An integrated decision making process:
An application on inventory, warehouse, production and distribution decisions

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GÉRALDINE STRACK

Promoteurs: FOUAD RIANE (UCL)
             BERNARD FORTZ (ULB)
Comité scientifique: PHILIPPE CHEVALIER (UCL)
                      EL-HOUSSAINE AGHEZZAF (GHENT UNIVERSITY)
Jury: MATHIEU VAN VYVE (UCL)
      JEAN-SÉBASTIEN TANCRED (UCL)
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Abstract

The complexity of supply chain management problems forces managers of today to seek for more integrated and coordinated decision tools. Coordination mechanisms become then a fundamental component of the decision strategy. Eliminating barriers between decision levels, and between different stages of the decision making process in business is crucial in order to improve the efficiency of the supply chain. This coordination component is even more important when a core resource is scarce. Our aim in this thesis is to propose coordinated decision tools which improve the management of the supply chain and of core resources.

After a short analysis of the different existing coordinated techniques, supply chain integration is analyzed in more depth. Supply chain integration considers the entire supply chain with its links and inter-dependencies by optimizing objectives of all parties involved in the collaboration. From this analysis, we propose an integrated methodology based on a decomposition technique where the coordination link between decisions is respected. This decomposition methodology is used, in two case studies, to propose integrated decision tools.

The first case study deals with inventory and warehouse decisions in the case of limited warehouse capacity. Those decisions are traditionally considered separately and optimized independently. In this research, we propose two integrated decision tools based on a decomposition of the global warehouse and inventory model in two sub-models. Those two methodologies offer different levels of coordination between warehouse and inventory decisions. Each of these is tested on a real case study where the coordination strength is measured by comparing inventory and warehouse costs. Those computational tests highlight also the influence on integration of warehouse and inventory costs as well as warehouse capacity.

In the second case study, production and distribution decisions are considered. We have examined a particular setting where reusable resources are shared between those two departments. Our aim is to analyze the impact of integrating those
decisions on the management of resusable resources. Therefore, we have built a global model and applied three different decomposition procedures to solve it. Those decomposition techniques differ in the way coordination is achieved and reusable resources are considered. Computational tests have been performed and highlight, among other, the importance of integration when reusable resources are available in limited quantity.
Résumé

De nos jours, les mécanismes de coordination sont devenus une composante essentielle à l’élaboration d’outils d’aide à la décision. Éliminer les barrières entre les différents niveaux décisionnels, les différents départements ainsi qu’entre les différentes entreprises est crucial afin d’améliorer l’efficacité dans la chaîne logistique. La coordination est d’autant plus importante si des ressources rares entrent en jeu. Notre but dans cette thèse est de proposer des outils d’aide à la décision basés sur la coordination afin d’améliorer la gestion de la chaîne logistique ainsi que la gestion des ressources rares.

Après une courte analyse des différentes techniques de coordination existantes, le concept d’intégration dans la chaîne logistique est étudié. L’intégration dans la chaîne logistique permet de prendre en compte l’entièreté du système ainsi que l’ensemble des liens et des interdépendances tout en optimisant les objectifs de toutes les parties impliquées dans la collaboration. Suite à cette analyse, nous proposons une méthode intégrée basée sur une approche de décomposition où le lien entre les décisions est respecté. Cette méthode va être utilisée, dans deux études de cas, pour proposer des outils d’aide à la décision intégrés.


Dans la deuxième étude de cas, les décisions de production et de distribu-
List of Publications

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Chapter 1

Introduction

Decision making issues are often so complex that current decision support systems are inappropriate to provide decision makers with satisfactory answers to their strategic, tactical or operational problems. Those decision support systems tackle decision makers’ issues with poor consideration if any for the necessary coordination between decision making, supply chain stages and departments. This shortfall leads to malfunctions which induce poor efficiency. Nowadays, it is generally accepted that improvements in supply chain coordination increase efficiency more than optimizing individual functional areas [45]. In addition, the development of more precise information and technological tools provide a better coordination for the development of decision making systems tools. Following those facts, the objective of this thesis is to propose coordinated decision tools in various areas of the supply chain and to analyze the impact of those decision tool on the efficiency of the supply chain. Coordination can be defined, in a broad sense, as “managing dependencies between activities” [56] and can be characterized by various attributes: type of decision, stage of decision, entity of decision.

The first attribute concerns the type of the decisions that will be coordinated. Decisions in business management are taken under different time horizons, with different purposes and based on data of different aggregation levels. Strategic decisions (e.g. construction of a plant) have a long term impact while tactical decisions (e.g. production level for a product family) and operational decisions (e.g. production scheduling) have an impact on the medium and short term respectively. For example, at the operational level, daily decisions concerning detailed production schedule need to be taken. Questions such as “what are we going to produce?
Introduction

when and on which machine?” need to be answered. At the higher tactical level, aggregated planning decisions are considered such as the production volume for each family of product, the stock level for each family, the planning of resources (workforce and machines), etc. Aggregate decisions may be considered due to the lack of precise information (e.g. lack of information on the availability of human resources) and/or in order to reduce the complexity of some problems (e.g. the setup cost is not considered when elaborating the aggregate planning).

The second attribute concerns the stages of the supply chain where coordination can be achieved. Supply chain management can be defined as the coordination of material, information and financial flows within and across organizational units [19]. There are three stages in a supply chain (see Figure 1.1): procurement, production and distribution stages. Traditionally those stages have been handled independently with large buffers of inventories. Nowadays with the increased competitive pressure, coordination is needed between those stages in order to reduce costs and improve service level. Supply chain costs encompass not only the use of factory resources, i.e. the cost of transforming raw materials into finished products but also the cost of service to the customer (delivery costs and sales costs of marketing, customer care, etc.).

Coordination can be achieved between independent stages (procurement, production and distribution) of the supply chain but can also be achieved internally between entities or organizational units of the company. The paper of Kahn et al. [50] gives a first insight into the importance and the meaning of interdepartmental coordination. The authors defined the latter as a mix of interaction and collaboration. The importance of each of those factors in the mix will depend on the managerial situation (e.g. stable product lines versus product launches...). Interaction corresponds to exchange of information between departments through meetings or other similar activities, whereas collaboration places the emphasis on a strategic alignment of the objectives of different departments (shared vision, collective goals and joint rewards). Interdepartmental coordination ensures better performance, increases in “service level, better management of inventory levels, higher forecast accuracy and greater customer and employee satisfaction” [50].

Coordination can be achieved in different ways:

• same type of decision (strategic, tactical or operational level) at different stages in the supply chain

• in one supply stage between different entities
Figure 1.1: Stages in a supply chain [30]
• Type of decision, stage in the supply chain and entities of the company can differ and any combination can of course be envisioned.

Based on those attributes, three coordination mechanisms have been analyzed in order to understand how to optimize the coordination in the supply chain. We have decided to focus our analysis of coordination mechanisms on hierarchical planning, collaborative planning and supply chain integration. Those research streams have been selected because we believe that they are the most representative of collaboration techniques and give a good understanding of the issues linked to these techniques. Hierarchical planning is presented first because it was the forerunner in the field of coordination techniques. After that, we will present the two others theories which can be distinguished by the strength of the collaboration between the entities.

1.1 Hierarchical Planning

Hierarchical planning is the precursory research stream that deals with coordination in the supply chain. As stated by Bitran and Tirupati [10] ‘hierarchical planning represents a philosophy to address complex problems, rather than a specific solution technique.’

Hierarchical planning was elaborated in the perspective of multi-level decisions: strategic-tactical-operational. It was first introduced in 1975 by Hax and Meal [47]. They defined hierarchical planning as a tool that supports management at each decision level. The basic idea is that the overall planning is broken up into sub-tasks according to decision time frame (strategic, tactical and operational) and to management needs. Those sub-tasks are interrelated in a hierarchical manner [31]. The hierarchical planning follows the context of the organizational hierarchy by linking the higher and lower level decisions in the most effective way. The higher level decisions are considered as constraints for the lower level decisions. Reversely, detailed information from the lower decision levels flows back to the higher decision level where it will be taken into account when aggregate decision will be made. In this pyramidal top down structure, collaboration is enforced by a central planning authority.

The hierarchical nature of this environment drives the decision making process. Decisions at each level (be it strategic, tactical or operational) will be modeled considering the information available at that level (aggregate information at the strategic level and more precise information at the operational level). For example,
1.1 Hierarchical Planning

when the aggregate planning is elaborated, the main decision variables are the level of production and the stock level for each family of products. The optimal value of those decision variables will be used as data for the detailed production scheduling. Therefore, the success of this hierarchical procedure depends on the link between the different decision models and on the aggregation and disaggregation procedures that are used.

Hax and Meal [47] have illustrated the hierarchical coordination mechanism in a case study (see Figure 1.2) where the difference between tactical and operational decisions is highlighted. Tactical decisions deal with a certain level of aggregate information which is used as constraint at the operational level. In this case study, they elaborate a hierarchical planning and scheduling system for a multiple plant, multiple products and seasonal demand situation. They consider four levels of decision with different information aggregation. The first level concerns the assignment of products to a plant by means of a mixed-integer programming tool. This allows to fix long term capacity. The second level deals with capacity management. For each plant, capacity is allocated to various product types (products which have the same inventory cost) by means of a linear model. Thirdly, a detailed schedule for each product family (products in one family share the same setup cost) is elaborated. Lastly, for each item in a product family, individual lot sizing quantities are calculated using standard inventory models.

There are various advantages to a hierarchical planning method. The major advantage is a reduction in the costs of data collection and computational time requirement [79] when trying to solve business problems as a whole. In addition, forecasts will have a smaller error margin for aggregated product families than for individual products. Furthermore, aggregate information allows managers to interact more easily with the model which may not be the case when managers loose track in too much detailed information.

Beside those advantages, there is a major disadvantage to this approach which is the suboptimality resulting from the decomposition procedure. With two sub-problems, the second sub-problem will be optimized taking into account the optimal solution found to the first sub-problem. In addition, one can easily understand the feasibility problem which can result from the aggregation and disaggregation procedure. For example, when the aggregate planning is determined (tactical level), an aggregate machine capacity constraint is considered which does not take into account the setup time. The latter is an activity which consumes time and which therefore is not negligible. This aggregate capacity leads to production levels
Figure 1.2: An Overview of Hierarchical Method [10]
1.2 Collaborative planning

that may become impossible to reach when the set up time is taken into account. In the hierarchical planning, the setup time is considered only at the scheduling level when the production planning decisions have been taken. In an increasingly competitive environment, this local optimization approach has to be improved.

A recent example of implementation of the hierarchical planning method can be found in Ozdamar et al.’s paper [60]. They develop a hierarchical production and distribution planning for one year for a multi-national company that produces liquid and powder detergent. There is one factory which produces liquid and powder detergent on two separate lines and distributes them to four warehouses with long vehicle. The demand of customer is unknown and decisions must be taken based on forecast obtained by the warehouses. A hierarchical planning is developed composed of an aggregate planning and a disaggregate planning. The aggregate planning solves the production-distribution problem by aggregating product families in product type, by considering an aggregate time period of two months, setup are not taken into account but a percentage of capacity is reserved for it. This aggregate model is solved on a rolling horizon where at each time period the corresponding disaggregate model is solved and information for this disaggregate model is used to solve the aggregate model for the next period. In the disaggregate model, the time period is refine to 8 weeks, families of products are considered and no more product types and setups are taken into account. Backorders are included in the aggregate model as well as in the disaggregate model in order to avoid feasibility problem which can occur from the aggregation-disaggregation procedure. Their hierarchical planning was tested on the detergent company database and the authors report results by comparing the total cost obtained by the aggregate compare to disaggregate model. They report a difference of 3-4% in cost between the two. The problem is that the solution obtained contains backorders which are not allowed by the company policy. In addition, high inventory levels are observed.

1.2 Collaborative planning

The second approach to the improvement of collaboration in the supply chain is collaborative planning. Stadtler [70] is the authority in that field. He defines collaborative planning as ”a joint decision making process for aligning plans of individual supply chain members with the aim of achieving coordination in light of information asymmetry”. We will further elaborate on three aspects of this
Joint decision making process is a characteristic of collaborative planning. Whereas hierarchical planning offers a way to coordinate from a top down perspective with a central planning instance, collaborative planning aims at synchronizing operations either in a horizontal (collaboration achieved across different supply chains) or vertical (collaboration achieved in the same supply chain) perspective without a centralized decision maker enforcing collaboration. In the joint decision making process, there is no centralized decision maker enforcing collaboration. Coordination is based on objectives and rules between actors with equal decision rights.

The individual plans of different planning domains are aligned in order to create a common and mutually agreed upon plan. Collaborative planning is successful when the implemented plan leads to an improved plan for the supply chain as a whole compared to the original situation [69]. More precisely, a supply chain equilibrium is reached only if the gain obtained by the collaboration is distributed among all parties in such a way that nobody will want to deviate from the collaborative planning. This is called a “Nash equilibrium”. Due to individual rationality, a Nash equilibrium can become a supply chain optimum if it results in a win-win situation for each supply chain member. Various tools are available in order to ensure a fair distribution of the gain between all parties involved in the collaboration.

A last characteristic of collaborative planning is the information asymmetry. Information asymmetry means that supply chain members do not possess the same information. Collaboration can take place between parties that will share relevant but limited information on specific items. Technically, collaborative planning implies that collaborative decision tools will contain more detailed information regarding the object of collaboration. However, each participant’s objective will remain independent. The shared detailed information allows to optimize the decision process more accurately.

As an example of collaborative planning [70] consider the collaboration between an automotive manufacturer and its suppliers. The automotive manufacturer provides various car manufacturers with headlights. Headlights are made of two parts: a body and a glass cover. One of the automotive manufacturer’s supplier provides
1.2 Collaborative planning

the glass cover and another supplier provides bulbs. The car manufacturer wants a reliable supply of headlights. Therefore, he provides the headlight manufacturer with a forecast of the demand. He requests from the same manufacturer a commitment to fulfill the demand forecast and to provide him with information about maximum supply capabilities. This information is needed when the actual demand is greater than the forecast demand. On the other hand, the headlight manufacturer wants a minimum demand commitment. In this example, information collaboration has taken place on demand and capacity.

Collaborative planning can be of different types: demand collaboration, inventory collaboration, procurement collaboration. All of these collaborations are based on the exchange of information on demand and supply of materials. They are named material-related collaboration.

- Demand collaboration is the collaboration between a supplier and a customer who share historical data, promotion, marketing activities, etc. For example, customers can give information about their medium-term material requirement to their supplier.

- Inventory collaboration is a particular form of demand collaboration in which information on the level of inventory is exchanged between the customer and the supplier. Vendor managed inventory (VMI) is an example of Inventory collaboration. The supplier will control the replenishment of the stock of the customer based on a service level agreement between himself and the customer.

- Procurement collaboration is also a collaboration on demand and supply information. The main difference with demand collaboration is that the procurement collaboration is initiated by the customer.

Other types of collaborative planning are the service-related collaborations: capacity collaboration, transport collaboration. These types of collaboration are based on the exchange of information on demand and on the availability of production services.

- Capacity collaboration is the collaboration between a customer and a supplier who agree on the reservation for a specific amount of capacity. This type of collaboration is driven by the customer.

- Transportation collaboration considers the collaboration between a customer,
typically a manufacturer or a retailer, and the supplier, which is the transportation or logistics provider.

Note that it is possible to have a mix of material-related collaboration and service related collaboration.

A very well known example of information technology tool for collaborative planning is Collaborative Planning, Forecasting and Replenishment (CPFR)[31]. It is a tool which helps suppliers and retailers to manage inventory more effectively by sharing information on the replenishment of products through the supply chain. This results in a better management of supplier’s inventory which leads to better planning and demand satisfaction.

The paper of Tyan et al. [73] gives a practical example of implementation of a VMI system in the Taiwanese grocery industry between P&G and Wellcome supermarket chains stores. The authors present the success of VMI by measuring the service level and inventory level. The service level and the inventory level achieved is measured by inventory days and in-stock %. The targets are 10 days of inventory level and 95% of in stock %. Over the three time periods, the service level went from 92% to 98% so well above target. The number of inventory days is 24 before VMI implementation and drops to 13 in time periods 2 and to 16 in time periods 3.

1.3 Supply chain integration

A third stream of research on collaboration techniques is supply chain integration. With this newly research area, many conceptual definitions are available in the literature and there is no unique stream of thinking on the subject. Compared to collaborative planning (see Section 1.2), supply chain integration offers a higher level of coordination by completely merging the decisions of various supply chain partners. By merging, we mean that one unique objective is considered for the entire supply chain whereas in collaborative planning each individual supply chain member has his own objective that he wants to optimize.

Vijayasarathy’s definition [80] gives a first insight on the link between supply chain integration and collaboration. This definition is then completed with information from conceptual papers as well as articles dealing with the link between supply chain integration and performance.
Vijayasarathy's [80] has defined supply chain integration as following:

Supply chain integration, at its normative ideal, refers to the adoption and use of collaborative and coordinating structures, processes, technologies and practices among supply chain partners for building and maintaining a seamless conduit for the precise and timely flow of information, materials and finished goods.

