consists of one conventional unit, one renewable supplier and one load. The conventional unit has a capacity of $P = 3$ MW, a startup cost of $K = $100, and a fuel cost of $C = 20$/MWh. The renewable supplier is equally likely ($\pi_1 = \pi_2 = 0.5$) to produce $W_1 = 96$ MW or $W_2 = 104$ MW. The load has a demand of $D = 100$ MW and a valuation of $V = 100$/MWh. We will assume that the conventional unit ownership is separated from the ownership of the renewable unit and that the conventional generator is risk-averse. We will also assume that the sys-
Figure 1: A simple system with one conventional unit, one renewable supplier and one load.

The system operator shares the same forecast for renewable supply as the conventional generator.

**Market Outcome.** Suppose that the conventional unit bids its true costs and capacity limit in the day-ahead energy market. In order to commit units in the day-ahead market, we assume that the system operator solves a deterministic equivalent unit commitment problem where the renewable supply forecast is equal to its average value, $\bar{W} = 100$ MW. Namely, the system operator solves the following unit commitment problem:

$$\begin{align*}
&\text{max} \quad 100d - 100u - 20p \\
&\quad p + W \geq d \\
&\quad 0 \leq p \leq 3u \\
&\quad u \in \{0, 1\}
\end{align*}$$

where $p$ is the power production of the conventional unit, $u$ is the commitment of the conventional unit and $d$ is the load consumption. Since the average renewable supply is enough to exactly satisfy demand, the operator will not commit the conventional unit. However, this implies that there will be unsatisfied demand in half of the occurrences. In order to clear the real-time market we assume that the system operator solves an economic dispatch model with the conventional unit committed to the optimal solution of the day-ahead model and uses the dual multiplier of the power balance constraint as the real-time price. The market price for each scenario $i \in \{1, 2\}$ is then obtained as the dual variable $\lambda_i$ of the following problem:

$$\begin{align*}
&\text{max} \quad 100d - 20p \\
&\quad p + W_i \geq d, (\lambda_i) \\
&\quad 0 \leq p \leq 3u^*
\end{align*}$$

where $u^* = 0$ is the optimal solution of the day-ahead unit commitment problem. Consequently, the real-time market price will be equal to $\lambda_1 = 100$/MWh for
the case of $W_1 = 96$ MW, and $\lambda_2 = 0$/MWh for the case of $W_2 = 104$ MW. Provided that the generator cannot place negative startup and marginal cost bids, the unit will be kept offline even if it understates its cost bids in order to increase its likelihood of being committed in the day-ahead market. Real-time prices in this market would be $\lambda_1 = 100$/MWh or $\lambda_2 = 0$/MWh with equal likelihood.

**Predicting the Market Outcome.** In large systems it is extremely challenging for market agents to estimate the outcome of the day-ahead and real-time market based on a bottom-up unit commitment and economic dispatch model. In a market with convex feasible sets and convex cost functions a generator could forecast its dispatch by solving its profit maximization problem against the market clearing price. Although no such theoretical guarantees are available in markets with non-convexities in the feasible set and costs, a proxy for a generator to predict its commitment in the day-ahead market is to solve its individual profit maximization problem against the day-ahead market price forecast. Assuming that the day-ahead price forecast is the weighted average of the real-time price, $\bar{\lambda} = 0.5 \cdot 100 + 0.5 \cdot 0 = 50$/MWh, the conventional unit in the example would then solve the following problem:

$$\max 50p - 20p - 100u$$

$$0 \leq p \leq 3u$$

$$u \in \{0, 1\}$$

Indeed, the optimal solution to this problem reproduces the outcome of the full day-ahead unit commitment model. The conventional unit is kept off and produces zero output.

**Self-commitment.** Suppose that, instead of bidding, the conventional unit self-commits. The conventional unit then solves the following problem:

$$\max 0.5(100p_1 - 20p_1) + 0.5(0p_2 - 20p_2) - 100u$$

$$0 \leq p_1 \leq 3u$$

$$0 \leq p_2 \leq 3u$$

$$u \in \{0, 1\}$$

where $p_i$ is the power production of the conventional unit in the case where scenario $i$ is realized. The optimal solution to this problem is to commit the unit. The unit receives an expected revenue of $0.5 \cdot (4\text{MW}) \cdot (100$/MWh) $= 200$/h from the market, incurs an expected fuel cost of $0.5 \cdot (4\text{MW}) \cdot (20$/MWh) $= 40$/h and incurs a certain startup cost of 100$. This is preferable to self-committing the unit to be off, in which case the profit is certain to be zero.

**Discussion.** We wish to highlight two observations from this example. The first observation is that generators can solve their profit maximization problem...
against the forecast day-ahead price as a proxy of the day-ahead market unit commitment. This is the assumption that we adopt in the model of Sect. 3.1.

The second observation is that the day-ahead market will not necessarily reproduce a unit commitment schedule that is desirable from the point of view of the conventional generator because the day-ahead market model is a simplified deterministic representation of a highly uncertain environment.

In order to simplify the discussion in the example, we have considered a single-period day-ahead unit commitment model without security constraints. We have also assumed that the renewable supply forecast will be equal to the average supply of renewable power, and that the generator does not account for the influence of its actions on price. Although these simplifying assumptions do not reflect realistic practice, they illuminate the difference between self-scheduling and bidding in the day-ahead market. In practice, a utility will likely use time series models of real-time electricity prices in order to construct price scenarios that will guide its decision whether or not to self-commit. Since a time-series model ignores the underlying unit commitment and economic dispatch problem, the simplifying assumptions that we adopt do not influence the first observation in the previous paragraph. The second observation is demonstrated in a realistic example in the case study of Sect. 5.