Flynn [34] has completed this definition by emphasizing that integration can take place between "supply chain partners" which belong to the same company (e.g. production and distribution departments) or to different businesses (e.g. suppliers and buyers).

Supply chain integration can be described along two directions: scope of integration and level of integration. Scope of integration concerns the number of supply chain areas in which cooperation is developed. There are four main areas namely flow of goods (e.g. VMI, common containers, etc.), planning and control (e.g. joint forecasting), organization (partnership) and flow of information (EDI, Internet). The level of integration indicates to what extent an integrative activity is developed [78]. Supply chain integration can be successfully implemented only if some structural factors are present. Those factors are at the number of four: Dependence Asymmetry, Trust, Commitment and Mutual Dependence [80]. Mutual dependence qualifies to what extent the partners involved in a collaboration depend on each other whereas dependence Asymmetry concerns the unbalanced structure of power between partners. Trust appears when partners involved in a collaboration find each other reliable and of good will whereas commitment refers to the fact that partners are willing to take a long term orientation for their collaboration.

Once supply chain integration has been enforced, performance can be measured. The link between supply chain integration and performance can be analyzed by using three different variables: attitude of buyers and/or suppliers toward each other, pattern (i.e. interaction between the focal enterprise and its supplier/buyer) and practice (tangible activities or technologies: e.g. EDI) [77]. Note that, supply chain integration performance is different when we have internal integration meaning integration within the focal enterprise or external integration (between customers or/and suppliers) [34]. In addition, integration can be explained along two axes (material integration and information integration) which offers different ways of analyzing the performance of an integrative process. Information integra-
tion concerns the sharing of information in the supply chain which is possible with information technology. This other vision of supply chain integration makes it possible to analyze the effects on performance of long-term relationships, information technology, information sharing and logistics integration [63].

Many example of successful supply chain integration can be found in the literature. One of them is the article of Lohatepanont et al. [55] where the authors propose to integrate airline scheduling design decisions with fleet assignment decisions. They develop integrated models and methodologies which are tested on a major US airline company dataset. They show that savings of as much as $ 200 million a year could be achieved by coordinating airline scheduling design decision with fleet assignment decisions.

1.4 Research motivation and structure of the Thesis

In today’s highly competitive world, businesses are forced to reorganize their internal processes. Companies are looking for better management decision tools which will enable them to optimize their procurement, production and distribution systems. The development of those decision tools is becoming possible due to the availability of more precise data processing and technological tools.

Those decision tools are based on modeling, optimization and simulation methods where coordination is a key factor. Coordination is generally recognized as a fundamental principle of decision making. In fact, eliminating boundaries between problems, tools and decision levels is essential when elaborating efficient global optimization methods.

Three supply chain coordination approaches were described in the previous sections

- Hierarchical planning
- Collaborative planning
- Supply chain integration

Their objective is to provide an answer to these new challenges each with its own advantages and disadvantages. We will briefly outline their advantages and disadvantages below.
1.4 Research motivation and structure of the Thesis

The traditional hierarchical planning theory has the advantage of reducing the computational complexity of global models by decomposing them in smaller sub-models. In addition, those smaller models are easier to understand. The weaknesses of this methodology are the feasibility as well as the suboptimality issue resulting from this decomposition. Furthermore, the decomposition procedure is achieved on decisions time length (strategic, tactical or operational) with poor consideration of relationship between decisions of different levels.

With collaborative planning, collaboration is achieved through the exchange of information on the subject of the collaboration. Information is exchanged in limited quantity and no global objective is considered. Each party involved in the collaboration optimizes its own objective without considering the other member’s objectives. Each party processes more detailed information because of the exchange which may increase the complexity of the problem but the output of the collaboration is of better quality.

An integrated approach allows to take into account the entire system, to evaluate the links between decisions taken at different levels in the enterprise and to reflect more precisely the needs of the management. This results in less sub optimality and feasibility problems. However, complexity increases and global models may be more difficult to understand by management. In the case of an integrated approach, the level of coordination is the highest. Due to the development in data processing and technological system, computational complexity is less a barrier to the development of integrated management decision tools. In addition, with the increase in competition, managers are seeking better management tools that will meet their requirements and respect their needs.

The aim of our research is to provide an answer to various questions linked to the coordination in supply chain and more precisely to the development of management tools aiming at a better modeling of supply chain coordination. The decision tools developed in this thesis are supply chain integration tools. Indeed, our aim is to model in the best way the entire system by considering one unique objective function as well as all the system’s constraints. We believe that this approach allows to coordinate the supply chain in the most effective way.

This thesis is divided in four chapters with three main, each of them is devoted to answering one of the research questions.

In Chapter 2, we analyze the integrated decision making process. Indeed, supply chain integration is a recent stream of research where the main focus has been on the development of a conceptual definition and on the link between integration
and performance. By integrated decision making process, we mean the various steps leading to the creation of decision tools aimed at a better management of supply chain coordination mechanisms. Therefore, in Chapter 2, we propose various approaches leading to integrated decision making and illustrate them by reviewing the methodologies used in the existing literature. We have focused our analysis on the following topics:

- Manufacturing production planning
- Tactical planning in the airline industry
- Vehicle routing for distribution

We conclude the chapter by suggesting a methodology to create integrated decision tools that we have named intelligent decomposition. This chapter can be summarized by the following research question:

*Question 1: Integration is an approach which can be applied in various ways: problem integration, model integration and tool integration. Is it then possible to suggest a taxonomy of these methods, based on a literature review, in order to improve the understanding of the benefits of integration?*

In Chapter 3, we develop an integrated decision making tool for tactical warehouse and inventory decisions. More precisely, we want to coordinate the replenishment decision at the inventory management level, the allocation of products to warehousing systems and the assignment of products to storage locations at the warehousing management level. A global mathematical model is presented and various methodologies based on intelligent decomposition are applied. Our methodologies are tested and compared in situations of excess and scare warehouse capacity. A sensibility analysis is performed in order to highlight factors and resources that influence the performance of the methodologies.

In Chapter 4, we analyze the coordination of tactical and operational production and distribution decisions when resources are shared between those two units. Our aim is to coordinate lot-sizing decisions at the distribution and production level and vehicle routing decisions at the operational level. A global mathematical model which encompasses these decisions is presented. Various intelligent decomposition methodologies are applied to solve the global model with different levels of integration of the decisions. Those methodologies are tested on a database and compared with each others. Factors and resources influencing the performance of the methodologies are highlighted. Finally, some sensitivity analysis is performed.
1.4 Research motivation and structure of the Thesis

The aim of those two chapters is to answer the following research questions:

**Question 2:** Monolithically constructed models are difficult to solve: their optimization consumes time and computing resources. Despite those difficulties, what are the evidences in favor of integration?

**Question 3:** How can the traditional sequential procedure still be applied in a more integrated environment?

**Question 4:** How can an integrated approach handle various aspects of resources management more accurately than what is currently done in the literature?

After summarizing the main idea of the thesis in the beginning of Chapter 5, we conclude by pointing out the main research findings, listing some managerial implications, limits to the research and topics for possible future research linked to the integrated decision tools presented in Chapters 3 and 4.
Chapter 2

Integrated decision making process

In Chapter 1, we have exposed the important need of decision tools based on collaboration and we have presented three different collaboration approaches namely hierarchical planning, collaborative planning and supply chain integration. From this analysis, we conclude that developing decision tools based on supply chain integration allows to offers a higher level of coordination of the supply chain by merging all parties objectives than hierarchical planning or collaborative planning. As a reminder, supply chain integration is defined as "the adoption and use of collaborative and coordinating structures, processes, technologies and practices among supply chain partners for building and maintaining a seamless conduit for the precise and timely flow of information, materials and finished goods"[80].

Our aim in this chapter is to survey articles where an integrated process is used to coordinate the supply chain. We want to formalize the processes leading to the elaboration of integrated decision making tools. Indeed, most of the conceptual papers about supply chain integration focus on the link between supply chain integration and performance, on the conceptualization of supply chain integration and on the factors which enable supply chain integration (see Section 1.3). To our knowledge, none have discussed how to include integration in the decision making process.

Broadly speaking, decision making processes can be represented as in Figure 2.1. First of all, decision makers identify problems through various viewpoints. These reflect the way in which they understand the situation and lead to a first
formulation of the problem in the form of questions: Where to install the production facilities? Which resources to use? How much to invest? etc. Once those questions are known and delimited, a more formal and structural formulation is achieved through mathematical modeling: Lot sizing model, Line formulation, Vehicle routing problem, Facility location, etc. Then, the right methodology is used in order to obtain a good solution: branch-and-bound, Benders decomposition, Ant heuristic, Genetic algorithm, Simulation, etc. Finally, those methodologies need to be implemented in computer programs in order to get answers to the original questions.

In general, a decomposition and hierarchical approach is applied to this decision making process. The various decision makers’ questions are classified according to their time horizon (strategic, tactical and operational). Each of those questions leads to a mathematical model which is solved using the best solution methodology. This decomposition methodology is used in order to reduce the complexity of problems. At each step in this hierarchical process, iterations can occur in order to adapt, modify the process to be as close as possible to the original decision makers’ questions. We propose to use the same structural process but with an integrated approach. This allows to take the link between decisions of different level and/or of different nature into account and results in a more efficient supply chain.

Three different decision integration approaches can be applied to the decision making process: model integration (Model I.), method integration (Method I.) and tool integration (Tool I.) (see Figure 2.1).

- Model integration consists in the development of a global model which en-
compasses management decisions at various stages and of various levels. This
global model represents the merging of all parties in the supply chain and
is solved globally. For example, production planning and scheduling deci-
sions can be considered simultaneously in a global model instead of being
tackled separately and independently. In a hierarchical setting, planning de-
cisions are considered first and take an aggregate capacity constraint into
account. Then scheduling decisions are solved considering the planning de-
cisions as fixed. With model integration, a global model is created where
production planning and scheduling costs appear in a unique objective func-
tion, production planning and scheduling constraints are modeled together
and the capacity constraint which links the two levels (planning and schedul-
ing) is fully detailed. This global model is then solved using the appropriate
methodology. This approach offers a very high level of coordination between
decisions.

- Method integration consists in the resolution of sub-models in a way that
  the link/coordination between the sub-models is taken into account in a
  better way than through an aggregation and disaggregation technique as in
  the hierarchical method. For example, the production and scheduling sub-
  models can be solved using an iterative loop which allows to update at each
  iteration the capacity constraint in order to avoid feasibility problems at the
  scheduling level. Compare to model integration, the level of coordination
  between decision is reduced due to the decomposition procedure.

- Tool integration deals with interoperability. Interoperability is the ability
  of a system (information technology product) to work with another system.
  For example, the production planning problem can be solved using an opti-
  mization tool whereas the scheduling problem is solved by a simulation tool.
  In that case, the link between those different tools must be made. Another
  well known example of interoperability is the ERP or SAP systems which
  are information tool which allow to connect different department (logistic,
  marketing, etc.) together.

Model integration and method integration will be validated through an analysis
of the most important articles in the field of supply chain integration. Even tough,
tool integration is a very important field, we have decided to not consider this
type of integration in our analysis. Tool integration is more related to information
technology research field and is therefore out of the scope of this thesis.
In this chapter, our aim is to answer the following research question:

**Question 1:** Integration is an approach which can be applied in various ways: problem integration, model integration and tool integration. Is it then possible to suggest a taxonomy of these methods, based on a literature review, in order to improve the understanding of the benefits of integration?

In order to answer this question, we analyze various articles dealing with an integrated approach. We concentrate our analysis on the approaches applied to coordinate the supply chain and propose a classification of those articles based on the three techniques exposed above (model integration, method integration, tool integration). Our aim is twofold. Firstly, we want to validate our classification. Secondly, we want to get some insights on the methodology to apply to create integrated decision tools.

We decided to concentrate our analysis on three different areas namely: tactical planning in the airline industry, manufacturing production planning and vehicle routing for distribution and on recent research performed in those domains. The remaining of this chapter is organized around those three areas. Those domains have been chosen for various reasons. Firstly, the airline industry has been chosen because it was the first industry where integrated approaches were developed. Secondly, production and distribution decisions were considered because we wanted to analyze the integration of decisions in the supply chain.

### 2.1 Manufacturing production planning

Production decisions have been widely analyzed and were traditionally solves in a hierarchical manner (see Figure 2.2).

At the strategic level, information on demand is unknown and therefore demand forecasts for each product family are calculated. This information combined with production costs for each plants, transportation costs between plants and plant capacity allows to allocate optimally product families to each plant. This decision will be reoptimized every year approximately (the exact time length depends on the industry).

At the tactical level, once every family of products is allocated to a plant, a production planning for each plant is elaborated. Therefore, demand forecasts for each product family combined with the capacity available at each plant is used to elaborate a production level (lot sizing decisions) as well as a stock level for each family of products. This production planning is re-evaluated every month.
2.1 Manufacturing production planning

Figure 2.2: Sequential production decisions [10]

approximately (the time length depends on the industry) in order to take into account the exact stock level as well as changes in the demand forecast.

At the operational level, the tactical decisions are detailed into a schedule. Indeed, scheduling problems determine the sequence in which jobs are performed on each machine, when and by whom in order to maximize the performance of the system (minimize the setup cost, the work in progress and the delivery time) while satisfying the production planning (delivery date, production level, etc.) and the limited storage capacity [26]. Those decisions are taken on the short term for example monthly (the time length depends on the industry) once the tactical production planning decisions have been made.

When the production plan is elaborated, aggregate decisions are taken without taking into account precise information [28]. Indeed, products are aggregated in product families depending on pre-determined characteristics. For example, products which have similar demand distribution functions are aggregated in the same product family, machines production capacity is aggregated in one global capacity or the global demand for all products is taken into account instead of individual product demand. Those optimal aggregate production planning decisions will act as constraints on decisions taken at a lower level (production scheduling). At the operational level, those aggregate decisions need to be disaggregated (machine capacity will be detailed, the demand by product family will be considered, etc.)

Consequently, decisions which are taken at the production planning level do not take into account the constraints related to production scheduling such as the setup time. Therefore the optimal solution obtained for the scheduling problem can be infeasible due to lack of capacity. In this situation, usually the production
planning is modified, ad hoc, in order to solve this feasibility problem.

Therefore, some authors have tried to integrate some of the decision levels regarding production decisions in order to avoid the feasibility problems related to the decomposition method. Hereafter, we present some papers dealing with that subject.

Lassere [53] proposes a global mathematical model which integrates production planning and scheduling decisions in a job shop. The author develops a global model with a unique objective function where production planning costs (storage and production cost) as well as scheduling costs (setup cost) are taken into account. In addition, their model takes into account the usual constraint of production planning and scheduling decisions (flow balance constraints, sequencing constraints, etc.) where the capacity constraint is fully detailed and takes into account setup times (not aggregate as in the hierarchical method). This global model is solved based on a decomposition technique where the global model is divided in two sub-models: a production planning model and a scheduling model. The production planning model is obtained by fixing the scheduling variables in the global model. Conversely, the scheduling sub-model is obtained by fixing the production planning variables in the global model. Those two sub-models are solved iteratively. In this article, the authors use model and method integration techniques. Indeed, a global model which integrates production planning and scheduling decisions is formulated (Model integration) and is decomposed in two sub-models which are solved iteratively (method integration).

Riane et al. [64] propose a decision tool which integrates production planning and scheduling decisions in the case of a hybrid flow shop. The authors propose, based on the hierarchical structure used to solve those production planning and scheduling sub-models, a methodological tool which allows to avoid the feasibility problems linked to the hierarchical approach. Indeed, the authors propose a feedback mechanism where the aggregate capacity constraint of the production planning model is adapted. This adjustment depends on the needs calculated through the simulation of the global model, global model which takes into account all the characteristics of the system. In this article, the authors use an approach based on method integration and tool integration. Indeed, planning and scheduling sub-models are solved sequentially with a feedback mechanism (method integration) using an optimization combined with a simulation tool (tool integration).

In parallel to production planning and scheduling decisions, authors have analyzed the integration of production scheduling and human resources decisions. One
of the main hypotheses in production scheduling is that human resources are available in the right quantity at the right time. In reality, the production scheduling aims at ordering the various jobs which need to be executed as well as the human resources needed to perform those jobs.

Daniels and Mazzola [26] propose a global model which integrates the production scheduling decisions as well as the flexible and renewable resources allocation problem (multi skill human resources) in a flow shop. They define the flexibility of resources as the ability to move resources from one machine to another when bottlenecks occur. The global model aims at minimizing the total completion time while taking into account scheduling constraints. In those linking constraints, each job processing time depends on the human resources availability. The authors develop an optimal branch-and-bound methodology for small size problems and a heuristic method for problems of industrial size. The heuristic method decomposes the global model into two sub-models which are solved iteratively. At each iteration, the execution time of each job is updated until the performance of the system can not be improved.

In another work, Daniels and Mazzola [27] study the case where resources (workers) are partially flexible in a flow shop. The aim is to analyze the link between performance (minimization of the total completion time) of the system and the degree of flexibility of resources. To do so, the authors develop a global model which is close to the one proposed in [26] but where the allocation of resources on machines must respect the ability matrix of each worker. This ability matrix depends on the training of each worker and on the number of workers trained at each time unit. The authors identify some competences which improved the performance of the system. As in their previous paper [26], they develop an optimal branch-and-bound approach for small size instances as well as a heuristic which decomposes the global model in three sub-models: the selection of a skill matrix, the sequencing of jobs, and the scheduling of operations. Those sub-models are solved iteratively. Numerical experiments are performed and show that a small investment in the training of workers can have a positive impact on the performance of the system. In addition, the way in which flexibility is allocated to the workers is also important for the performance of the system. In both of the papers propose by the authors, two approaches are proposed to integrate decisions: model and method integration. Model integration is realized by solving the global model using a branch and bound technique. The heuristics propose by the authors is an example of method integration where two sub-models are solved iteratively.
The simultaneous analysis of production scheduling and resource planning has also been analyzed by Giard et al. [41]. They studied the case of a mail sort out office where the machines are structured as a flow shop and the capacity of each machine depends on the number of workers allocated to it. They define a global model where the level of production depends on the number of workers allocated to the machine. The objective of the model is to minimize the variation of the workforce on each machine. They propose a methodology to solve this global model which is based on artificial intelligence. In this study, a global model is developed by the authors to solve production scheduling and resource planning decisions and is solved as a whole. This methodology is close to a model integration approach.

Chen [16] analyzes the integration of production scheduling and human resources decisions in the case of parallel machines. The author proposes two global models which differ on the way to calculate the production scheduling performance. The objective function of the first model is composed of the completion time of tasks and the cost of resource allocation whereas the objective function of the second model contains the number of late tasks and the total cost of resource allocation. Those models are solved using a branch-and-bound method where the relaxation of the global model is solved at each node. This linear relaxation is solved using the column generation method. With this method, the author manages to solve problems of medium size. In this paper, the approach used by the authors is a model integration approach where a global model is solved globally.

Benbouzid-Sitayeb et al. [9] propose a model which takes into account the production scheduling model as well as the maintenance problem. The authors develop a global model where the objective function takes into account production scheduling as well as maintenance decisions. This integrated model is solved using an adaptation of the ant colony optimization algorithms and is compared to the traditional sequential methodology. The approach used by the authors is model integration where a global model is developed and solved globally.

Another extension of production scheduling integration deals with the integration of cutting stock and sequencing decisions [82]. Cutting stock problems consist of cutting small pieces in a specific size and in the right quantity from a larger piece in order to optimize some objective function (e.g. minimize production cost, waste, maximize profit, etc.). Sequencing issues linked to the cutting stock problem can arise. Those sequencing questions concern the order of the patterns to be cut such that some objective function is optimized. For example, the pieces to be cut come in general from stack which are stored close by the cutting machine.
In that case, the objective function can be formulated as to minimize the number of stacks which remain open can be used. Usually those decisions are taken sequentially: first the cutting stock problem is solved then the scheduling problem is tackled. The authors, Yanasse et al. [82] develop a global model where those two decisions are integrated in the objective function as well as in the constraints. They obtain an integer programming model which is solved using a decomposition method. First a Lagrangean relaxation is solved where the binding constraints linking the cutting and scheduling problem are relaxed. This relaxation allows to decompose the model in two: a cutting problem and a scheduling problem. The binding constraints are taken into account through the Lagrangean multipliers. The two sub-models are solved iteratively. The methodology applied by the authors (global model combined with a decomposition technique) is a mix of model (global model) and method integration (decomposition approach to solve the global model).

Table 2.1 concludes this analysis by proposing a classification of those articles with respect to the three integrated techniques namely model integration, method integration and tool integration exposed in the introduction of this chapter.

<table>
<thead>
<tr>
<th>Decisions integrated</th>
<th>Production planning &amp; scheduling decisions</th>
<th>Production scheduling &amp; resources management</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Integration Types</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model I.</td>
<td>Lassere [53], Giard et al. [41], Chen [16], Daniels and Mazzola [26], [27]</td>
<td></td>
<td>Benbouzid, Sitayeb et al. [9]</td>
</tr>
<tr>
<td>Method I.</td>
<td>Lassere [53], Riane et al. [64], Daniels and Mazzola [26], [27]</td>
<td></td>
<td>Yanasse et al. [82]</td>
</tr>
<tr>
<td>Tool I.</td>
<td>Riane et al. [64]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.1: Manufacturing production planning classification.

Several remarks can be made on this summary table. First of all, all the articles analyzed fall in the model or the method integration category. One paper [64] makes reference to tool integration. Model integration is performed by authors in three ways. Either the global model is reformulated in order to reduce its
26 Integrated decision making process

complexity. Or, some authors have used a combination of methodologies which allow to solve the global model. Lastly, the global model is solved on a short horizon or for small size instances.

Secondly, some of the articles were difficult to classify and therefore were inserted in two different categories. It is the case of Lassere’s article [53] which develops a global model and then applies a decomposition methodology. This decomposition scheme does not make use of the traditional hierarchical sub-models. As already mentioned, in the case of Riane et al.’s article, two integrated techniques are used: method integration and tool integration. The authors solve two sub-models using a combination of optimization and simulation tool.

2.2 Tactical planning in the airline industry

Since 1950, the airline industry has been a very active domain for OR applications [51]. The mathematical models linked to airline management are more and more complex. This increase in complexity reflects the increase in the size of airlines, the increase in demand as well as the need for those companies to be more profitable. Those trends have encouraged researchers to analyze the link between decisions at different levels.

For airlines, strategic decisions concern the size of the fleet and the type of airplane used. Usually, those decisions are dealt with independently and sequentially due to the combinatorial complexity of the problems tackled [52]. Those decisions are taken 12 months before the execution phase and over a horizon of 9 months from the execution phase.

At the tactical level [52, 42](see Figure 2.3), the planning is created and pairs of origin/destination as well as arrival and departure times of each flight are determined. This planning is realized for a period of 3 months. Once this planning is achieved, a type of plane is allocated to each flight in order to maximize revenue (plane type assignment problem). Then for each type of plane, the schedule is calculated (the successive flights that the same plane realizes) taking into account maintenance periods.

Finally, for each route, a crew pairing problem is solved. The aim of the crew pairing problem is to create a collection of pairings for crews such that each flight is covered and workforce demand is satisfied. Those pairings are anonymous, last for 2 to 5 days and begin and end at the same location. Each pairing created by the crew pairing problem will be combined to form a chain in order to create a monthly
schedule for each crew and will be assigned to individual crew members while taking into account scheduled activities, trainings, vacation, rules and regulations, etc. (Crew assignment problem). The crew pairing problem combined with the crew assignment problem form the crew scheduling problem.

The tactical planning can be modified and adapted daily by taking into account operational decisions such as climatic perturbation, flight traffic jam, non-planned maintenance, etc.

Individual models have been developed for each of those tactical levels. A literature review has been realized by Klabjan [51] on the various models available for the plane type assignment problem and of flight scheduling. Barnhart et al. [8] review the literature on the problem of crew scheduling and Gamache and Soumis [38] have analyzed the problem of determining monthly personalized crew scheduling.

At the tactical level, some authors have tried to integrate decisions which have been dealt with independently and sequentially. The sequential procedure can lead to suboptimal solutions and to feasibility issues due to a lack of flexibility [23]. Hereafter, we report on articles in the field of airline management which integrate various tactical decisions with the aim of improving efficiency in the supply chain. For each of those articles, we focus on the methodology applied in order to propose decision tools that improve the coordination mechanism.

Lohatepanont and Barnhart [55] propose a daily model that integrates the tactical decisions of planning and of plane type assignment. This global model is solved using a combination of methods such as column/line generation and branch-and-bound techniques. Preliminary results are given which show the gain
achieved by using a more integrated technique. Their approach is close to a model integration approach where an integrated model is developed and solved globally without the use of a decomposition methodology.

The simultaneous resolution of the scheduling problem and of the plane type assignment has been considered by Barnhart et al.[7]. The authors propose a string based model where a string represents a sequence of flights beginning and ending at a maintenance station, respecting the flow constraints and which respects maintenance periods. They propose to solve their global model by a combination of branch-and-bound and column generation techniques. As in the previous paper, the author use a model integration approach where a global model is developed and solved as a whole.

In the airline industry, the main costs are operational costs linked to the fleet, in other words to fuel and workers. Those costs appear at the flight scheduling and crew scheduling level. Therefore, some authors have concentrated their work on how to manage more effectively those two decisions [23]. The integration of flight scheduling decisions as well as crew scheduling decisions is justified by the time link (connection time between flights) which exists between those two decisions. Indeed, connection times can be reduced if the same crew and the same plane is used for various flights. Therefore, the number of feasible crew schedules depends on flight scheduling decisions.

A first attempt of integration of flight scheduling and crew scheduling decisions has been realized by Klabjan et al.[52]. Their aim is to solve the problem of crew pairing without the restriction resulting from the flight scheduling problem. Therefore, they include in the crew pairing problem, feasibility constraints linked to the flight scheduling problem (the number of planes available). They take also into account flight windows for the arrival and departure of planes which allow to improve the profitability of the company. Their methodology is based on the fact that they solve the crew pairing problem before the flight scheduling problem (opposite procedure as the one presented in Figure 2.3). Nevertheless, in this procedure, the authors do not guarantee that they will find a feasible flight schedule in terms of maintenance. In this approach, the authors use a decomposition approach in order to solved the flight scheduling and crew scheduling problem (method integration approach).

Cordeau et al. [23] propose a global model which takes simultaneously into account the flight scheduling problem as well as the crew pairing problem. This global model contains the constraints of each of the sub-models with in addition
linking constraints such as the connection time between two flights. The authors propose to solve the problem using Benders decomposition where each of the sub-problems (the master problem is the flight scheduling problem and the sub-problem is the crew rostering problem) is solved using the column generation technique. Integer solutions are obtained through a heuristic. They manage to solve real world cases of about 500 flights. Even though, Benders decomposition decomposes the global model in a master problem and a sub problem, the approach use by the authors is a model integration approach. In order to be a method integration approach, the decomposition of the global model has to be performed by the authors and not by the methodology applied.

Cohn and Barnhart [20] developed a model of crew pairing which takes into account the flight scheduling constraints. They propose an exact solution methodology which is a branch-and-bound technique combined with a column generation methodology. Additionally, a heuristic is developed which decomposes the global model in two sub-models: a flight scheduling problem and a crew pairing problem. Those two sub-models are solved iteratively. This heuristic allows to have good solutions in a short computational time. Computational results show that only some of the maintenance constraints of the flight scheduling problem have an impact on the crew pairing model. In this papers, the author propose through their exact methodology a model integration approach and with their heuristic they propose a method integration technique.

The integration of crew pairing decisions and the crew assignment problem has been analyzed by Guo et al.[42]. Solving those two problems sequentially can be problematic because the crew pairings determined can be infeasible when realizing crew rosters (chaining pairings together taking rest periods into account) and assigning them to crew while taking into account individually planned activity. This implies that some pairings must be broken up at the crew assignment level and reconstructed according to those individual planned activities. Therefore, the authors propose to take into account some information of the crew assignment problem when solving the crew pairing problem such as crew capacity at each home base. In addition, they propose to create anonymous crew pairing chains instead of only pairings. A Crew pairing chain is a sequence of pairings respecting flight plan and weekly rest. Those pairing chains will be created at the crew pairing problem level and respect individual planned activity and home-base restriction but will still be anonymous. In a the second phase, those pre-pairing chains will be combined to form a roster for each crew. Including more information in the
first step of the crew scheduling problem allows to reduce drastically the risk of infeasibility in the second step. Consequently, they develop a flow model where those modified crew pairing chains are established (crew pairing problem). Then in a second step, a multi-weight based heuristic is used for the personalized rostering (crew assignment problem). In this second phase, the anonymous pairing chains are combined in order to respect a global balancing constraint over all home bases and also between crews. In this paper, the authors do not developed a global model. They apply a decomposition methodology which is adapted in order to coordinate better crew pairing and crew assignment decisions. This approach is closed to a method integration technique.

Medard and Sawhney [57] solve simultaneously the crew pairing problem and the crew assignment problem in order to improve daily management. The aim of the authors is to obtain a good solution at the operational level. This short horizon reduces the size of the model. Consequently, they propose a model which integrates completely the two problems with a rolling horizon and is therefore an example of model integration approach.

Dandhu and Klabjan [25] propose a model which integrates three phases of the tactical decisions: the plane type assignment, the flight scheduling and the crew pairing problem. Their model completely integrates the plane type assignment problem and the crew pairing problem while assuring the feasibility of the flight scheduling problem. The maintenance constraints are not taken into account. As the global model is complex to solve, the authors propose two solution methodologies. The first solution methodology is based on the Benders decomposition method. The linear relaxation of the crew pairing model is solved and the information from the dual of this model is used to derive Benders cut. Those cuts are added to the plane type assignment model. The second methodology solves the integrated model only for a small number of the crew pairings at the time. Both of the methodologies proposed by the authors aim at solving the integrated model globally and therefore corresponds to a model integration approach.

Table 2.2 shows the classification of the articles according to the three integrated techniques. Most of the papers use model integration and no article has discussed the issue of tool integration. The methodologies for model integration is comparable to the one applied in the previous section. We want to point out the paper of Guo et al.[42] which gives a different approach to integrated techniques. They propose to solve two sub-models that are different from the one used in the hierarchical methodology. Those sub-models are created in order to avoid
2.3 Vehicle routing problem

Hereafter, we survey articles that aim at coordinating decisions including the vehicle routing problem (VRP). The Vehicle routing problem determines the best route for various vehicles leaving from one or several depots to several customers. The objective is to minimize the total cost of traveling for a fixed and known amount of demand. Various extensions of this model are studied in the literature: VRP with limited transportation capacity, VRP with time windows, etc. This problem is formulated as an integer model and is NP-hard. The sequential structure of the distribution decisions is shown in figure 2.4.

Haase et al. [43] propose a model which determines simultaneously the vehicle and the drivers assignment problem based on a predefined timetable in a urban bus system. The model is based on the case of a unique depot containing an homogeneous fleet of vehicles. The drivers have all the same characteristics but vary in number depending on their availability. The travel times are known. The authors propose to solve those two problems (vehicle and driver assignment problem) in the opposite way: first the driver assignment problem is solved then the vehicle assignment problem is considered. Consequently, the authors developed
Integrated decision making process

Figure 2.4: Sequential distribution decisions

a mathematical model which calculates the schedule of drivers while taking into
account constraints on the availability of vehicles (side constraints). The objective
function takes bus and driver’s cost into account which ensure that both problem
are optimize. Those side constraints are necessary in order to obtain a feasible
solution when solving the vehicle assignment problem. The methodology used is
close to the one developed by Klabjan et al.[52] for the plane scheduling problem
combined with the crew pairing problem mentioned in 2.2. In this papers, the au-
thors have develop a global model which allows to appreciate the link between the
vehicle and drivers assignment problem. This approach is a model integration ap-
proach. Nevertheless, the authors decompose the global model in two sub-models
which are solved iteratively. This methodology is closed to a method integration
approach.

Freling et al. [36] propose a new mathematical model where vehicle and human
resource planning decisions are integrated in the case of a unique depot and a
homogeneous fleet of vehicles. Their global model is composed of an objective
function which encompasses costs of both problems and constraints concerning
those two decisions. Their model is solved using a combination of methods such as
the Lagrangean relaxation and column generation. They have tested their solution
methodology and their model on instances of industrial size. Unlike Hasse et al.
[43], the authors propose a model and methodology which can be applied in real
situations. In this article, the authors develop an integrated model which is solved
globally. This methodology is a model integration approach.

Hollis et al. [48] develop a model that solves simultaneously the vehicle and
driver assignment problems. Their study is based on the Australian post office
2.3 Vehicle routing problem

In this context, multiple depots with various types of vehicles as well as multi-skilled drivers are considered. In order to highlight the importance of the trade-off to realize, the authors propose a model that takes into account in its objective function fixed and variable costs of human resources and of vehicles. In addition, their model contains the usual constraints of the vehicle and drivers assignment problem where the link between those two problems is highlighted. Their solution methodology is based on a combination of a heuristic with a column generation approach. The heuristic allows to find an initial solution and column generation is used to improve this initial solution. Once the column generation procedure ends, an integral solution is found heuristically using a branch-and-bound technique. In their paper, the authors have used model integration in order to coordinate vehicle and driver assignment problems.

Sarmiento and Nagi [67] review the field of integrated Vehicle Routing Problem and production planning (lot sizing decisions and production scheduling decisions).

Recently, Chen and Vairaktarakis [17] studied the integration of production scheduling decisions with VRP decisions. The authors consider a just-in-time environment. The service level depends on the waiting time of customers and increases with the number of vehicles used. The increase in the number of vehicles used will increase the distribution cost. Therefore, there exists a trade-off between the service level and the distribution cost (in the objective function). The authors propose a global model where production and distribution scheduling decisions as well as VRP decisions are integrated. The authors consider two means to calculate the service level (the average delivery time and the maximal delivery time) as well as two types of configuration (one machine and machines working in parallel). This global model is solved optimally and heuristically. The heuristic developed solves a reduced version of the global model and proves to give near optimal solution. Through their study, the authors apply a model integration approach where a global model is developed and solved as one.