3 Model

The decision of a generator on whether or not to bid in the day-ahead market depends on its assessment of the risks associated with either decision, and is depicted in Fig. 2. Throughout the paper we assume that the single source of uncertainty is the real-time price of electricity, denoted \( \lambda_{st} \). Here \( s \in S \) indexes a discrete set of scenarios that represents the possible realizations of uncertainty and \( t \in T \) indexes the set of time periods over which the evaluation of the self-scheduling decision is performed. We consider two stages of decision making. In the first stage generators commit their units, either through the market or through self-commitment. Subsequently, generators observe the realization of real-time prices and in the second stage generators dispatch their units, given the fixed commitment decisions, against the realized real-time price of electricity.

Generators quantify the benefits of bidding in the market by committing combined cycle units against their expectation of the day-ahead price, assumed equal to the average real-time price, \( \lambda_t = \sum_{s \in S} \pi_s \lambda_{st} \). The day-ahead market model is presented in Sect. 3.1. The day-ahead market model also determines the day-ahead profit of generators, which is a secured profit that can only increase if the generator identifies profit opportunities in the real-time market. This hedging function of the day-ahead market is represented in Fig. 2 by the max operator acting on the day-ahead and real-time profits. The real-time profits are computed by running the real-time dispatch model, which is presented in Sect. 3.3. The profits are computed over a large number of samples, and the resulting distribution of profits is transformed through the CVaR risk criterion in order to compute the risked profits. For risk level \( a \) and random costs
(negative profits), the conditional value at risk is the expected cost conditional on costs being within the higher $a$-percentile of the distribution (equivalently, profits being in the lower $a$-percentile of the distribution).

The computational advantage of using the CVaR risk criterion, demonstrated by Rockafellar and Uryasev [18], is the fact that the CVaR and value at risk, VaR, can be computed by solving a linear program. In addition, as we will show in Sect. 4, this linear program can be incorporated within a Bender’s decomposition scheme for solving the risk-constrained self-commitment problem. The risk-constrained self-commitment problem is presented in Sect. 3.2.

### 3.1 Market Model

The notation used in this section is summarized in the appendix. A feature that makes the modeling of combined cycle units complex is the fact that the components of the units can only be fired in sequence. This can be depicted through a state transition diagram which represents the sets of permissible transitions, as in Fig. 3. The set of permissible states $x \in X$ and transitions $a \in A$ in this diagram correspond to a generating unit that consists of three combustion turbines. Within each state there are various operating constraints that need to be respected. Costs are incurred for operating within each state and also transitioning between states. Transitions between states are modeled through the indicator variable $v_{at}$, equal to 1 if there is a transition over arc $a$ in period $t$ and zero otherwise. The indicator variable $u_{xt}$ indicates whether the unit is in state $x$ or not in period $t$.

Each state obeys a non-linear incremental heat rate curve, such as the one depicted in Fig. [1]. The entire operating range of a certain state $x$ is separated in a set of segments $\{1, \ldots, M\}$. The incremental heat rate is constant within each segment and the total production $p_{x}$ within the specific state $x$ is equal to the minimum load of the specific state $BP_{x}$ plus the production within each segment $p_{xmt}$. Fig. [3] explains the notation that is used in the mixed integer linear program. We note that the heat rate curve needs to be increasing in order for the model to be valid. This is not always obeyed in practice, in which case we ‘convexify’ the marginal cost curve before solving the model. This is
Figure 3: The state transition diagram for a combined cycle unit with three combustion turbines.

Figure 4: The incremental heat rate curve within each operating state of a combined cycle unit.

demonstrated in Fig. 4, where the incremental heat rate of segment 2 is lifted in order to obtain an increasing heat rate curve.

The objective function of the market model is given by Eq. 1. The cost of operating a combined cycle unit consists of fuel costs, variable operating and maintenance (VOM) costs, fixed operating costs and transition costs. Fuel costs are computed using the non-linear heat rate curve of Fig. 4, where $F$ is the price of fuel, $HR_{xm}$ is the heat rate in segment $m$ of state $x$ and $BP_{xm}$ is the upper breakpoint of segment $m$ in the heat rate curve. The fixed operating charge that is incurred every hour that a unit runs is denoted as $OC_x$. The fixed cost of transitioning over an arc $a$ from a certain state $F(a)$ to another state $T(a)$ is denoted as $TC_a$. $VOM_x$ is the VOM cost in state $x$. The generator also has the option of buying ($b_t > 0$) or selling ($b_t < 0$) energy at the day-ahead price $\bar{\lambda}_t$. 

9
\[
\begin{align*}
\min & \sum_{t \in T} \lambda_t b_t + \sum_{x \in X, m \in 1 \ldots M-1, t \in T} \bar{\lambda}_t \bar{b}_t \cdot \bar{p}_{xmt} \\
& + \sum_{x \in X, t \in T} \text{VOM}_x p_{xt} \\
& + \sum_{x \in X, t \in T} (\text{OC}_x + F \cdot \text{HR}_{x,1} \cdot \text{BP}_{x,1}) u_{xt} \\
& + \sum_{a \in A, t \in T} \text{TC}_a v_{at} \\
\end{align*}
\]

The total power output in a certain state \( x \) is calculated as in Eq. (2).

\[
p_{xt} = u_{xt} \text{BP}_{x,1} + \sum_{m=1}^{M-1} p_{xmt}, x \in X, t \in T
\]

Limits on the production of each segment are imposed in Eq. (3).