Chiang and Russell [18] develop a global model which integrates buying decisions for propane gas with a VRP between the regional depot and customers. The objective of their approach is to take into account the variation in the propane gas price between the various regional depots. Therefore, there exists a trade-off between the propane price in the various regional depots, the travel distance to reach those regional depots, the transportation cost, the maximum legislative travel time and length. The solution methodology developed by the authors is based on set partitioning and tabu search. With this method, they obtain opti-
mal and near-optimal solutions. The authors test their model and method on a real world database and prove that cost reductions of millions of dollars can be realized. Their model and methodology can also be applied in other fields. In this article, the authors develop an integrated model which is solved globally (model integration approach).

Erkut and Alp [32] analyze the particular situation where hazardous material is transported. In order to reduce the risk of accidents, the authors take simultaneously into account the VRP and the time of travel. Indeed, the probability of having an accident depends highly on the time of the day where the journey is realized (the probability of having an accident at night is higher than during the day). The authors formulate the probability of having an accident as a function of time in order to be able to integrate VRP decisions and time of travel. The authors propose four different types of global models which encompass constraints which are more and more realistic. Those models are solved using pseudopolynomial dynamic algorithms. They implemented their methodology on a real case and good solutions were obtained in reasonable computational time. In this study, the authors use model integration where an integrated model is created and solved globally.

Cordeau et al. [21] solve simultaneously the assignment of cars and of locomotives to passenger trains at VIA Rail Canada. In general, those problems are solved sequentially: first the number of cars is calculated then the right locomotive is assigned (one that has enough power). This sequential procedure was adopted due to the important size of the model and to the variability of the demand. The model developed in this paper aims at solving VIA Rail Canada issue and therefore additional constraints are considered in order to take into account a realistic situation. The problem is formulated as a multicommodity network flow-based model. This problem is solved using a heuristic branch-and-bound method in which the linear relaxations are solved by column generation. The issue of this heuristic is that the running time increases a lot with the size of the problem and that it has little general applicability. They propose, as well, a second model which is based on a more general setting that can be adapted to various situations. They propose to solve this model with Benders decomposition.

The authors proposed in another paper [22] various extensions to the original model presented in the paper [21]. For example, they take into account maintenance constraints and penalize cars switching, etc. They develop a new methodology to solve the multicommodity network flow-based model which is based on
2.4 Conclusion

a branch-and-bound algorithm that solves, by Benders decomposition, a mixed-integer problem at each node of the tree. This adapted methodology allows to solve problems of industrial size. In their two papers ([21] and [22]), they use an approach based on model integration where a global model is developed and solved using various methodology (branch-and-bound, Benders decomposition and a mix of both).

<table>
<thead>
<tr>
<th>Integration Types</th>
<th>Vehicle type and Driver assignment problem</th>
<th>Production scheduling and VRP</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Freling et al. [36]</td>
<td></td>
<td>Erkutand Alp [32]</td>
</tr>
<tr>
<td></td>
<td>Hollis et al. [48]</td>
<td></td>
<td>Cordeau et al. [21, 22]</td>
</tr>
<tr>
<td>Method I.</td>
<td>Haase et al. [43]</td>
<td></td>
<td></td>
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<tr>
<td>Tool I.</td>
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</table>

Table 2.3: Classification of Vehicle routing problem for distribution.

Table 2.3 shows the classification of the articles according to the three integrated techniques. As in the case the airline industry, most of the papers use model integration and no article has discuss the issue of tool integration.

2.4 Conclusion

In this chapter, we focus on coordination techniques based on supply chain integration. Our aim is to understand how to create integrated decision tools. More precisely, we suggest three different techniques: model integration, method integration and tool integration which are analyzed through a literature review. Our literature review concentrate on three areas: tactical planning in the airline industry, manufacturing production planning and vehicle routing for distribution. From this literature review, we learned that most papers achieve model integration and that tool integration is very scarcely considered. Tool integration is an important issue which needs to be considered in order to practically implement those integrated tool in business life.
From this analysis, we think that integrated decision making tools based on the ideas underlying model integration and method integration are suitable for our objectives. Indeed, our aim is to take advantage from those two approaches: we want to propose decision tools which offer a very high level of coordination and also are computationally efficient. With model integration, a global model which integrated completely all decisions is solved. This offers a very high level of coordination but is computationally costly. With method integration, each hierarchical sub-model is solved taking, at best, the coordination link between decisions into account. This offers computational efficiency but less coordination between decisions. Therefore, taking those characteristics into account, we propose to develop integrated decision tools which are based on the decomposition of the global model in sub-models but not the same sub-models than the one of the traditional hierarchical planning. Sub-models are created with the objective of reducing the loss in coordination due to decomposition. In other words, intelligent decomposition is performed. The creation of those sub-models is dependent on the decisions (problem type and time length) considered as well as on the formulation of the global model. Our aim through for the rest of this thesis is to analyze how the various techniques proposed in this chapter can be applied on real situations and how they perform in term of coordination of decisions. Therefore, this model/method integration technique is applied and tested on two case studies in Chapter 3 and in Chapter 4. The case studies concern integrated decision tools in the area of inventory and warehouse decisions on one hand and production and distribution decisions on the other hand. We have selected those two case study because they deal with decisions of different stages in the supply chain and different time horizon (tactical and operational).

The first integrated decision tool is presented in Chapter 3 and aims at coordinating inventory and warehouse decisions. In order to create this integrated decision tool, model integration is used and a global warehouse and inventory model is developed. This global mathematical model has been tested on an industrial size database and turned out to be complex to solve for large instances. Therefore, in order to solve this mathematical model, we have decided to apply method integration. Two solution methodologies have been developed which offer different levels of integration of warehouse and inventory decisions. Those solution methodologies decompose the global model in sub-models with the aim of maintaining the level of coordination as strong as possible. Computational tests are performed on a real world database using multiple scenarios differing by the
warehouse capacity limits and the warehouse and inventory costs. Our aim is to evaluate the coordination value (by cost and benefit analysis) of our various integrated decision tools developed as well as the impact of the warehouse capacity limit on the performance of our integrated decision tools. Our observation is that the total cost of the inventory and warehouse systems can be reduced drastically by taking into account the warehouse capacity restrictions in the inventory planning decisions, in an aggregate way. Moreover additional inventory and warehouse savings can be achieved by using more sophisticated integration methods for inventory and warehouse decisions.

In Chapter 4, we present an integrated decision tool for tactical-operational production and distribution decisions in a reusable resources environment. More precisely, we aim at coordinating lot sizing production and distribution decisions with vehicle routing decisions. Based on model integration technique, we have developed a global multi-period multi-item multi-vehicle model where a capacity constraint models the link between production and distribution decisions. This global mathematical model is complex to solve for large instances due to the vehicle routing decisions. By the mean of method integration technique, we have developed three heuristics which allows to reduce the complexity of the global model by decomposing it in sub-models which destroy the less the coordination link. The first two are based on a decomposition approach of the global model in production and distribution sub-models which are independent of each other and are solved sequentially. The third heuristic use production and distribution sub-models but offers a higher level of integration by taking into account transportation decisions in the production planning sub-model. Computational tests are performed in order to evaluate the coordination strength achieved in the various integrated decision tools. In addition, we have shown that the performance of the integrated decision tools depends on the amount of reusable resources in the system, the type of customer demand but not on the weight of the production cost against the distribution cost. Moreover, the three integrated decision tools allow to tackle problems of larger size than an optimal solution approach. In addition, in order to implement those integrated decision tools, we have used tool integration. Indeed, commercial MIP solver XPRESS-MP combined with the publicly available TSP solver Concorde [1, 2] are used for the implementation of those integrated decision tools.
Chapter 3

An Integrated Model for Warehouse and Inventory Planning

3.1 Introduction

The aim of this chapter is to present our first case study where inventory and warehouse decisions are analyzed and coordination mechanisms are proposed in order to improve the efficiency of the supply chain. Through this analysis, we want to answer three of our research questions which are the following:

- **Question 2**: Monolithically constructed models are difficult to solve: their optimization consumes time and computing resources. Despite those difficulties, what are the evidences in favor of integration?

- **Question 3**: How can the traditional sequential procedure still be applied in a more integrated environment?

- **Question 4**: How can an integrated approach handle various aspects of resources management more accurately than what is currently done in the literature?

We restrict our analysis to tactical inventory and warehouse decisions. Those decisions are usually dealt with separately and optimized independently. We propose to coordinate those decisions and to analyze the benefit of coordination.
Therefore, we develop a global tactical warehouse and inventory model which al-
lows to highlight the link between those two decisions. Two solution methodologies
are developed based on an intelligent decomposition of the global model in various
sub-models. Indeed, those sub-models are created by keeping the link between in-
ventory and warehouse decisions as strong as possible. Finally, our methodologies
are tested and the value of supply chain integration is calculated.

This chapter is organized as follows. In Section 3.2, warehouse and inventory
decisions are defined and the field of research is exposed. A brief review of the lit-
erature on tactical inventory and warehouse decisions is performed in Section 3.3.
In Section 3.4, we describe our model with the assumptions and formulation. The
two solution methodologies are presented in Section 3.5 and finally computational
experiments are performed on an industrial test case.

3.2 Motivations

This research has been inspired by a real case study. The business under consid-
eration sells mass market products all around the world. We are only interested
in the Belgian market activity. For the distribution of their product in Belgium,
they operate from a distribution center located near by Brussels. This distribu-
tion center is replenished by production units located all around the world and is
used for storage and order preparation of the products. Their customers are the
various distribution chains active on the Belgian Market. Our main interest is the
storage activity taking place in this distribution center. The distribution center
is organized in two areas: a forward area and a reserve area. The forward area
is a small area used for broken-case and full-case picking. When the inventory of
an item stored in the forward area reaches its reorder point an internal replen-
ishment is performed (from the reserve area to the forward area) or an external
replenishment is achieved when the product is not available in the reserve area.
The main issues related to this area concerns the choice of the products which will
be stored in the forward area. Indeed, if all products are located in the forward
area, the size of this area increases and the advantage of lower order picking cost
vanishes. Some of the other decisions are to allocate space in the forward area
for the different products, the location of the product in the forward area, etc..
The reserve area is a large area where pallet picking activity are realized as well
as reserve storage. The products stored in the reserve area are handled through a
dedicated storage policy: each product is assigned to a specific location and each
location in the reserve area is composed of homogeneous palettes: there is only one product on each palette. The reserve area is divided into "a base" which concerns the level "0" and "1" which are accessible easily (without any specific equipment). The upper levels are called the "reserve" and are accessible only with a clark. The boxes/palettes are picked from the "base" and the replenishment of the "base" is performed from the "reserve" area. The capacity of the reserve area is considered as unlimited because external storage is possible. Nevertheless, the base of the reserve area is considered of limited capacity. Some of the decisions to consider are the level of the external supply quantities, the product to assign to the reserve area and the location of the product in the reserve area.

The decisions considered for the forward area and the reserve area concerns inventory decisions (supply of products to the warehouse) and warehouse decisions (location of the products in the warehouse). Traditionally those decisions have been considered independently and separately. With the improvement in information technology, it becomes possible to develop tools which can help managers to handle warehouse and inventory issues more efficiently.

At all classical levels of decision (strategic, tactical and operational) [74, 76, 65], warehouse managers have to tackle problems which can be divided into two broad classes: warehouse management and inventory management problems.

Regarding warehouse management issues, managers have to decide where to assign the products inside the warehouse. Strategic decisions concern issues such as the size of the warehouse and the technical specifications of the warehouse. Tactical decisions concern issues such as the layout of the warehouse and the sizing of the various areas inside the warehouse [81, 14]. Finally, operational decisions deal with control policies and routing problems. Concerning inventory management, managers must decide which product, and how much of each product need to be stored in the warehouse. In this class, strategic decisions concern the size and the design of the warehouse (a common decision with warehouse management) and more specific decisions such as the configuration of the inventory management decision systems. The tactical decisions concern issues such as the operating hours, the replenishment policies and work force size. On the last level (operational), problems such as "what to produce or deliver", "when" and "on which machine or by whom" are considered. (see also [59] for more details)

All those decisions are interrelated but are dealt with independently [65]. Up to now, those issues (strategic, tactical and operational decisions) are handled in a pyramidal top-down approach where the flexibility of decisions decreases from top
to bottom. Strategic decisions are first taken and then create limits to decisions taken at the tactical and operational levels. For example, once the size and the design of the warehouse are fixed, these decisions will have to be respected when replenishment policies have to be designed as well as when the size of the different warehousing areas has to be optimized (see [65], [46] for more examples).

On top of this, decisions taken at each level of the pyramid are also handled independently and sequentially [74]. For example, concerning warehousing decisions taken at the tactical level, Jeroem P. van den Berg [74] has introduced a classification of the different problems faced by managers. He has proposed four different classes of decisions.

The first class tackles issues related to the assignment of products across warehousing systems. Warehousing systems differ by the level of automation used. The author [74] gives three levels of automation: manual order pickers where pickers ride along the picking position (picker to product systems), AS/RS order picker or carousel where products are picked by machines and sorted by people (product to picker systems) and totally automated order pickers (robot technology). In addition, warehouses are often divided in areas according to the unit load retrieval. Usually, a forward area is used for order picking of units of items frequently ordered and a reserve area is used either for replenishment of the forward area or for order picking of cartons or pallets of items or for products not ordered frequently enough.

The second class of decisions concerns the clustering of correlated products in such a way that products that are frequently ordered together are assigned to locations close to each other. The third class of problems concerns the workload balancing problem across the warehouse and the last class concerns the assignment of products to storage locations in order to minimize the distance traveled for order replenishment and picking.

We propose a tactical model which integrates more phases of the decision process: the replenishment decision in the inventory management, the allocation of products to warehousing systems and the assignment of products to storage locations in the warehousing management. We consider a picker to product distribution warehouse which is divided in a forward and a reserve area. Our objective is to minimize all relevant warehousing and inventory costs by optimizing the quantity of each product allocated to the forward area (by reducing the work load related to order picking), the location of the product inside the forward area and the inventory replenishment policies. Our tactical model takes the size of the warehousing
systems as given (strategic decision level). Our aim is to test whether or not an integrated approach to take these inventory and warehousing decisions has some additional value, compared to the classical sequential approach.

3.3 Literature review

We give references to the different models in the field of warehouse and inventory management available in the literature. As written in Section 3.2, most tactical issues in warehouse and inventory management are tackled independently and sequentially. In consequence, the models developed in those two fields are presented separately.

3.3.1 Forward-reserve models

The Forward-reserve problem (FRP) is the problem of assigning products to the forward and reserve areas in order to reduce the overall work content in order picking [75]. Nowadays, most warehouses are divided in two areas: forward and reserve. The forward area is used for broken-case and full-case picking and the reserve area is used for pallet picking and reserve storage. Once a product is stored in the forward area (respectively the reserve area), all picks must be performed from the forward area (respectively the reserve area). When the inventory of an item stored in the forward area reaches its reorder point an internal replenishment is performed (from the reserve area to the forward area). The forward area is smaller than the reserve area where order picking takes less time and is then less costly. The critical decision concerns the choice of the products which will be stored in the forward area. Indeed, if all products are located in the forward area, the size of this area increases and the advantage of lower order picking cost vanishes. The other decision is to allocate space in the forward area for the different products.

Hackman and Rosenblatt [44] were the first to address the issues of deciding which product to store in the forward area (assignment issue) and how much to store (allocation issue). They considered a warehouse composed of a small area (forward area) where picking of products is based on an efficient (less time consuming) AS/RS automated storage and retrieval system. The reserve area is a large area (infinite capacity) handled through an inefficient manual/semi-automated material handling system. Reception of products is made through the manual/semi automated area and can be used to satisfy customers orders or to make internal replenishment of the AS/RS area. The question tackled in this
article is to decide which and how much product must be stored in the forward area taking into account picking costs and internal replenishment costs and the capacity constraint of the forward area. They solve the problem through a greedy heuristic where the products are assigned to the forward area according to some a priori ranking of the products until the space is full. This ranking is based on the comparison of the savings due to efficient picking in the AS/RS area and the cost of internal replenishment. They prove a sufficient condition for optimality.

Frazelle et al. [35] propose a model that tackles three warehouse decisions: the size of the forward area and the allocation/assignment of products to the forward area. They propose a model which minimizes the total warehousing costs, which depends on the size of the forward area (replenishment cost of the forward area, reserve/forward picking cost and the cost of capital (shelvings)), under forward capacity and congestion constraints. Firstly, they show that the congestion constraint is redundant. Consequently, the optimal quantity assignment/allocation solution can be found based on the work of Hackman and Rosenblatt [44]. Secondly, they show that the optimal assignment for the products (forward or reserve area), considering the linear relaxation of their model, is the ranking given by Hackman and Rosenblatt 1990 [44] which is independent of the size of the forward area. They proposed an algorithm which gives a near optimal solution to their model based on the linear relaxation of their model.

van den Berg et al.[75] propose a model to solve the forward-reserve problem in the case of unit load replenishment. Those replenishments can occur during busy or idle picking periods. The objective is to minimize the number of urgent or concurrent replenishments of the forward area arriving during the busy periods. Such replenishments are needed in case of shortage during the busy period but should be avoided because congestion can result. Instead, replenishment activities are encouraged to take place during the previous idle period. The resulting forward-reserve model is a binary programming problem which is solved using a greedy knapsack heuristic procedure based on a linear relaxation of the initial model. In a second part of the article, they modify the model to incorporate a limit on the amount of concurrent replenishment.

### 3.3.2 Inventory models

The aim of inventory management is to minimize total operating costs while satisfying customer service requirements [40]. In order to accomplish this objective, an optimal ordering policy will be determined by answering to questions such as
when to order and how much to order. The operating costs taken into account are
the procurement costs, the holding costs and the shortage costs which are incurred
when the demand of a client can not be satisfied (either lost sales costs or back-
order costs)[40, 49]. There exists different inventory policies [49] : the periodic
review policy and the continuous review policy. The first policy implies that the
stock level will be checked after a fixed period of time and an ordering decision
will be made in order to complete the stock to an upper limit (order up to point),
if necessary. In the second policy, the stock level will be monitored continuously.
A fixed quantity will be ordered when the stock level reaches a reorder point. The
order quantity will only be delivered after a fixed lead time and shortage can ex-
st if the inventory is exhausted before the receipt of the order quantity. Those
basic policies can be adapted to take into account special situation such as single
or multi item models, single or multi period models, deterministic or stochastic
demands, lost sales or backorder...(see [49, 59, 40] for more details and examples)

3.4 Model formulation

3.4.1 Problem Description

We consider a warehouse composed of a reserve area and a forward area. The
forward area is divided into locations and each product in the forward area is
allocated to a number of locations. Before the picking period, the forward area
is replenished through advance replenishments from the reserve area. The level
of the advance replenishment for each product corresponds to the filling of the
allocated space in the forward area. Nevertheless, if the stock level in the forward
area reaches some reorder point, to avoid shortages, concurrent replenishments
will take place. Meanwhile, the company receives external supply for all products.
The issues that we address simultaneously are the decision of the routes taken by
the different products in the warehouse (external supplies to the reserve area or
directly to the forward area) and the quantity of the products allocated to the
forward and/or the reserve area (warehouse management issues). In addition, the
optimal frequency of the external supplies will be optimized as well as the safety
stocks required to offer an adequate customer service level (inventory management
issues). These issues are interrelated because the external supply cost will depend
on the routes taken by the product on one hand and the location of the product
in the warehouse will depend on the quantity ordered on the other hand.
3.4.2 Assumptions

First of all, we assume that the layout of the warehouse is given. By this, we mean that the size of the warehouse and of the different warehouse systems are given (forward and reserve areas). Nevertheless, we suppose that it is possible to rent external storage space if the space available in the warehouse is not sufficient to store all the products. This additional capacity is rented at a higher cost than the cost of the internal warehouse capacity. We suppose also that this additional capacity implies the same costs (reception cost, storage cost...) than the ones linked to the reserve area.

Different storage policies may exist: random storage policy and dedicated storage policy [40]. In a random storage policy, products are randomly assigned to a location in the warehouse. In a dedicated storage policy, each product is assigned to a specific location. In the latter case, if the product is not available in the warehouse then the location of this product will be empty and there will be unused space. The forward area, due to its purpose, will be handled through a dedicated storage policy. Concerning the reserve area, we will consider the two storage policies, the dedicated or the random.

During the picking period, different activities can occur. We have considered six main activities: concurrent and advance replenishments of the forward area from the reserve area, picking from the forward area and the reserve area, external supply of the forward area and the reserve area. We formulate some assumptions for each of those activities.

Concerning the advance replenishment of the forward area, we suppose that this activity occurs during some idle period just before the picking period and does not imply any congestion cost. Whereas, concurrent replenishments occur during the picking period when there are not enough items of a product in the forward area to satisfy the demand of that product and therefore induce congestion. Concurrent replenishment will be performed immediately when the reorder point is reached. The level of the reorder point corresponds to the average demand during the concurrent replenishment lead time, plus some safety stock. We suppose that the internal safety stock is fixed and known for all products.

Concerning the picking activity, we suppose that the time it takes to pick a product from the forward area (respectively the reserve area) does not depend on the location of the product inside the forward area (respectively the reserve area) because products are typically picked during standard picking tours through the whole areas. Therefore, we won’t make a distinction between the various locations
inside the different areas. Nevertheless, the number of products that we can put in
a location of the forward area will depend on the size or volume of that product.
Each product can be picked either from the forward area or from the reserve area.
If the product has been allocated to the forward area (respectively the reserve
area) then all the picks have to be performed from the forward area (respectively
the reserve area). Several units of a product can be picked in a single pick. The
cost of picking is proportional to the number of picks.

Finally, concerning the external supply of the products, we assume that the
warehouse manager want to use an inventory control policy based on continuous
review policy (reorder point system). Therefore the order quantity is constant, the
reorder point is constant, the delivery time is fixed and the demand of the various
products during the supply lead time is probabilistic.

3.4.3 Model

The indexes used are $i : 1, \ldots, I$ to denote products and $j : 1, \ldots, J$ to denote a
number of locations in the forward area.

Next, we describe the data and variables used in the model. For each element,
we give the units of measure between brackets.

**Data:**

$\alpha_i$ : number of units of product $i$ that can be stored in a single location of the
forward area [units].

$CostRepA$ : cost of advance replenishment [euros/replenishment].

$CostRepC$ : cost of concurrent replenishment [euros/replenishment].

$PickCostF$ : picking cost in the forward area [euros/pick].

$PickCostR$ : picking cost in the reserve area [euros/pick].

$SSI$ : internal safety stock for products in the forward area which are replenished
through the reserve area [units].

$CostR$ : reception cost for the reserve area [euros/reception].

$CostF$ : reception cost for the forward area [euros/reception].

$CostCar_i$ : inventory carrying cost of product $i$ [euros/units/picking period].

$CostAcqu_i$ : acquisition cost of product $i$ [euros/units].

$CostShort_i$ : shortage cost of product $i$ [euros/units].
An Integrated Model for Warehouse and Inventory Planning

*CostCapasupp*: additional capacity cost [euros/units].

*L*: supply lead time [picking periods].

*CapaF*: capacity of the forward area [locations].

*CapaR*: capacity of the reserve area [units].

*U*<sub>*i*</sub>: random variable representing the demand of product *i* during one picking period with expected value *E[U*<sub>*i*</sub>][units].

*d*<sub>*i*</sub>: random variable representing the demand of product *i* during the supply lead time [units].

*σ*<sub>*i*</sub>: standard deviation of demand of product *i* during the supply lead time [units].

*µ*<sub>*i*</sub>: average demand of product *i* during the supply lead time [units].

*p*<sub>*i*</sub>: random variable representing the number of picks of product *i* per picking period with expected value *E[p*<sub>*i*</sub>][picks].

*δ*<sub>*ij*</sub>: expected number of concurrent replenishments of product *i* per picking period, if *j* locations are allocated to product *i* in the forward area. Then *δ*<sub>*ij*</sub> can be computed as *δ*<sub>*ij*</sub> = ∑<sub>*s=1</sub>∞ *P(U*<sub>*i*</sub> ≥ *s*(*j µ*<sub>*i*</sub> − *SSI*)) because there is one concurrent replenishment each time that (*j µ*<sub>*i*</sub> − *SSI*) units of products have been picked.

*u*<sub>*ij*</sub>: Using the definition of variable *δ*<sub>*ij*</sub>, we define *u*<sub>*ij*</sub> = *δ*<sub>*ij*</sub> − *δ*<sub>*ij*</sub>−1. It gives the increase in the expected number of replenishment of product *i* if we allocate an additional location from *j* − 1 to *j* in the forward area to product *i*.

**Variables:**

\[
x_{ij} = \begin{cases} 
1 & \text{if the product } i \text{ is supplied to the reserve area, picked from the forward area} \\
0 & \text{and if } j \text{ locations at least are allocated to product } i \text{ in the forward area} \\
0 & \text{otherwise}
\end{cases}
\]

\[
y_{i} = \begin{cases} 
1 & \text{if the product } i \text{ is supplied directly to the forward area from the suppliers} \\
0 & \text{and picked from the forward area only} \\
0 & \text{otherwise}
\end{cases}
\]

\[
z_{i} = \begin{cases} 
1 & \text{if the product } i \text{ is assigned to the reserve area and picked} \\
0 & \text{from the reserve area only} \\
0 & \text{otherwise}
\end{cases}
\]
3.4 Model formulation

\[ \text{Capasupp} = \text{number of external storage location rented [units]. (These are identical to the location in the reserve area)} \]

\[ Q_i = \text{replenishment quantity of product } i \text{ [units]} \]

\[ r_i = \text{reorder point of product } i \text{ [units]} \]

The objective function is the expected warehouse and inventory costs per picking period and is defined as follows:

\[
\min \sum_{i=1}^{I} \text{CostRepA} \times x_{i1} + \sum_{i=1}^{I} \sum_{j=1}^{J} \text{CostRepC} \times x_{ij} \times u_{ij} + \sum_{i=1}^{I} \text{CostR} \times z_i \times \frac{E(U_i)}{Q_i} + \sum_{i=1}^{I} \text{CostF} \times y_i \times \frac{E(U_i)}{Q_i} + \sum_{i=1}^{I} \text{PickCostF} \times E(p_i) \times (x_{i1} + y_i) + \sum_{i=1}^{I} \text{PickCostR} \times E(p_i) \times z_i + \text{CostCapasupp} \times \text{Capasupp} + \sum_{i=1}^{I} \text{CostCar}_i \times \left( \frac{Q_i}{2} + r_i - \mu^L_i \right) + \sum_{i=1}^{I} \text{CostAcqu}_i \times \left( \frac{E(U_i)}{Q_i} \right) \times Q_i + \sum_{i=1}^{I} \text{CostShort}_i \times \left( \frac{E(U_i)}{Q_i} \right) \times \int_{r_i}^{\infty} (d_i - r_i) f(d_i) dd_i.
\]

Concerning the warehouse costs, following our assumptions, we have taken into account the cost of advance replenishment of the forward area (3.1) and the cost of concurrent replenishment of the forward area (3.2). The cost of advance replenishments of product \( i \) occurs once per picking period if product \( i \) is assigned to the forward area (i.e., if \( x_{i1} = 1 \)). The cost of concurrent replenishment depends on the number of concurrent replenishments which occur during the picking period. The latter depends on the number of locations assigned to each product in the forward area and on the demand of each product. Therefore, we have used the definition of \( u_{ij} \) to obtain the concurrent replenishment cost as expressed in (3.2). The warehouse cost contains also picking cost in the forward area (3.4) (respectively the reserve area (3.5)) which depends on the expected number of picks during the picking period. The rental cost of the additional storage capacity is expressed in (3.6).
The traditional inventory costs are composed of inventory carrying cost (3.7), acquisition cost (3.8) and shortage cost (3.9). We have also reception costs as defined in (3.3). The reception cost depends on the location (forward or reserve area) of the product in the warehouse. The reception cost involves the warehouse \((z_i, x_i, 1)\) and the inventory \((Q_i)\) variables highlighting the link between warehouse and inventory decisions and making the objective function non linear.

Constraints :

\[ x_{ij} \leq x_{ij-1} \quad \forall i, j : j \geq 2 \quad (3.10) \]
\[ \sum_{i=1}^{I} \left( \sum_{j=1}^{J} x_{ij} \right) + \left( \frac{Q_i + r_i - \mu^L_i}{\alpha_i} \right) y_i \leq CapaF \quad (3.11) \]
\[ \sum_{i=1}^{I} \left( \left( \frac{Q_i}{2} + r_i - \mu^R_i \right) z_i + \left( \frac{Q_i}{2} + r_i - \mu^F_i \right) x_{i1} - \sum_{j=1}^{J} \alpha_i x_{ij} \right) \leq CapaR + Capasupp \quad (3.12) \]
\[ \sum_{i=1}^{I} (x_{i1} + z_i + y_i) = 1 \quad (3.13) \]
\[ Capasupp \geq 0 \quad (3.14) \]
\[ x_{ij}, y_i, z_i, Q_i, r_i \geq 0 \quad \forall i, j \quad (3.15) \]
\[ x_{ij}, y_i, z_i \in 0, 1 \quad \forall i, j \quad (3.16) \]

There are sequencing constraints (3.10) specifying that a \(j^{th}\) location can be assigned to product \(i\) only if \(j-1\) locations have already been assigned. The number of products stored in the different areas (forward and reserve area) must not exceed the total amount of space available and depends on the storage policy: (3.11) for the dedicated storage in the forward area, (3.12) in case of random storage policy in the reserve area. In addition, following the assumptions, the product can only follow one route in the warehouse (3.13). Finally, all the variables must be non negative (3.14)/(3.15).

As stated in section 3.4.2, the reserve area can, instead of being handled through a random storage policy, be managed through a dedicated storage policy. In this case, the reserve capacity constraint (3.12) will be replaced by the following:

\[ \sum_{i=1}^{I} \left( \left( \frac{Q_i}{2} + r_i - \mu^R_i \right) z_i + \left( \frac{Q_i}{2} + r_i - \mu^F_i \right) x_{i1} - \sum_{j=1}^{J} \alpha_i x_{ij} \right) \leq CapaR + Capasupp \quad (3.17) \]
3.5 Methodology

The global model composed of the warehouse and the inventory decisions and constraints presented in Section 3.4.3 is a mixed integer non linear model. Given the complexity of solving this model to optimality, our aim is to find a procedure to solve heuristically this model in order to integrate decisions concerning the inventory and warehouse fields. We propose two heuristic methods to solve this problem offering different levels of decisions integration. The first method is a heuristic sequential solution procedure. The second method gives a higher level of integration and is similar to the method used in the iterative procedure proposed by C.J. Vidal and M. Goetschalchx [79] for solving bilinear models.

3.5.1 Heuristic Sequential Solution

In this heuristic, we decompose our global model in an inventory sub-model and a warehouse sub-model. Those two sub-models are solved sequentially: first the inventory sub-model is solved and then the optimal value of the inventory variables are fixed and used to solve the warehouse sub-model. The inventory sub-model is obtained by eliminating from our global model costs and constraints related to the warehouse problem. We obtain a non linear inventory model with storage and inventory constraints. In this sub-model, warehouse variables still appear in order to model the ordering cost and the capacity constraints. To render this inventory sub-model independent of warehouse decisions, the reception costs are approximated and the two capacity constraints are relaxed into one global capacity constraint. We obtain a linear mixed integer multi-item inventory control model with one capacity constraint. In order to solve this inventory sub-model, we dualize the inventory and storage constraints and obtain the lagrangian function of this inventory control model. As the problem is convex, the Karush-Kuhn-Tucker Optimality Conditions are sufficient to solve this problem. The optimal values of the inventory variables are then used to solve the warehouse sub-model. This warehouse sub-model is obtained by eliminating costs and constraints related to the inventory problem in the global model and by fixing the value of the inventory variables to the value obtained when solving the inventory sub-model. We obtain a mixed integer model where the two capacity constraints (one for each warehouse area) are considered in order to obtain a feasible solution. We solve this model with a Branch&Bound procedure.

Hereafter, we present in more details each of the sub-models and the solution
methodology applied to solve them.

The inventory sub-model (i.e., the original model without costs and constraints related to the warehouse problem) is a multi-item inventory control model with two capacity constraints defined by the minimization of inventory costs (reception inventory, carrying, shortage costs) under inventory and storage capacity constraints. The formulation of the inventory sub-model is therefore:

$$\text{min} \sum_{i=1}^{I} \text{CostCar}_i \times \left( \frac{Q_i}{2} + r_i - \mu_i L \right) + \sum_{i=1}^{I} \text{CostAcqu}_i \times \left( \frac{E(U_i)}{Q_i} \right) \times Q_i$$

$$+ \sum_{i=1}^{I} \text{CostShort}_i \times \left( \frac{E(U_i)}{Q_i} \right) \times \int_{r_i}^{\infty} (d_i - r_i) f(d_i) dd_i$$

$$+ \sum_{i=1}^{I} \text{CostR} \times z_i \times \frac{E(U_i)}{Q_i} + \sum_{i=1}^{I} \text{CostR} \times x_i \times \frac{E(U_i)}{Q_i} + \sum_{i=1}^{I} \text{CostF} \times y_i \times \frac{E(U_i)}{Q_i}$$

$$+ \text{CostCapasupp} \times \text{Capasupp}$$

under the constraints

$$\sum_{i=1}^{I} \left( \sum_{j=1}^{J} x_{ij} \right) + \left( \frac{Q_i + r_i - \mu_i L}{\alpha_i} \right) y_i \leq \text{CapaF}$$

$$\sum_{i=1}^{I} \left( \frac{Q_i}{2} + r_i - \mu_i L \right) z_i + \left( \frac{Q_i}{2} + r_i - \mu_i L \right) x_i - \sum_{j=1}^{J} \alpha_i x_{ij} \leq \text{CapaR} + \text{Capasupp}$$

$$\text{Capasupp} \geq 0$$

$$x_{ij}, y_i, z_i, Q_i, r_i \geq 0 \quad \forall i, j$$

$$x_{ij}, y_i, z_i \in 0, 1 \quad \forall i, j$$

This is a non linear mixed integer model where the warehouse variables still appear in order to model the ordering/reception costs and capacity constraints. To render this model independent of the warehouse decisions variables, we will perform an approximation and a relaxation.