\[
p_{xmt} \leq (\text{BP}_{x,m+1} - \text{BP}_{xm}) u_{xt}, x \in X, 1 \leq m \leq M - 1, t \in T
\]

Eqs. (4), (5) impose ramp rate limits in each state. \( R^+_x \) and \( R^-_x \) are the ramp up and ramp down rates of mode \( x \) respectively. The ramp rate constraints apply both for transitions from state to state as well as ramp rates within a state.

\[
\begin{align*}
p_{xt} - p_{x,t-1} & \leq (2 - u_{x,t-1} - u_{xt}) \text{BP}_{x,1} + (1 + u_{x,t-1} - u_{xt}) R^+_x, x \in X, 2 \leq t \leq N \\
p_{x,t-1} - p_{xst} & \leq (2 - u_{x,t-1} - u_{xst}) \text{BP}_{x,1} + (1 + u_{x,t-1} - u_{xt}) R^-_x, x \in X, 2 \leq t \leq N
\end{align*}
\]

Eq. (6) dictates that the combined cycle unit can perform no more than one state transition from period to period.

\[
\sum_{a \in A} v_{at} \leq 1, t \in T
\]

Eq. (7) describes the dynamics of the transition.

\[
u_{xt} = u_{x,t-1} + \sum_{a \in A: T(a)=x} v_{at} - \sum_{a \in A: F(a)=x} v_{at}, x \in X, 2 \leq t \leq N
\]

Eq. (8) requires that at each period the unit be in exactly one state.

\[
\sum_{x \in X} u_{xt} = 1, t \in T
\]

10
Minimum up and down times for each state are enforced in Eqs. (9), (10). The minimum up and down times of state $x$ are denoted $UT_x$, $DT_x$ respectively.

$$\sum_{\tau = t - UT_x + 1}^{t} \sum_{a \in A} v_{at} \leq u_{xt}, x \in X, UT_x \leq t \leq N \quad (9)$$

$$\sum_{\tau = t + 1}^{t + DT_x} \sum_{a \in A} v_{at} \leq 1 - u_{xt}, 1 \leq t \leq N - DT_x \quad (10)$$

Eq. (11) defines a binary variable $u_t$ that indicates whether the combined cycle unit is off or not, and Eq. (12) defines a binary variable $v_t$ that indicates whether the unit has started up or not.

$$u_t = \sum_{x \in X - \{‘Off’\}} u_{xt}, t \in T \quad (11)$$

$$v_t = \sum_{a \in A} v_{at}, t \in T \quad (12)$$

Using these variables we define overall minimum up and down time constraints in Eqs. (13), (14), where $UT$ and $DT$ denote the minimum up and down times respectively.

$$\sum_{\tau = t - UT + 1}^{t} v_{\tau} \leq u_t, UT \leq t \leq N \quad (13)$$

$$\sum_{\tau = t + 1}^{t + DT} v_{\tau} \leq u_t, 1 \leq t \leq N - DT \quad (14)$$

Lower and upper bounds and integrality constraints are defined in Eqs. (16), (17).

$$v_{at} \leq 1, a \in A, t \in T \quad (16)$$

$$u_{xt} \in \{0, 1\}, u_t, v_t, v_{at}, p_{xt}, p_{xmt} \geq 0, x \in X, a \in A, t \in T \quad (17)$$

Given a unit commitment and transition schedule $u_{xt}^0$, $v_{at}^0$ for hour $t$ of the previous day, the minimum up and down time constraints of Eqs. (9), (10) for the first hours of the day are given in Eqs. (18), (19).

$$\sum_{\tau = 1}^{t} \sum_{a \in A} v_{at} + \sum_{\tau = N - UT_x + t + 1}^{N} \sum_{a \in A} v_{at}^0 \leq u_{xt}, x \in X, 1 \leq t \leq UT_x - 1 \quad (18)$$

$$\sum_{\tau = t + 1}^{N} \sum_{a \in A} v_{at}^0 + \sum_{\tau = 1}^{t} \sum_{a \in A} v_{at} \leq 1 - u_{xt}, x \in X, N - DT_x + 1 \leq t \leq N \quad (19)$$
Similarly, given commitment and startup schedule $u^0_t, v^0_t$ for hour $t$ of the previous day, the minimum up and down time constraints of Eqs. (13), (14) for the first hours of the day are given by Eqs. (20), (21).

$$\sum_{\tau=1}^{t} v_{\tau} + \sum_{\tau=N-UTx+1}^{N} v^0_{\tau} \leq u_t, 1 \leq t \leq UTx-1$$  \hfill (20)

$$\sum_{\tau=1}^{\tau+i+1} v^0_{\tau} + \sum_{\tau=1}^{N} v_{\tau} \leq 1 - u_t, N - DT_x + 1 \leq t \leq N$$  \hfill (21)

### 3.2 Self-Commitment Model

Consider a risk-averse generator that evaluates the real-time market payoff $Q(w, \lambda_s)$ over a set of price scenarios $s \in S$ according to the CVaR criterion, where $w$ is the set of first-stage commitment decisions and $\lambda_s = (\lambda_{st}, t \in T)$ is the vector of real-time electricity prices for scenario $s$. Theorem 10 of Rockafellar and Uryasev [18] guarantees that the CVaR of the random payoff $Q(w, \lambda_s)$ can be computed as the optimal objective function value of the following optimization:

$$\min_{\zeta} \zeta + \frac{1}{a} \sum_{s \in S} \pi_s(Q(w, \xi_s) - \zeta)^+,$$  \hfill (22)

where $(x)^+ = \max(x, 0)$. In addition, the theorem guarantees that the VaR of the random payoff $Q(w, \lambda_s)$ is given by the optimal value of $\zeta$.