First of all we approximate the objective function. The reception cost, which depends on the routes taken by the products, is approximated by the following

1The constraint concerning the storage capacity of the reserve area depends on the storage policy of the reserve. We will develop in this section the methodology concerning the random storage policy. The methodology is easily adaptable in case of dedicated storage policy.
3.5 Methodology

\[ \sum_{i=1}^{I} \text{CostRecp} \times \frac{E(U_i)}{Q_i} \]

where CostRecp is the cost of reception which is independent on the route taken by the product and is defined as an average of the historical reserve and forward reception cost.

Secondly, we relax the capacity constraints (reserve and forward constraints). By this, we mean that instead of having two capacity constraints for the different areas in the warehouse, we will consider only one global capacity constraint for the entire warehouse. This global capacity constraint is the aggregation of the forward and the reserve capacity constraint, and is defined as follows:

\[ \sum_{i=1}^{I} (Q_i + r_i - \mu_i^L) \leq \text{Capa} + \text{Capasupp} \]

The new value \( \text{Capa} \) is the global aggregated warehouse capacity and is defined as the sum of \( \pi \text{CapaF} \) and \( \text{CapaR} \), where \( \pi \) is the average historical number of products in one location of the forward area.

The objective function so obtained is independent of the routes taken by the various products and therefore is independent of the warehouse decisions taken. The global capacity constraint is also independent of the routes taken by the different products in the warehouse. Nevertheless, the ordering quantity and reorder point of each product will be dependent on the amount of space globally available in the warehouse but not on the size of the different areas in the warehouse, the number of locations allocated to each product in the forward area and on the routes of the various products. This inventory model is therefore integrating a decision from inventory and warehouse fields (through the global capacity constraint).

By dualizing the global capacity constraint with multiplier \( \lambda \) and the additional capacity non-negativity constraint with multiplier \( \mu \), we obtain the lagrangian of this multi product inventory model with three unknown elements, \( Q_i \), \( r_i \) for all \( i=1..I \) and \( \text{CapaSupp} \) and no constraint:
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\[ L(\lambda, \mu) = \min \sum_{i=1}^{I} \text{CostCar}_i \times \left( \frac{Q_i}{2} + r_i - \mu L_i \right) \]

\[ + \sum_{i=1}^{I} \text{CostAcqu}_i \times \left( \frac{E(U_i)}{Q_i} \right) \times Q_i \]

\[ + \sum_{i=1}^{I} \text{CostShort}_i \times \left( \frac{E(U_i)}{Q_i} \right) \times \int_{r_i}^{\infty} (d_i - r_i) f(d_i) dd_i \]

\[ + \sum_{i=1}^{I} \text{CostRecep} \times \frac{E(U_i)}{Q_i} \]

\[ + \text{CostCapasupp} \times \text{Capasupp} \]

\[ - \lambda \times \left( \text{Capa} + \text{Capasupp} - \left( \sum_{i=1}^{I} (Q_i + r_i - \mu L_i) \right) \right) \]

\[ - \mu \times \text{Capasupp} \]

The first order necessary conditions are used to derive the optimal value for the ordering quantity \((Q_i)\), the reorder point \((r_i)\) for all \(i = 1..I\) and the additional capacity \((\text{Capasupp})\) for fixed \(\lambda\) and \(\mu\)[68]. The standard necessary conditions for optimality give the following results²:

\[ Q_i = \sqrt{\frac{2 \times E(U_i) \times (\text{CostRecep} + \text{CostShort}_i \times \int_{r_i}^{\infty} (d_i - r_i) f(d_i) dd_i)}{(\text{CostCar}_i + 2 \times \lambda)}} \]  (3.18)

\[ \text{Prob}(d_i \geq r_i) = \frac{Q_i \times (\text{CostCar}_i + \lambda)}{\text{CostShort}_i \times E(U_i)} \]  (3.19)

\[ \text{CostCapasupp} - \lambda - \mu = 0 \]  (3.20)

We omit the non negativity constraints on \(Q_i\) and \(r_i\) because they are implied by the first order conditions. The complementary slackness and feasibility conditions are used to determine the optimal value of the additional capacity \((\text{Capasupp})\) and the Lagrangian multipliers \((\lambda, \mu)\):

²In the rest of the paper, for notational convenience, the optimal value of the variables will be indicated by an upper bar
3.5 Methodology

\[ \lambda \times \left( \text{Capa} + \text{Capasupp} - \left( \sum_{i=1}^{I} (Q_i + \tau_i - \mu_i^L) \right) \right) = 0 \] (3.21)

\[ \mu \times \text{Capasupp} = 0 \] (3.22)

\[ \sum_{i=1}^{I} (Q_i + \tau_i - \mu_i^L) \leq \text{Capa} + \text{Capasupp} \] (3.23)

\[ \text{Capasupp} \geq 0 \] (3.24)

\[ \mu \geq 0 \] (3.25)

\[ \lambda \geq 0 \] (3.26)

By combining the necessary optimality condition (3.20), the complementary slackness condition (3.22) and the feasibility condition (3.25) we obtain:

\[ (\text{CostCapasupp} - \lambda) \times \text{Capasupp} = 0 \] (3.27)

\[ \lambda \leq \text{CostCapasupp} \] (3.28)

which replaces (3.20), (3.22) and (3.25).

The resulting first order necessary conditions are defined as:

\[ Q_i = \sqrt{2 \times \left( \frac{E(U_i) \times (\text{CostRecp} + \text{CostShort} \times \int_{r_i}^{\infty} (d_i - \tau_i) f(d_i) dd_i)}{(\text{CostCar}_i + 2 \times \lambda)} \right)} \] (3.29)

\[ \text{Prob}(d_i \geq \tau_i) = \frac{Q_i}{\text{CostCar}_i + \lambda} \] (3.30)

\[ \lambda \times \left( \text{Capa} + \text{Capasupp} - \left( \sum_{i=1}^{I} (Q_i + \tau_i - \mu_i^L) \right) \right) = 0 \] (3.31)

\[ \sum_{i=1}^{I} (Q_i + \tau_i - \mu_i^L) \leq \text{Capa} + \text{Capasupp} \] (3.32)

\[ 0 \leq \lambda \leq \text{CostCapasupp} \] (3.33)

\[ \text{Capasupp} \geq 0 \] (3.34)

\[ (\text{CostCapasupp} - \lambda) \times \text{Capasupp} = 0 \] (3.35)

To find all possible solutions to (3.29) - (3.35), we must distinguish three possible cases:
1. $\lambda = 0$

we obtain the following system of equations:

$$Q_i = \sqrt{\frac{2 \times E(U_i) \times (\text{CostRecp} + \text{CostShort}_i \times \int_{r_i}^{\infty} (d_i - r_i) f(d_i) dd_i)}{\text{CostCar}_i}}$$

$$\text{Prob}(d_i \geq r_i) = \frac{Q_i \times (\text{CostCar}_i)}{\text{CostShort}_i \times E(U_i)}$$

$$\text{Capasupp} = 0$$

$$\sum_{i=1}^{l} (Q_i + r_i - \mu_i^L) \leq \text{Capa}$$ (3.36)

If no optimal value for $Q_i$ and $r_i$ can be found so that constraint (3.36) is satisfied then there is no solution with $\lambda=0$. If constraint (3.36) is satisfied, we have a solution to the lagrangian.

2. $\lambda > 0$

We distinguish two sub-cases:

(a) $\lambda = \text{CostCapasupp}$

we obtain the following system of equations:

$$Q_i = \sqrt{\frac{2 \times E(U_i) \times (\text{CostRecp} + \text{CostShort}_i \times \int_{r_i}^{\infty} (d_i - r_i) f(d_i) dd_i)}{\text{CostCar}_i + 2 \times \text{CostCapasupp}}}$$

$$\text{Prob}(d_i \geq r_i) = \frac{Q_i \times (\text{CostCar}_i + \text{CostCapasupp})}{\text{CostShort}_i \times E(U_i)}$$

$$\text{Capasupp} \geq 0$$ (3.37)

$$\sum_{i=1}^{l} (Q_i + r_i - \mu_i^L) \leq \text{Capa} + \text{Capasupp}$$ (3.38)

If no optimal value for $Q_i$ and $r_i$ can be found so that constraint (3.37) and (3.38) are satisfied then there is no solution with $\lambda = \text{CostCapasupp}$. If constraint (3.37) and (3.38) are satisfied, we have a solution to the lagrangian.

(b) $0 < \lambda < \text{CostCapasupp}$
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In this case, we have that $\overline{Capasupp} = 0$ and we obtain a system composed of three equations (the three necessary conditions for optimality ([68],[59])) with three unknown elements ($Q_i$, $r_i$ and $\lambda$).

$$Q_i = \sqrt{\frac{2 \times E(U_i) \times (CostRep + CostShort_i \times \int_{\tau_i}^{\infty} (d_i - \tau_i) f(d_i) dd_i)}{(CostCar_i + 2 \times \lambda)}}$$

$$Prob(d_i \geq \tau_i) = \frac{Q_i \times (CostCar_i + \bar{X})}{CostShort_i \times E(U_i)}$$

$$\sum_{i=1}^{I} (Q_i + \tau_i - \mu_i) = Capa$$

(3.39)

Those three possible solution cases will be analyzed, a possible solution will be calculated and the best one will be selected (i.e. the one which minimizes the objective function of the first sub-problem).

When the optimal solution of the inventory model is obtained, the optimal order and reorder quantities are used to solve the warehouse model. The resulting warehouse model (where the values of the inventory variables ($Q_i$, $r_i$ for all $i = 1...I$) are fixed based on the solution of the inventory model with one capacity constraint) is a mixed integer problem which is solved using a Branch-and-Bound procedure. The warehouse model is defined as follows:

$$\min \sum_{i=1}^{I} CostRepA \times x_{i1}$$

$$+ \sum_{i=1}^{I} \sum_{j=1}^{J} CostRepC \times x_{ij} \times u_{ij}$$

$$+ \sum_{i=1}^{I} CostR \times z_i \times \frac{E(U_i)}{Q_i} + \sum_{i=1}^{I} CostR \times x_{i1} \times \frac{E(U_i)}{Q_i} + \sum_{i=1}^{I} CostF \times y_i \times \frac{E(U_i)}{Q_i}$$

$$+ \sum_{i=1}^{I} PickCostF \times E(p_i) \times (x_{i1} + y_i)$$

$$+ \sum_{i=1}^{I} PickCostR \times E(p_i) \times z_i$$

$$+ CostCapasupp \times Capasupp$$
under the following constraints:

\[
x_{ij} \leq x_{ij-1} \quad \forall i, j : j \geq 2
\]

\[
\sum_{i=1}^{I} \left( \sum_{j=1}^{J} x_{ij} \right) + \left( \frac{Q_i + r_i - \mu^L_i}{\alpha_i} \right) y_i \leq \text{CapaF}
\]

\[
\sum_{i=1}^{I} \left( \frac{Q_i}{2} + r_i - \mu^L_i \right) z_i + \left( \frac{Q_i}{2} + r_i - \mu^L_i \right) x_{i1} - \sum_{j=1}^{J} \alpha_i x_{ij} \leq \text{CapaR} + \text{Capasupp}
\]

\[
\sum_{i=1}^{I} (x_{i1} + z_i + y_i) = 1
\]

\[
\text{CapaSupp} \geq 0
\]

\[
x_{ij}, y_i, z_i \geq 0 \quad \forall i, j
\]

\[
x_{ij}, y_i, z_i \in \{0, 1\} \quad \forall i, j
\]

In the warehouse model, the two capacity constraints are taken into account in order to obtain a feasible solution to the warehouse model and the optimal value of the additional capacity (CapaSupp) is reoptimized.

### 3.5.2 Integrated Heuristic method

According to C.J. Vidal and M. Goetschalch [79], global optimization for bi-linear problems is only possible for small instances. Medium and large scale supply chain problems such as warehouse planning and inventory management problems need to be optimized through a heuristic approach. They propose a heuristic based on an iterative solution of the two linear sub-problems. We use the same heuristic approach to solve our non-linear MIP model. In this heuristic, as in the sequential heuristic, we decompose our global model in two sub-models: an inventory sub-model and a warehouse sub-model. In the previous heuristic, those two sub-models were solved sequentially whereas in this method they are solved in an iterative manner. The inventory sub-model is composed of costs and constraints related to the inventory decisions. The value of the warehouse variables in this sub-model is fixed by the values of those variables obtained at the previous iteration. We obtain a linear mixed integer multi-item inventory control model with two capacity constraints, one for each area in the warehouse. Those inventory constraints are dualized and the corresponding lagrangian is solved. The warehouse sub-model is composed of costs and constraints related to the warehouse decisions. The value of the inventory variables are fixed by the value of those variables obtained at the
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previous iteration. We obtain the same mixed integer warehouse model as the one in the sequential heuristic. This iterative procedure is run for a limited number of iteration until the value of the warehouse and inventory cost stay stable. Hereafter, we present with more details each of the sub-models and the solution methodology applied to solve them.

The first sub-problem is composed of the inventory variables and constraints and the values of the warehouse variables is fixed (by the value obtained at the previous iteration). Consequently, the ordering cost and the location of the products inside the warehouse is fixed by the value of the warehouse variables. We obtain a classical multi product inventory model with two capacity constraints for each area in the warehouse (forward and reserve area).

The objective function of the first inventory sub-problem is :

\[
\min \sum_{i=1}^{I} \text{CostCar}_i \times \left( \frac{Q_i}{2} + r_i - \mu_i^L \right) \\
+ \sum_{i=1}^{I} \text{CostAcqu}_i \times \left( \frac{E(U_i)}{Q_i} \right) \times Q_i \\
+ \sum_{i=1}^{I} \text{CostShort}_i \times \left( \frac{E(U_i)}{Q_i} \right) \times \int_{r_i}^{\infty} (d_i - r_i) f(d_i) dd_i \\
+ \sum_{i=1}^{I} \text{CostR} \times \frac{E(U_i)}{Q_i} + \sum_{i=1}^{I} \text{CostF} \times \frac{E(U_i)}{Q_i} \\
+ \text{CostCapasupp} \times \text{Capasupp}
\]

Under the constraints :

\[
\sum_{i=1}^{I} \left( \sum_{j=1}^{J} x_{ij} \right) + \left( \frac{Q_i + r_i - \mu_i^L}{\alpha_i} \right) \frac{1}{\text{Capasupp}} \leq \text{CapaF} \\
\sum_{i=1}^{I} \left( \frac{Q_i}{2} + r_i - \mu_i^L \right) \frac{1}{\text{Capasupp}} + \sum_{j=1}^{J} \frac{Q_i^2}{2} + r_i - \mu_i^L \frac{1}{\text{Capasupp}} - \sum_{j=1}^{J} \alpha_i x_{ij} \right) \leq \text{CapaR} + \text{Capasupp}
\]

\[
\text{Capasupp} \geq 0 \\
Q_i, r_i \geq 0 \ \forall i, j
\]

By dualizing the two capacity constraints with multipliers \( \lambda^F \) (forward capacity constraint) and \( \lambda^R \) (reserve capacity constraint) and the non-negativity additional
capacity constraint with multiplier $\mu$, the lagrangian can be written as follows:

$$L(\lambda^F, \lambda^R, \mu) =$$

$$\min \sum_{i=1}^I \text{CostCar}_i \times \left( \frac{Q_i}{2} + r_i - \mu^L_i \right)$$

$$+ \sum_{i=1}^I \text{CostAcqu}_i \times \left( \frac{E(U_i)}{Q_i} \right) \times Q_i$$

$$+ \sum_{i=1}^I \text{CostShort}_i \times \left( \frac{E(U_i)}{Q_i} \right) \int_{r_i}^\infty (d_i - r_i) f(d_i) dd_i$$

$$+ \sum_{i=1}^I \text{CostR} \times \pi \times \frac{E(U_i)}{Q_i} + \sum_{i=1}^I \text{CostF} \times y_i \times \frac{E(U_i)}{Q_i}$$

$$+ \text{CostCapasupp} \times \text{Capasupp}$$

$$- \lambda^F \times \left( \text{CapaF} - \left( \sum_{i=1}^I \left( \sum_{j=1}^J \alpha_i x_{ij} \right) + \left( \frac{Q_i + r_i - \mu^L_i}{\alpha_i} \right) \frac{1}{y_i} \right) \right)$$

$$- \lambda^R \times \left( \text{CapaR} + \text{Capasupp} - \left( \sum_{i=1}^I \left( \frac{Q_i}{2} + r_i - \mu^L_i \right) \pi_i + \left( \frac{Q_i}{2} + r_i - \mu^L_i \right) \pi_{i1} - \sum_{j=1}^J \alpha_i x_{ij} \right) \right)$$

$$- \mu \times \text{Capasupp}$$

First of all, as we have done in the sequential heuristic solution procedure, the necessary optimality condition on the variable $\text{Capasupp}$ and the complementary slackness and feasibility conditions on the additional capacity non-negativity constraint can be used to derive the different possible values of the variable $\text{Capasupp}$ and the lagrangian multipliers $\mu$ and $\lambda^R$.

$$\text{CostCapasupp} - \lambda^R - \bar{\mu} = 0 \quad (3.40)$$

$$\bar{\mu} \times \text{Capasupp} = 0 \quad (3.41)$$

$$\bar{\mu} \geq 0 \quad (3.42)$$

$$\text{Capasupp} \geq 0 \quad (3.43)$$

From the above equations, we can eliminate $\mu$ by substitution using 3.40 and we derive the following:

$$\text{Capasupp} \geq 0 \quad (3.44)$$

$$(\text{CostCapasupp} - \lambda^R) \times \text{Capasupp} = 0 \quad (3.45)$$

$$0 \leq \lambda^R \leq \text{CostCapasupp} \quad (3.46)$$

This system (3.45) and (3.46) gives the different possible value for $\lambda^R$ (and consequently $\mu$) and $\text{Capasupp}$ which solve the lagrangian. We can then write the
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first order necessary optimality conditions defining the optimal order quantity and reorder point for each product for a fixed value of $\lambda^R$ and $\lambda^F$:

$$Q_i = \sqrt{\frac{2 \times E(U_i) \times \text{CostR} \times \pi_i + \text{CostF} \times \pi_i + \text{CostShort}_i \times \int_{\pi_i}^{\infty} (d_i - \pi_i) f(d_i) dd_i}{2 \times \frac{\lambda^F}{\alpha_i} \times \pi_i + \lambda^R \times \pi_i}}$$

(3.47)

$$\text{prob}(d_i \geq \pi_i) = \frac{Q_i}{\text{CostShort}_i \times E(U_i)} \times \left(\frac{\lambda^F}{\alpha_i} \times \pi_i + \lambda^R \times \pi_i + \lambda^R \times \pi_i\right)$$

(3.48)

In order to solve (3.44) - (3.48), we must distinguish three possible situations:

1. $\lambda^R = 0$

   We obtain a lagrangian where one capacity constraint (forward capacity constraint) has been dualized. This lagrangian can be solved by the same methodology used in the sequential heuristic procedure.

2. $\lambda^R > 0$

   (a) $\lambda^R = \text{CostCapasupp}$

   As the value of $\lambda^R$ is fixed, we obtain again a lagrangian with one capacity constraint (forward capacity constraint) which can be solved based on the sequential heuristic procedure.

   (b) $0 < \lambda^R < \text{CostCapasupp}$

   The optimal value of the lagrangian multipliers ($\lambda^R$, $\lambda^F$) will be determined by lagrangian relaxation where the two capacity constraints (forward and reserve constraints) will be dualized. The optimal value of the two lagrangian multipliers will be found by applying the subgradient optimization algorithm on the lagrangian dual. The resulting lagrangian dual is as follows:

   $$W_{LD} = \text{Max} \{ L(\lambda^F, \lambda^R) : \lambda^F, \lambda^R \geq 0 \}$$

   Then, the optimal order quantity and reorder point will be calculated with the system (3.47) and (3.48) for fixed values of $\lambda^R$ and $\lambda^F$.

   We have decided that, due to the non monotonicity of the objective function, we will stop the subgradient optimization phase when we encounter ten iterations.
without an improvement in the solution obtained.

In the second sub-problem, the warehouse variables will be optimized taking into account the warehouse capacity, allocation and the routing constraints while the values of the inventory variables will be fixed. The model obtained is a mixed integer problem where the reorder point and the ordering quantity are fixed (at the value obtained at the previous iteration). This problem is solved by Branch & Bound. This mixed integer model is the same as the one used to solve the model with the sequential heuristic procedure. Those two sub-problems are solved iteratively (Figure 3.1), where the output of one of the sub-problem will become the input of the other sub-problem at the next iteration.

We decide to stop the iterative process after a limited number of iterations where the best inventory and warehouse cost has been recorded at each iteration. Indeed, this stopping criterion is based on two facts. First of all, we want to keep a reasonable computing time. Secondly, we have observed, in preliminary tests, that the improvement in warehouse and inventory cost was occurring in the first steps of the iterative process.

This procedure offers a higher level of integration of warehouse and inventory decisions because we do not only take into account the size of the warehouse in the inventory model but also the size of the different areas in the warehouse, the routes taken by the products and the size of the location inside each areas of the warehouse.

3.6 Computational results

Our aim in this section is to prove the value of integrating tactical warehouse and inventory decisions.

The various solution methodologies presented in the previous section vary according to the level of integration of inventory and warehouse decisions. Indeed, the integrated heuristic offers a higher level of integration of inventory and warehouse decisions than the sequential heuristic. Those solution methodologies are going to be implemented and tested on a real world database. This will allow us to prove the value of integrating warehouse and inventory decisions by analyzing the reduction in warehouse and inventory costs obtained by using a more integrated
3.6 Computational results

In order to highlight even more the importance of integrating warehouse and inventory decisions, the sequential and integrated heuristic solution are benchmarked against the traditional uncapacitated heuristic. This uncapacitated heuristic deals with inventory and warehouse decision in an uncoordinated manner: inventory decisions are taken without any warehouse considerations. Therefore, an uncapacitated inventory model is solved for each item, where the ordering quantity and the reorder point is calculated without taking into account the size of the warehouse, the size of the different areas in the warehouse and the routes of the products. The inventory decisions are taken independently of the warehouse decisions, based on ordering and inventory costs trade-offs. Once the uncapacitated multi items inven-
ory control model is solved, the warehouse model is solved based on the value of the inventory variables found. This warehouse model is the same as the one used for the sequential heuristic procedure and the integrated method.

### 3.6.1 Database

The evaluation of the heuristics proposed in Section 3.5 will be realized through tests which will be performed on 400 products coming from real world data of the retail sector. In order to be able to implement the heuristics, some information is needed concerning the products. Concerning the demands of the products, we have assumed that picking periods demand of product $i$ in the warehouse follows a normal distribution $N(\mu_i, \sigma_i)$. This assumption was made because, in order to derive the demand probability distribution, we needed the histogram of the expeditions per picking period and this information was not available. In addition, those products are delivered regularly which supports the assumption.

Secondly, the inventory model that we have considered is a continuous review, multi-item reorder point with lost sales. This lost sales assumption is made because, in case of shortage, an urgent order is expedited from another warehouse (so the order is lost for the warehouse under study) in order to obtain a 100% service level.

The different costs composing the objective function were not available in the company. Those costs have been fixed according to the assumptions made in the model description.

### 3.6.2 Description of the computational results

For the presentation of the results obtained with the three different methods (sequential, integrated and uncapacitated heuristic) we give the difference in cost (expressed in percentage) when we use a more integrated method. For ease of presentation, we are going to introduce two new definitions:

- First integration savings
  
The improvement obtained in the inventory and warehouse costs when we

---

3 For more information on the dataset see the appendix
4 A sensitivity analysis is performed in Section 3.6.3 in order to analyse the impact of changes in the objective function costs
3.6 Computational results

use, on the same dataset, with the same capacity and storage policy, the heuristic sequential procedure instead of the uncapacitated procedure. This improvement will be expressed in percentage. A positive value for the improvement means an improvement in cost (decrease in cost) when using the sequential procedure instead of the uncapacitated procedure.

- Second integration savings

The improvement obtained in the inventory and warehouse costs when we use, on the same dataset, with the same capacity and storage policy, the integrated procedure instead of the uncapacitated procedure. This improvement will be expressed in percentage. A positive value for the improvement means an improvement in cost (decrease in cost) when using the integrated procedure instead of the uncapacitated procedure.

Unlimited reserve capacity

When there is unlimited capacity in the warehouse (unlimited reserve area, but limited size of the forward area), the sequential heuristic procedure and the uncapacitated method give, as expected, the same results for the inventory and the warehouse decisions. Indeed, the only difference between the two models is that the sequential heuristic procedure takes into account a global capacity constraint which is redundant when there is enough capacity in the warehouse. We can conclude that the first integration gives 0% improvement in cost. The only improvement in warehouse and inventory cost is observed when the integrated procedure is used. Therefore, Tables 3.1 and 3.2 show the results for the second integration.

<table>
<thead>
<tr>
<th>Warehouse Cost Savings (%)</th>
<th>Replen. cost &amp;recep.cost</th>
<th>Res. Pick. &amp;recep.cost</th>
<th>For. Pick. &amp;recep.cost</th>
<th>Total War. cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Second integ.</td>
<td>0.877</td>
<td>-0.584</td>
<td>1.960</td>
<td>0.989</td>
</tr>
</tbody>
</table>

Table 3.1: Unlimited capacity warehouse results (expressed in percentage).

From Table 3.2, we observe that the reception cost has decreased and that the storage and shortage cost have increased. Those observations can be explained by the increase in the optimal order quantities. This increase is a consequence of using more appropriate reception cost in the integrated procedure than in the sequential
heuristic procedure (see Section 3.5.1). Concerning the shortage cost, an increase in the order quantity implies a decrease in the number of replenishment cycles. Nevertheless, this decrease is compensated by a decrease in the safety factor which increases the expected shortage per replenishment cycle.

In addition, the increase in the optimal order quantities has an impact on the optimal routes of products in the warehouse. Indeed, some of the products which were supplied directly to the forward area must be supplied through the reserve area because there is not enough space in the forward area. Consequently, more product are picked from the reserve area which decreases the replenishment cost and decreases the forward reception and picking cost (see Table 3.1).

Concerning the capacity utilization in the warehouse, the amount of space used in the reserve area has increased due to the shift in routes. The forward capacity is used totally and better with the integrated procedure because each location in this area is filled totally.

**Limited warehouse capacity**

In the case of limited warehouse capacity (in the forward and in the reserve area), inventory and warehouse savings can be obtained by using the first and the second integration.

<table>
<thead>
<tr>
<th></th>
<th>Warehouse Cost Savings (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Replen. cost</td>
</tr>
<tr>
<td>First integ.</td>
<td>18.056</td>
</tr>
<tr>
<td>Second integ.</td>
<td>15.641</td>
</tr>
</tbody>
</table>

Table 3.3: Limited capacity warehouse results (expressed in percentage).
Table 3.4: Limited capacity inventory results (expressed in percentage).

The improvement obtained by the first integration
As there is limited space in the reserve area, a reduction in the optimal order quantity is observed. This observation can be explained by two facts. First of all, the solution methodology used in the sequential heuristic. Indeed, the mathematical model of the sequential heuristic solution is solved using a lagrangian relaxation procedure: the global capacity constraint is associated to a lagrangian multiplier and is dualized in the objective function (see Section 3.5.1). This lagrangian multiplier can be interpreted as an implicit storage cost. Therefore, when there is not enough capacity in the warehouse, the lagrangian multiplier increases which increases the carrying cost (see equation 3.29 in Section 3.5.1). This results in a decrease in the order quantity. Secondly, we notice that the reduction in the order quantity is drastic. This is explained by the way in which the global capacity constraint in the sequential heuristic solution is constructed (see Section 3.5.1). Indeed, the global capacity constraint is based on a dedicated storage policy which is more restrictive than the real capacity constraints.

Firstly, from Table 3.4, this order quantity reduction implies higher reception and shortage cost. Nevertheless, this increase in reception and shortage cost is compensated by the decrease in carrying cost and mostly the decrease of additional capacity cost. Also observe that the carrying becomes so expensive (the lagrangian multiplier increase because of limited capacity) that it is more interesting to have higher shortage cost (reducing service level).

Secondly, this reduction in the order quantity implies that some product can be supplied directly in the forward area (when the reduction in the order quantity allows it) instead of being supplied through the reserve area. Consequently, we observe, from Table 3.3, a drastic decrease in the cost of renting additional capacity and replenishment cost. Those reductions are compensated by increases in the forward and reserve reception and picking costs. This is explained by the use of a global capacity constraint in the
sequential heuristic solution which does not allow to represent appropriately the available capacity in the warehouse.

The improvement obtained by the second integration.

With the integrated procedure, the amount of products allocated to each location in the forward area is optimized. This results in a decrease in the order quantity compared to the uncapacitated method. Therefore, we observe from Table 3.4 an increase in the reception cost and shortage cost combined with a decrease in the carrying cost. This decrease in the order quantity is less important than the one obtained with the first integration because the capacity constraints are more adequately represented in the integrated method. Consequently, the available space in the warehouse is more appropriately used.

In addition, the reduction in the optimal order quantity leads to a reoptimization of the routes in the warehouse. This results in more direct reception through the forward area and less reception through the reserve area. Therefore, from Table 3.3, we observe lower forward picking and reception costs and higher reserve picking and reception costs. Consequently, the replenishment cost decreases compared to the uncapacitated method.

Value of integration

<table>
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<tr>
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</thead>
<tbody>
<tr>
<td>First integ.</td>
<td>8.817</td>
<td>0</td>
</tr>
<tr>
<td>Second integ.</td>
<td>12.022</td>
<td>0.079</td>
</tr>
</tbody>
</table>

Table 3.5: Total savings (in percentage) with limited and unlimited capacity.

Table 3.5 summarizes the percentage of total savings which can be obtained by integrating more decisions of the warehouse and inventory fields. The amount of savings which can be achieved depends on the capacity available in the warehouse. When there is enough capacity, only the integrated model allows one to improve the inventory and warehouse costs due to a better management of the different locations in the forward area. This involves a reoptimization of the routes. Never-
theless, the amount of savings realized is relatively small compared to the savings obtained in the case of limited capacity. When the capacity available in the warehouse is limited, the major part of the savings is incurred by using the heuristic sequential procedure. This is mainly due to the decrease in additional storage capacity rented. With the integrated method, space is better utilized than when the heuristic sequential method.

3.6.3 Sensitivity analysis

The computational tests have been performed without having the real value of the different cost coefficients composing the objective function. In addition, in business practice, changes in the objective function cost coefficients can occur. For example, changes in the productivity of the pickers which will influence the picking cost or changes in the carrying cost due to changes in the value of the product [72]. Therefore, enterprises may be interested in knowing if those changes will have an impact on the optimal inventory and warehouse configuration (i.e. optimal order quantity, reorder point and optimal routes for each product).

In order to answer those questions, a sensitivity analysis is performed. This sensitivity analysis will be performed by testing various scenarios under limited and unlimited reserve capacity. The scenarios will differ from one another by the variation applied to the warehouse and inventory cost coefficients. In order to have results which can be interpreted, the variation applied to the different cost coefficients will be no more than 20%.

Warehouse sensitivity test

The first sensitivity test concerns the impact of changes in the warehouse cost coefficients on the total savings and on the warehouse and inventory configuration. There exists a dependency between the warehouse cost coefficients. Therefore, the value of the warehouse objective function cost coefficients have been fixed according to their definition and significance.

The warehouse objective function is composed of forward and reserve reception costs, forward and reserve picking costs and advanced and concurrent replenishment costs. We know that the forward picking cost is lower than the reserve picking cost. Indeed the forward area is a smaller area than the reserve area where the picking activity can be performed faster (cheaper) than in the reserve area.

5The appendix gives a complete description of the different scenarios
On the other hand, the reserve reception cost is lower than the forward reception cost because the replenishment of the reserve area can be technically achieved more easily than in the forward area (by larger sizes or full pallets). Concerning the replenishment activity, advance replenishment is less costly than concurrent replenishment because this activity does not result in creating congestion. Those relations are important otherwise it would not be interesting to have a forward and a reserve area. In addition, the assumptions made on our model would not be valid. (see Section 3.4.2 for more details)

Therefore, in the sensitivity analysis, we are interested in the impact on the warehouse and inventory cost and on the inventory and warehouse configuration of the relative cost difference between the forward and reserve reception costs, the forward and reserve picking cost and the advance and concurrent replenishment costs. Therefore, we have three relative cost differences in our first sensitivity scenario, each taking three possible values. For each of these twenty seven sensitivity scenarios, we have decided to consider two extreme values.

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>First integ.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Min</td>
<td>0</td>
<td>8.67</td>
</tr>
<tr>
<td>average</td>
<td>0</td>
<td>8.90</td>
</tr>
<tr>
<td>Max</td>
<td>0</td>
<td>9.10</td>
</tr>
<tr>
<td>Second integ.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Min</td>
<td>0.06</td>
<td>11.00</td>
</tr>
<tr>
<td>average</td>
<td>0.096</td>
<td>13.89</td>
</tr>
<tr>
<td>Max</td>
<td>0.16</td>
<td>15.60</td>
</tr>
</tbody>
</table>

Table 3.6: Results of the warehouse sensitivity analysis.