In the case of the self-commitment model, the above math program is a linear program that represents the reaction of generators in the real-time market, given a day-ahead self-scheduling decision $w = (u_{xt}, v_{at}, u_t, v_t)$. In particular, the risk-averse self-scheduling model can be described by the following optimization problem:

$$\min_{\zeta} \zeta + \sum_{x \in X, t \in T} (OC_x + F \cdot HR_{x1} \cdot BP_{x1})u_{xt}$$

$$+ \sum_{a \in A, t \in T} TC_a v_{at} + \frac{1}{a} \sum_{s \in S} \pi_s(Q(w, \lambda_s) - \zeta)^+$$  \hfill (23)

subject to (6), (7), (8), (10), (11), (12), (13), (14), (16), (17), (18), (19), (20), (21),

where the second-stage cost given first-stage decision $w$ and price realization $\lambda_s$ is computed by solving the following linear program:

$$Q(w, \lambda_s) = \sum_{t \in T} \min_{a \in A} \lambda_{st} b_t + \sum_{x \in X, m \in 1...M-1, t \in T} HR_{x,m+1} \cdot F \cdot p_{xmt}$$

$$+ \sum_{x \in X, t \in T} VOM_{x} p_{xt}$$  \hfill (24)

subject to (2), (3), (4), (5), (25).
Note that the market procurement \( b_t \), total production \( p_t \), state production \( p_{x,t} \) and segment production \( p_{x,m,t+1} \) decision variables are now contingent on scenario \( s \) since they are second-stage decisions. When the self-scheduling problem is solved over a large number of scenarios, it is impossible to solve the problem in extended form. In Sect. 4 we present a decomposition algorithm for solving the problem.

### 3.3 Evaluation Model

**Market-based dispatch.** Given a day-ahead unit commitment schedule produced by the market, generators incur the following cost:

\[
C^{DA} = \min \sum_{t \in T} \lambda_t b_t + \sum_{x \in X, m \in 1 \ldots M - 1, t \in T} HR_{x,m+1} \cdot F \cdot p_{x,m,t+1}^* \\
+ \sum_{x \in X, t \in T} VOM_x^* p_{x,t}^* \\
+ \sum_{x \in X, t \in T} \left( OC_x + F \cdot HR_{x,1} \cdot BP_{x,1} \right) u_{x,t}^* \\
+ \sum_{a \in A, t \in T} TC_a v_{a,t}^* 
\]  

(26)

where the optimal decisions of the market model presented in Sect. 3.1 are indicated by a star superscript.

A generator will respond to real-time profit opportunities by changing its dispatch only if the real-time prices are high enough to yield an increased profit. In order to compute the real-time profit opportunity of generators we solve the following problem:

\[
C^{RT}_s = \min \sum_{t \in T} \lambda_{st} b_t + \sum_{x \in X, m \in 1 \ldots M - 1, t \in T} HR_{x,m+1} \cdot F \cdot p_{x,m,t+1} \\
+ \sum_{x \in X, t \in T} VOM_x^* p_{x,t} \\
+ \sum_{x \in X, t \in T} \left( OC_x + F \cdot HR_{x,1} \cdot BP_{x,1} \right) u_{x,t} \\
+ \sum_{a \in A, t \in T} TC_a v_{a,t} 
\]  

(27)

s.t. (2), (3), (4), (5)

\[
u_{x,t} = u_{x,t}^*, v_{a,t} = v_{a,t}^*, u_t = u_t^*, v_t = v_t^*, x \in X, a \in A, t \in T
\]  

(28)

Eqs. (28) fix the commitment and transition decisions to their day-ahead values. This implies that only the production of units can be rescheduled in the real-time market. Generators will only respond to real-time prices if they can improve their hedged position. It follows that when generators participate in the day-ahead market, they incur the following cost:

\[
C^M_s = \min(C^{RT}_s, C^{DA})
\]  

(29)
Self-commitment-based dispatch. In the case where generators self-commit their units, they cannot enjoy the hedge of the day-ahead market profits. Instead, generators solve the commitment problem independently in order to bring themselves in a better position of accruing higher gains in the real-time market than those that would have been possible by participating in the day-ahead market. The cost of self-commitment is computed as:

\[ C_{SC}^s = \min \sum_{t \in T} \lambda_{st} b_t + \sum_{x \in X, m \in 1\ldots M-1, t \in T} HR_{x,m+1} \cdot F \cdot p_{xmt} \]

\[ + \sum_{x \in X, t \in T} VOM_x p_{xt} \]

\[ + \sum_{x \in X, t \in T} (OC_x + F \cdot HR_{x1} \cdot BP_{x1}) u_{xt} \]

\[ + \sum_{a \in A, t \in T} TC_a v_{at} \]

\[ s.t. \quad (2), (3), (4), (5) \]

\[ u_{xt} = u_{xt}^*, v_{at} = v_{at}^*, u_t = u_t^*, v_t = v_t^*, x \in X, a \in A, t \in T \]  

where the starred parameters are now the optimal solutions of the optimization problem presented in Sect. 3.2.

Risked Costs. In order to compare the payoff of self-committing versus bidding in the market, we compute a distribution of costs over a set of price samples, \( O \). We then obtain a vector of sample costs \( C_{SC}^s = (C_{SC}^s, s \in O) \), \( C_M^s = (C_M^s, s \in O) \) and apply the CVaR operator:

\[ R^M = CVaR_a(C_M) \]  

\[ R^{SC} = CVaR_a(C^{SC}) \]

In the case of a discrete set of outcomes the CVaR operator simply averages the \( \alpha \% \) worse outcomes. The generator then decides to self-commit if \( R^{SC} \leq R^M \) and to participate in the day-ahead market otherwise.

4 Solution Methodology

In this section we present a decomposition algorithm for solving the problem in Sect. 3.2 which cannot be solved in extended form when a large number of scenarios is accounted for. We first use elementary arguments to prove the convexity of the value function in the first-stage decisions and compute the subgradient of the value function with respect to first-stage decisions. We then use these results to apply Benders’ decomposition on the problem.

The self-commitment problem has the following form:

\[ \min c^T w + \frac{1}{a} \sum_{s \in S} \pi_s(Q(w, \lambda_s) - \zeta)^+ \]  

14
where $W$ is a set of polyhedral and integer constraints, $w$ is a set of first-stage decision variables and $c$ is a vector of cost coefficients. The second-stage cost for a given realization is given by

$$(P2_s): Q(w, \lambda_s) = \min_q q^T z \quad (36)$$

subject to

$$Aw + Bz = h \quad (37)$$
$$z \geq 0 \quad (38)$$

where $z$ are second-stage continuous decision variables, $A,B$ are matrices of appropriate dimension, $q_s$ is a vector of appropriate dimension that depends on $\lambda_s$, and $h$ is a vector of appropriate dimension. Note that the sole source of uncertainty is the coefficient vector of the objective function.

**Proposition 4.1** The value function $V(w, \zeta) = \sum_{s \in S} \pi_s (Q(w, \lambda_s) - \zeta)^+$ is a convex function of $(w, z)$.

**Proof** Consider any $t \in (0,1)$ and two candidate first-stage solutions $(w_1, \zeta_1)$, $(w_2, \zeta_2)$. Define

$$w_t = tw_1 + (1-t)w_2 \quad (39)$$
$$\zeta_t = t\zeta_1 + (1-t)\zeta_2 \quad (40)$$

According to theorem 2, paragraph 3.1 of [2], we have that $Q(w, \lambda_s)$ is a convex function of $w$. We therefore get

$$Q(w_t, \lambda_s) - \zeta_t \leq t(Q(w_1, \lambda_s) - \zeta_1) + (1-t)(Q(w_2, \lambda_s) - \zeta_2) \quad (41)$$

We then have

$$(Q(w_t, \lambda_s) - \zeta_t)^+ \leq (t(Q(w_1, \lambda_s) - \zeta_1) + (1-t)(Q(w_2, \lambda_s) - \zeta_2))^+ \quad (42)$$

$$\leq t(Q(w_1, \lambda_s) - \zeta_1)^+ + (1-t)(Q(w_2, \lambda_s) - \zeta_2)^+ \quad (43)$$

The first inequality follows from the fact that $(x)^+$ is an increasing function, while the second inequality follows from $(a + b)^+ \leq a^+ + b^+$. The desired conclusion follows from the fact that the expectation operator preserves convexity.

**Proposition 4.2** The subgradient of $V(w, \zeta)$ at $(w, \zeta)$ is given by

$$\partial V(w, \zeta) = \sum_{s \in S} \pi_s 1_s \begin{bmatrix} -\pi_s^T A \\ -1 \end{bmatrix} \quad (44)$$

where $1_s = 1_Q(w, \lambda_s) > \zeta$ and $\pi_s$ are the dual optimal multipliers of the coupling constraints in Eq. (37).
Proof It suffices to show that for any \((w', \zeta') \neq (w, \zeta)\)

\[
(Q(w', \lambda_s) - \zeta')^+ \geq 1_s[(Q(w, \lambda_s) - \zeta)^+ - \pi_s^T A(w' - w) - (\zeta' - \zeta)] \quad (45)
\]

Suppose \(Q(w, \lambda_s) \geq \zeta\). For any \((w', \zeta') \neq (w, \zeta)\) we have that

\[
1_s[(Q(w, \lambda_s) - \zeta)^+ - \pi_s^T A(w' - w) - (\zeta' - \zeta)] = Q(w, \lambda_s) - \pi_s^T A(w' - w) - \zeta' \quad (46)
\]

\[
\leq Q(w', \lambda_s) - \zeta' \quad (47)
\]

\[
\leq (Q(w', \lambda_s) - \zeta')^+ \quad (48)
\]

The first inequality follows from the fact that \(-\pi_s^T A\) is a subgradient of \(Q(w, \lambda_s)\) at \(w\) \[2\]. On the other hand, if \(Q(w, \lambda_s) < \zeta\) we have

\[
1_s[(Q(w, \lambda_s) - \zeta)^+ - \pi_s^T A(w' - w) - (\zeta' - \zeta)] = 0 \quad (49)
\]

\[
\leq (Q(w', \lambda_s) - \zeta')^+ \quad (50)
\]

We can now apply Benders’ decomposition to the problem. We formulate a relaxation of the first-stage problem by introducing an auxiliary variable \(\theta\):

\[
(P1) : \min_c c^T w + \zeta + \frac{1}{a} \theta
\]

\[
\theta \geq D^l \begin{bmatrix} w \\ \zeta \end{bmatrix} + d_l, 1 \leq l \leq k
\]

\[
w \in W, \theta \geq 0, \zeta \geq \zeta_{LB}
\]

where \(\zeta_{LB}\) is a lower bound on the value at risk, which is easily computable from problem data and prevents unboundedness of \((P1)\). Using the previous results, we propose the following algorithm:

**Step 0**: Set \(k = 1\). Initialize \(\hat{\theta}^1 = -\infty\), and \((\hat{w}^1, \hat{\zeta}^1)\). Go to step 2.

**Step 1**: Solve \((P1)\). Set \((\hat{w}^k, \hat{\zeta}^k)\) equal to the optimal first-stage solution. Go to step 2.

**Step 2**: For all \(s \in S\), solve \((P2_s)\) using \(\hat{w}^k\) as input. Set \(\hat{\pi}_s^k\) equal to the dual optimal multipliers of the coupling constraints in Eq. (37). Set \(1^k_s = 1_{(\hat{\pi}_s^k)^T (h - A\hat{w}^k) \geq \hat{\zeta}^k}\). Go to step 3.

**Step 3**: Set

\[
D^k = \sum_{s \in S} \pi_s 1_s^k(-(\hat{\pi}_s^k)^T A, -1) \quad (54)
\]

\[
d^k = \sum_{s \in S} \pi_s 1_s^k(\hat{\pi}_s^k)^T h \quad (55)
\]

If \(\hat{\theta}^k = \sum_{s \in S} \pi_s 1_s^k((\hat{\pi}_s^k)^T (h - A\hat{w}^k) - \hat{\zeta}^k)\) then exit with \((\hat{w}^k, \hat{\zeta}^k)\) as the optimal solution. Otherwise, set \(k = k + 1\) and go to step 1.
Table 1: Incremental heat rate curve of the combined cycle unit used in the case study. Breakpoints are in MW and heat rates are in MMBtu/MWh.

<table>
<thead>
<tr>
<th></th>
<th>Seg. 1</th>
<th>Seg. 2</th>
<th>Seg. 3</th>
<th>Seg. 4</th>
<th>Seg. 5</th>
<th>Seg. 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>BP_1</td>
<td>15</td>
<td>72.2</td>
<td>129.4</td>
<td>186.6</td>
<td>243.8</td>
<td>301</td>
</tr>
<tr>
<td>BP_2</td>
<td>316</td>
<td>373.2</td>
<td>430.4</td>
<td>487.6</td>
<td>544.8</td>
<td>602</td>
</tr>
<tr>
<td>BP_3</td>
<td>624.5</td>
<td>710.14</td>
<td>795.78</td>
<td>881.42</td>
<td>967.06</td>
<td>1052.7</td>
</tr>
<tr>
<td>HR_1</td>
<td>9.04</td>
<td>8.55</td>
<td>8.88</td>
<td>9.21</td>
<td>9.54</td>
<td>9.87</td>
</tr>
<tr>
<td>HR_2</td>
<td>9.25</td>
<td>8.75</td>
<td>9.08</td>
<td>9.42</td>
<td>9.76</td>
<td>10.09</td>
</tr>
<tr>
<td>HR_3</td>
<td>8.87</td>
<td>8.39</td>
<td>8.71</td>
<td>9.04</td>
<td>9.36</td>
<td>9.68</td>
</tr>
</tbody>
</table>

5 Case Study

We present results from a case study performed on a combined cycle unit with three combustion turbines. We use the technical and cost characteristics of units ALAMIT3 - ALAMIT5 of the WECC 240 bus model [17], assuming that the three generators are connected in a 3 × 1 configuration. The minimum up and down times of the units have been set equal to 4 hours, and the overall unit up/down times have been set equal to 6 hours. The heat rate curve of the unit is shown in Table 1.

We run our case study for a 48-hour horizon. We analyze two seasons, Spring and Summer, for two typical planning horizons, Friday-Saturday and Saturday-Sunday. We use 2007 fuel price data for gas and real-time electricity price data from the San Francisco (SF) bus, which is publicly available at the California ISO website. We use this data to fit a time series model which is subsequently used for generating scenarios and simulation samples. We also note that we use a 24-hour lag for the generation of scenarios for stochastic unit commitment and simulation. This implies that we assume a 24-hour interval between the moment that we are required to make a first-stage decision to the first hour of operations.

5.1 Impact of Risk Aversion

The decomposition algorithm of Sect. 4 is applied to a stochastic program with 100 scenarios. In order to investigate the impact of risk aversion on the model, we run each problem against a CVaR_a risk criterion with a = 25%, 50%, 75% and 100%.

The risk-adjusted profits of self-commitment versus bidding in the day-ahead market are shown in Table 2. For example, the day-ahead market commitment produces the same average profit for every level a of risk aversion, however the payoff of the profit distribution has a different value depending on the level of risk aversion, which is the result shown in the table. Bold figures indicate the preferable option from the point of view of the generator.

We can draw several interesting conclusions from Table 2. From observing
Table 2: Profits (in $ over the 48-hour horizon) of self-scheduling versus day-ahead bidding.

<table>
<thead>
<tr>
<th></th>
<th>$a = 100%$</th>
<th>$75%$</th>
<th>$50%$</th>
<th>$25%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SpringFriSat</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self-commit</td>
<td>51,500</td>
<td>22,474</td>
<td>5,347</td>
<td>0</td>
</tr>
<tr>
<td>Market</td>
<td>25,159</td>
<td>9,273</td>
<td>1,495</td>
<td>938</td>
</tr>
<tr>
<td>SpringSatSun</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self-commit</td>
<td>25,504</td>
<td>-1,274</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Market</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SummerFriSat</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self-commit</td>
<td>994,490</td>
<td>864,210</td>
<td>775,060</td>
<td>657,960</td>
</tr>
<tr>
<td>Market</td>
<td>1,097,800</td>
<td>1,001,900</td>
<td>981,100</td>
<td>981,100</td>
</tr>
<tr>
<td>SummerSatSun</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self-commit</td>
<td>33,200</td>
<td>18,167</td>
<td>6,624</td>
<td>0</td>
</tr>
<tr>
<td>Market</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3: The unit commitment schedule (MW) for self-scheduling ($a = 50\%, 75\%, 100\%$) versus day-ahead market bidding for Spring Friday-Saturday.

<table>
<thead>
<tr>
<th>Hours</th>
<th>1-6</th>
<th>7 - 9</th>
<th>10 - 24</th>
<th>25 - 48</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market</td>
<td>0</td>
<td>0</td>
<td>301</td>
<td>0</td>
</tr>
<tr>
<td>Self-commit</td>
<td>0</td>
<td>301</td>
<td>301</td>
<td>301</td>
</tr>
</tbody>
</table>

the outcome of Spring Friday-Saturday, we note that self-commitment produces a better outcome for generators for all but the most risk averse case ($a = 25\%$). This can be understood by comparing the unit commitment schedules of the two policies, shown in Table 3. The day-ahead market will start the unit up at hour 10 on Friday and will keep the generator on a $1 \times 1$ configuration (301 MW of capacity) until the end of Friday. The market will shut the unit down for the full duration of Saturday. This is due to the fact that the market model does not recognize the profit opportunities that arise from high prices. Instead, the optimal self-commitment is to start the unit up on hour 7 of Friday, and keep the unit running on $1 \times 1$ mode until the end of Saturday. In the most risk averse case, the self-scheduling model will shut the unit down, which results in zero profits. Instead, the day-ahead market will deliver a risked profit margin of $938 from the day-ahead market which makes market bidding preferable.

In the case of Spring Saturday-Sunday, we observe that for $a = 100\%$ self-commitment identifies a better schedule than the market schedule, which is to keep the unit down for both days. For the case of $a = 75\%$, we observe that self-commitment results in negative risked profit. This can happen because the set of samples $O$ used for the evaluation of the policy is different from the set of scenarios $S$ that are input in the stochastic optimization model. For the case of $a = 50\%$ and $a = 25\%$ both approaches keep the unit off for both days, and neither policy performs better. In general, if both models produce the same unit commitment schedule, self-commitment cannot dominate bidding in the
market.

In the case of Summer Friday-Saturday, the market provides its intended hedging function. The optimal schedule produced by both approaches is identical, namely keeping the unit at $3 \times 1$ configuration for the full duration of both days. The hedge of the day-ahead market, however, provides a secure profit which can be improved by participating in the real-time market. The self-scheduling approach enjoys no such protection. In the case of Spring Saturday-Sunday for $a = 25\%$ and $50\%$ both policies produce the same solution of keeping the unit off and are indistinguishable since they both produce 0 risked profit. Instead, the market is preferable for $a = 75\%$ and $100\%$ even though both policies produce the same commitment.

The case of Summer Saturday-Sunday resembles that of Spring Friday-Saturday. The self-commitment policy identifies profit opportunities and keeps the unit on, whereas the market keeps the unit off for both days. For all but the most risk averse case, $a = 25\%$, the self-commitment approach is preferable. For the latter case the self-commitment approach also shuts the unit down for both days.

We observe that the incentive for generators to self-commit is enhanced by less risk aversion. This is due to the fact that increased risk aversion reduces the differences of near-optimal unit commitment schedules. The results of Table 2 are somewhat alarming. We notice that in most instances the market is outperformed by the self-commitment approach. This undermines the depth and very purpose of the day-ahead market.

5.2 Impact of Price Volatility

In order to investigate the impact of price volatility we run the same model against a set of price scenarios whose spread around the average hourly value is 150\% the spread of the original price data. The results are shown in Table 4. We observe that the increased price volatility increases the incentive to self-commitment in most cases, with the exception of summer Friday-Saturday in which case it instead enhances the incentive to submit a bid to the market. This may be somewhat counterintuitive since higher price volatility implies higher risk which would make the day-ahead market more desirable. On the other hand, higher volatility also increases the value of explicitly accounting for uncertainty in a stochastic self-commitment model, which drives the result.

The results for Summer Friday-Saturday can be understood easily. Both self-commitment and bidding in the market result in a commitment of the plant at $3 \times 1$ mode for the full horizon of the problem, since both models identify the high profit margins during this high-demand period. Bidding in the market will secure a day-ahead profit of $981,102. Subsequent realizations of the real-time market will only increase this profit. Self-commitment enjoys no such hedging, and real-time market realizations may result in profits well below $981,102.

In contrast, consider the case of Spring Friday-Saturday. The unit commitment schedule for both self-commitment as well as market bidding is identical in the case of volatile prices as in the case of reference (non-volatile) prices, shown
Table 4: Profits (in $ over the 48-hour horizon) of self-commitment versus day-ahead bidding for the case of high price volatility.

<table>
<thead>
<tr>
<th></th>
<th>100%</th>
<th>75%</th>
<th>50%</th>
<th>25%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SpringFriSat</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self-commit</td>
<td>95,776</td>
<td>50,280</td>
<td>24,130</td>
<td>-825</td>
</tr>
<tr>
<td>Market</td>
<td>40,754</td>
<td>15,947</td>
<td>2,706</td>
<td>938</td>
</tr>
<tr>
<td><strong>SpringSatSun</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self-commit</td>
<td>66,333</td>
<td>23,936</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Market</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>SummerFriSat</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self-commit</td>
<td>1,026,700</td>
<td>834,410</td>
<td>704,080</td>
<td>535,690</td>
</tr>
<tr>
<td>Market</td>
<td>1,165,200</td>
<td>1,019,000</td>
<td>981,120</td>
<td>981,100</td>
</tr>
<tr>
<td><strong>SummerSatSun</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self-commit</td>
<td>59,327</td>
<td>36,538</td>
<td>18,844</td>
<td>-1,241</td>
</tr>
<tr>
<td>Market</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 5: The distribution of profits for Spring Friday-Saturday for market bidding with reference prices (upper left), self-commitment with reference prices (upper right), market bidding with volatile prices (lower left) and self-commitment with volatile prices (lower right).
in Table 3. The distributions of the profits for the case of bidding in the market versus self-commitment are shown in Fig. 5 for both the case of reference (non-volatile) prices as well as volatile prices. The results for the case of market bidding can be understood easily. A significant portion of the distribution is concentrated around $938, which is the day-ahead profit. This is due to the fact that for the given commitment, a large number of realizations cannot make a profit in real time and only accrue the day-ahead market profit. It is also intuitive that the spread of the profit distribution should increase in the positive direction as price volatility increases. This stems from the fact that high revenue outcomes under reference prices result in even higher revenues under volatile prices. Instead, low revenue outcomes will result in a profit no less than the profit accrued in the day-ahead market, which results in a significant concentration of mass around the day-ahead profit. In the case of self-commitment, what is less intuitive is the fact that the increased price volatility increases the positive spread of the profit distribution in the lower right panel, but leaves the negative spread of the distribution nearly unaffected. The positive bias resulting from increased price volatility can be understood by the fact that periods of low prices correspond to low generator output, with a minor influence on revenue, whereas periods of high prices correspond to periods of high output with a major influence on revenue. This drives the observation that increased volatility makes self-commitment more appealing than bidding in the market. As renewable resources are increasingly integrated in the system, resulting in increased real-time price volatility, this undermines the liquidity of the day-ahead market.

5.3 Running Time

The problems are solved using CPLEX 12.5.0.0 on a Macbook (2.4 GHz Intel Core i5, 8GB 1333 MHz DDR3). The running times and number of optimality cuts generated in each problem instance are shown in Table 5. Running times range between 5 seconds and 2.5 minutes. The summer problem instances were generally easier to solve due to the fact that the prices were sufficiently high that the optimal policy, a $3 \times 1$ configuration, was easy to identify without exploring too many alternative solutions through optimality cuts.

6 Conclusions

We have presented a risk-averse stochastic model for self-committing combined cycle units under real-time electricity price uncertainty. We have used the model in order to investigate the impact of risk aversion and price volatility on the incentive for generators to submit bids in the day-ahead market versus self-committing their units.

The additional intelligence of a stochastic model in terms of accounting for uncertainty and a longer horizon can yield sufficient benefits to outweigh the insurance of the forward day-ahead market. This undermines the depth and very purpose of the day-ahead market.
The incentive for generators to self-commit is enhanced by less risk aversion. This is due to the fact that increased risk aversion reduces the differences of different near-optimal unit commitment schedules.

The day-ahead market will be preferable whenever the decision dictated by both self-commitment and the market is identical. This happens, for example, in situations where margins are high (summer) and the optimal decision is to maximize output in both models. In conditions of lower profit margins self-commitment will likely produce different decisions.

Price volatility resulting from renewable energy integration enhances the incentive of generators to self-commit instead of mitigating it, at least for low to moderate risk aversion levels. This is counter-intuitive since the day-ahead market hedges against risk, which is greater in conditions of higher price volatility. The result is driven by the fact that self-commitment explicitly accounts for the increased uncertainty resulting from higher price volatility and arrives at more profitable unit commitment schedules due to the dominant role of high-price periods on the profit distribution.

The results are somewhat alarming and suggest that additional incentives should be provided to flexible generating units for making their capacity available in day-ahead scheduling. Ancillary services, capacity payments, resource adequacy obligations or other mechanisms could be mobilized in order to provide additional incentives for the provision of balancing capacity \[14\]. This is work that we intend to explore in future research.

### A Nomenclature

**Sets**

- \( S \): set of scenarios
- \( T = \{1, \ldots, N\} \): set of time periods

**Decision variables**

- \( b_{st} \): buy in scenario \( s \), period \( t \)
\( p_{xst}, u_{xst} \): production/commitment of unit in state \( x \), scenario \( s \), period \( t \)
\( v_{ast} \): transition of unit in arc \( a \) for scenario \( s \), period \( t \)
\( p_{xmsl} \): incremental production for unit in segment \( m \) of state \( x \) in state \( s \), period \( t \)
\( u_{xt} \): first-stage commitment of unit state \( x \) in period \( t \)
\( v_{at} \): first-stage transition of unit on arc \( a \) in period \( t \)
\( u_t, v_t \): commitment, startup of unit in period \( t \)

**Parameters**

- \( N \): number of periods in horizon
- \( M \): number of segments in incremental heat rate curve of MSG unit
- \( \pi_s \): probability of scenario \( s \)
- \( UT, DT \): minimum up/down time of unit
- \( UT_x, DT_x \): minimum up/down time of state \( x \)
- \( HR_{xm}, BP_{xm} \): incremental heat rate/breakpoint in segment \( m \) of state \( x \)
- \( VOM_x, R_x^+, R_x^- \): variable O&M cost/ramp up rate/ramp down rate of state \( x \)
- \( TC_a \): transition cost of MSG unit for arc \( a \)
- \( F(a), T(a) \): set of states that arc \( a \) transitions from/to

**References**


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