Table 3.6 gives the minimum, average and maximum total savings, in percentage, obtained with the different sensitivity scenarios (i.e. by varying the value of the warehouse objective function cost coefficients) and with different capacity limits. Most of the difference in the total savings (except in the case of the second integration with limited capacity) are relatively stable with no more than 1% difference between the maximum and minimum total savings for a certain capacity limit and a certain type of integration. By analyzing in more details the results obtained with the different sensitivity scenarios, we observe that the optimal order quantity and reorder point do not change from one scenario to the other. Only in
3.6 Computational results

The case of limited capacity, we can observe that the routes taken by the various products are different from one sensitivity scenario to the other. An exception can be observed in the case of limited capacity when the integrated heuristic solution procedure is used. In this case, we can observe a difference of more or less 4% between the minimum and maximum total savings realized. This observation can be explained by two remarks. First of all, the methodology used in the heuristic integrated procedure. Indeed, in order to solve the inventory model, we have developed a procedure which uses the subgradient algorithm (see Section 3.5.2 for more details). As said in Section 3.5.2, the subgradient algorithm will be stopped after a finite number of iteration without improvement in the value of the objective function. This stopping criteria does not guarantee that the lagrangian relaxation of the inventory problem is solved to optimality which can explain the result obtained with the different sensitivity scenarios. Secondly, with the heuristic integrated solution procedure, the changes in the forward and reserve reception cost are taken into account at the inventory and the warehouse level. Indeed, this change is updating the forward and reserve reception cost and the reserve and forward capacity constraints in the inventory model during the successive iterations. Consequently, minor changes of the optimal ordered and reorder point (sometime no changes are remarked) are observed combined with changes in the optimal route for each product.

Inventory and warehouse sensitivity test

The second sensitivity test that we have considered is the relative importance of the total inventory cost compared to the total warehouse cost (and conversely). The aim is to analyze the impact of those changes on the value of the total savings realized with the first and the second integration and on the warehouse and the inventory configuration (i.e. the optimal order quantity, reorder point and route for each product). During computational tests, we have observed that there is no impact on the optimal order quantity, optimal reorder point and the optimal routes for each product but there is an impact on the costs. Table 3.7 gives the minimum, average and maximum impact, in percentage, on the total savings realized with the first and the second integration.
Inventory sensitivity test

The last sensitivity test analyses the impact on the inventory, warehouse configuration and on the total savings of changes in the inventory objective function cost coefficients. This sensitivity analysis is important. On the one hand, changes in the inventory carrying cost and acquisition cost can occur because the value of the product changes (e.g. reevaluation of taxes, insurance ...). On the other hand, the shortage and carrying cost is difficult to estimate [72].

Computational tests have been performed and several remarks can be made.

<table>
<thead>
<tr>
<th>Inventory sensitivity test</th>
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<tbody>
<tr>
<td>The last sensitivity test analyses the impact on the inventory, warehouse configuration and on the total savings of changes in the inventory objective function cost coefficients. This sensitivity analysis is important. On the one hand, changes in the inventory carrying cost and acquisition cost can occur because the value of the product changes (e.g. reevaluation of taxes, insurance ...). On the other hand, the shortage and carrying cost is difficult to estimate [72]. Computational tests have been performed and several remarks can be made.</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>First integ.</td>
<td>Min 0</td>
<td>25.83</td>
</tr>
<tr>
<td></td>
<td>average 0</td>
<td>36.51</td>
</tr>
<tr>
<td></td>
<td>Max 0</td>
<td>43.96</td>
</tr>
<tr>
<td>Second integ.</td>
<td>Min 0.24</td>
<td>30.08</td>
</tr>
<tr>
<td></td>
<td>average 0.41</td>
<td>39.64</td>
</tr>
<tr>
<td></td>
<td>Max 0.54</td>
<td>46.31</td>
</tr>
</tbody>
</table>

Table 3.7: Results of the inventory and warehouse sensitivity analysis.

In the case of unlimited reserve capacity, we know that the inventory cost function is relatively flat around the optimal ordered quantity. This observation means that small changes in the optimal order quantity and reorder point will not have a significant impact on the total savings. This is confirmed by the value...
obtained with the sensitivity test in the case of unlimited reserve capacity (see table 3.8).

In the case of limited capacity, the impact on the total savings of changes in the inventory cost coefficients can be much more important. Indeed, modifications in the inventory cost coefficients will have an impact on the optimal order quantity, reorder point and optimal route of each product. This modification will also have an indirect impact on the value of the lagrangian coefficients through the capacity constraint (see Section 3.5 for more details) which leads to a larger modification of the order quantity and a larger impact on costs. Two particular situations can be pointed out. First of all, in some cases, no savings are obtained with the second integration. This can be observed when the modification in the inventory cost coefficients are such that the reserve capacity limit becomes unlimited (due to a decrease in the optimal order quantity and reorder point for each product). Secondly, we can observe in Table 3.8 that the minimum value obtained for the total savings with the first integration is negative. This is due to the approximation made with the global capacity constraint of the heuristic sequential procedure. In fact, this global capacity constraint overestimates the need of capacity. Therefore, sometimes, the heuristic sequential procedure estimates that capacity is missing whereas in reality (when the forward and reserve capacity are considered separately) there is enough capacity. In this particular case the heuristic uncapacitated method performs better than the heuristic sequential method.

### 3.7 Conclusion

Currently, most of the tactical warehouse and inventory issues are tackled independently and sequentially. Our aim through this chapter was to show the value of integrating more decisions of the warehouse and inventory fields. Consequently, we have presented a global model which takes into account the replenishment decision at the inventory management level, the allocation of products to warehouse systems and the assignment of products to storage locations at the warehouse management level. In order to solve this global model, we have presented two heuristics which illustrate two possible levels of integration.

In the first heuristic solution procedure, the inventory and warehouse models are solved sequentially and the inventory model is taking into account a global capacity constraint for the entire warehouse and approximated reception cost. This global capacity constraint reflects the limited space in the warehouse but neither
the routes taken by the products nor the number and capacity of the locations in the forward area are considered in the inventory model.

In the second heuristic solution procedure, a higher level of integration is achieved by considering two capacity constraints, one for each area in the warehouse (forward and reserve area) and reception cost for each product which depends on the routes taken by those products. This made possible to take into account the routes taken by the products, the number and capacity of the locations in the forward area. This second heuristic solution procedure is based on an iterative loop of the two sub-problems (inventory and warehouse sub-model) where the output of one of the model at one iteration becomes the input of the other model at the next iteration.

Computational results were obtained under limited and unlimited capacity situation in the warehouse. It has been shown that under unlimited capacity (limited forward capacity and unlimited reserve capacity), savings could be achieved only by using the second level of integration (second heuristic solution procedure) whereas in the case of limited capacity the major savings were achieved through the first level of integration (first heuristic solution procedure), although the higher level of integration could achieve some additional savings.

A sensitivity analysis was performed to observe the impact of changes in the objective cost coefficients on the total savings realized and on the warehouse and the inventory configuration. Three sensitivity tests were realized. The first test analyses the impact of changes in the warehouse cost coefficients. The second test analyzes the impact of changes in the relative importance of the total warehouse cost compare to the total inventory cost. Lastly, changes in the inventory cost coefficients were performed and analyzed. The results show that in most cases changes in the objective cost coefficients had no significant impact on the warehouse and the inventory configuration and the total savings realized. Nevertheless, two exceptions were observed. The first exception occurred when the warehouse cost coefficients were modified. In the case of limited capacity, the results obtained for the inventory model with the heuristic integrated method were not very stable. This is due to the methodology used to solve the inventory model and to the higher level of integration between the inventory and the warehouse models. The second exception is discovered when the inventory cost coefficients are changed and when the reserve capacity is limited. In this case, sometimes, the integrated procedure is not better than the heuristic uncapacited method and the heuristic sequential procedure was performing worse than the heuristic uncapacitated method.
3.7 Conclusion

By this first case study, our aim was to present a situation where coordination mechanisms allow to improve the efficiency of the supply chain. As said in Chapter 2, coordination can be achieved by realizing an intelligent decomposition. In this case study, a global model which takes into account tactical inventory and warehouse decisions is created. By analyzing this global model, a solution methodology is developed which is based on a decomposition of the global model in sub-models which destroys the less the link between those two decisions. Through our computational experiments, we have seen that those methodologies allow to keep the coordination strong while reducing the computational time. In addition, computational results prove that the capacity of the warehouse (the scarce resource) is handled more efficiently with an integrated view.
Chapter 4

An Integrated model for Production and Distribution decision with reusable resources management

4.1 Introduction

In this chapter, we present a new case study where an integrated decision tool is proposed in order to improve the efficiency of the supply chain. In this chapter, we want to confirm the results obtained in the previous chapter and therefore answer the same three research questions.

In this case study, we analyze the coordination mechanism between tactical and operational production and distribution decisions in the case where reusable resources are shared between those two departments. After strongly motivating the importance of collaboration between those decisions, a global model is developed which takes into account all the particularities of the environment. Intelligent decomposition methods are considered in order to solve this integrated model. Therefore, the solution methodologies are elaborated in order to avoid destroying the link which exists between tactical and operational production and distribution decisions in a reusable resources environment. Each of the methodologies proposed decomposes the global model differently and allows to appreciate the importance
An Integrated model for Production and Distribution decision with reusable resources management

of a good decomposition of the global model. Those methodologies are finally tested on a database and the value of integration is highlighted.

This chapter will be decomposed as follows. In Section 4.2, the importance of production and distribution decisions integration is exposed. In Section 4.3, we make a brief review of the literature available on the subject. The third section introduces the model formulation and the various assumptions made. A description of the various heuristics used to solve the integrated model follows in Section 4.6 and the computational tests and sensitivity analysis are presented in Section 4.7.

4.2 Motivations

Production and distribution activities can be handled independently using inventories as buffers between them, but this leads to important holding costs and longer product cycle time in the supply chain. Reducing inventories and product cycle time can be achieved by coordinating production and distribution decisions. Management decision tools based on modeling, optimization and simulation methods, where integration of decisions is an important component, help in coordinating those functional areas.

The integration of production and distribution decision making can be achieved at all levels namely strategic, tactical and operational. Strategic production and distribution decisions include issues of design and decisions in the supply chain such as location, plant capacity and transportation channels. At the tactical level, production and distribution decisions deal with questions such as how much to produce, how much to ship in a time period, how long the production cycle / distribution cycle should be, how much inventory to keep, . . . Finally, operational production and distribution decisions concern problems of detailed scheduling: when and on which machine to process a job, when and by which vehicle to deliver a job, which route should the vehicle takes . . . We are interested in the integration of tactical and operational production and distribution decisions. More precisely, we have concentrated our work in three areas: at the tactical level lot sizing decisions in both production and distribution and at operational level the vehicle routing problem.

Our aim in this chapter is to analyze the benefits of coordination (sharing of information) between the production and the distribution department when tangible reusable resources are shared between those two functional areas. On
4.2 Motivations

the one hand, the production process is dependent on distribution decisions in order to ensure availability of the reusable resource at the right time. Without the reusable resource, the production process cannot take place. On the other hand, the distribution process is dependent on the production process to deliver the finished product (which is composed of a content and a reusable resource) to the customers at the right time. Without coordination between production and distribution decisions, reusable resources may not be at the right time in the right position in the supply chain.

To understand the importance of reusable resources management and coordination between production and distribution departments, we examine the gas filling industry. This illustration is based on a real case study and has inspired this research work. The company we consider specializes in industrial and medical gases and related services. It supplies oxygen, hydrogen and many other gases and services to a large number of customers that range from industry to hospitals, oil refineries and aerospace facilities. The company distributes its gas cylinders between its different filling centers, distribution centers and customers. These cylinders are an extremely practical method of supplying gas and are therefore a critical resource, the use of which creates many difficulties in production and distribution planning. A cylinder is defined by the nature of its contents (compressed gas, refrigerated liquid, etc.), the type of gas (\(O_2\), \(N_2\), \(CO_2\), \(H_2\), etc.), the gas purity and the size (volume and pressure). Cylinders can be supplied individually or in sets of 9 or 18 bundled together and emptied as a unit. Inventory management should provide a controlled access to cylinders and should allow for the return of empty gas containers as well as a precise tracking of cylinders to enable the supply of customers again when needed. Regulatory checks and retesting of cylinders have to be handled before any reuse. All these constraints make the management of these cylinders a complex task involving different departments.

In the previous example, large quantities of reusable resources need to be kept in inventories when there is no coordination between the production and distribution departments in order to have fluidity in the supply chain. When those reusable resources are expensive, it can lead to high holding costs. Consequently, our aim is to develop a global model at the tactical and operational level to optimize the use of reusable resources in production and distribution department. We also propose three different heuristics to solve this integrated model. The performance of our heuristics is evaluated by realizing computational tests. Finally, the advantages and disadvantages of our heuristics are highlighted.
4.3 Tactical and operational production and distribution models

In this section, we survey the literature dealing with the issues of coordination of production and distribution. As we are only interested in tactical and operational decisions in those fields, we concentrate our survey on the most important articles in that field. More precisely, we focus, in the first part of this review, on articles which are interesting due to the type of decision considered and the managerial environment involved. In the second part of this review, we cite articles where authors have concentrated on the development of a good solution methodology to solve this coordination problem.

The coordination of production and distribution activities at the operational and tactical level has been sparsely analyzed in the literature. Chandra et al. [15] consider the coordination of production and distribution scheduling (VRP problem). They study a 2-echelon, multi-product, multi-period and multi-retailer system with one plant and deterministic demand. They propose two solution methodologies to solve this global model. The first one is based on a decomposition approach of the global model in production and distribution sub-models. The second methodology consists in searching for cost reductions in the plan found by the first methodology. According to their computational results, savings of 3% to 20% can be achieved by coordinating those two functional areas and the value of integration increases with the length of the planning horizon and the number of products and customers.

Barbarosoglu et al. [3] study the potential benefit of coordinating production and distribution lot sizing decisions (no routing decisions are considered). They analyze a 3-echelon system, multi-product, multi-depot, multi-period and multi-customer, with deterministic demand. No production and distribution lead times are considered and no capacity limits are imposed on the inventories. They proposed to solve this global model by Lagrangian relaxation and subgradient optimization. This allows information to flow between those two models while keeping the advantage of a hierarchical structure. In addition, the distribution sub-model is solved by the mean of a forward heuristic. The authors report that their algorithm provides good bounds in short CPU times even for large instances.

Fumero et al. [37] analyze the integration of capacity management, inventory allocation and vehicle routing decisions. They consider a 2-echelon, multi-customer, multi-period system with one plant and deterministic demand. No lead times
are considered for the transportation of items from the plant to the customers. Their solution method is based on Lagrangean relaxation. Computational tests show that the value of coordination increases with the number of products and customers, with the available capacity for production and the fleet and with the number of time periods.

Ozdamar et al. [60] investigate a two-stage system composed of a factory and remote warehouses. No routing issues are considered in this problem. They propose a monolithic problem which is solved at different levels of aggregation based on a hierarchical structure. Backorders are allowed. An iterative solution methodology is proposed and tested on a database of detergent products.

Mohamed et al. [58] want to integrate the tactical production and distribution decisions for a multi-national company over a finite planning horizon considering a two-stage system composed of facilities and customers dispersed around the world. The issues considered are the location of product manufacturing, the assignment of markets to facilities and the influence of inflation and exchange rates on those decisions. Capacity at each facility is considered as a decision variable. Capacity can change from one period to another and this modification involves a capacity changing cost which is included in the objective function. No routing decisions are involved. Inventories are available at the factories and demands are deterministic and must be satisfied in a JIT way.

Dhaenens-Flipo and Finke [29] study a multi-facility, multi-product, multi-period industrial problem. They want to coordinate the distribution and the production function in a 3-echelon system composed of facilities (each with parallel production lines), warehouses and customers. Storage is allowed at the plants or at the warehouses and not at the customers. Customers’ demands must be satisfied in a JIT manner. They consider setup cost, fixed and variable transportation costs. They formulate the problem as a network flow problem. Numerical experiments show that this model can solve large real-life industrial problems in reasonable computing time.

There has been some recent articles published on the Production, Inventory, and Distribution Routing Problem (PIDRP) where the aim of the authors was to develop a good solution methodology to solve this NP-hard problem. The PIDRP considers decisions of production and distribution lot sizing level combined with vehicle routing decisions. In general terms, we can describe PIDRP as a single item problem involving one plant with multiple customers, deterministic demand, a fleet of homogeneous capacitated vehicle.
Lei et al. [54] were the first to propose a MIP formulation for the PIDRP. They considered a variant of the PIDRP where there are multiple plants and a heterogeneous fleet of vehicles. They proposed a solution methodology based on a two phase approach where, in the first phase, routing decisions are limited to direct shipments. In the second phase, consolidation of less than truckload shipments found in the first phase is investigated.

Boudia et al. [11], [12], [13] proposed a MIP formulation somewhat similar to the previous authors and proposed to solve this model by the mean of different solution methodologies: a memetic algorithm with dynamic population management (MA|PM), a reactive GRASP and two improved mechanisms based on a reactive mechanism and a path relinking methodology and finally a combined greedy heuristics with a local search procedure. Computational results confirm the importance of integrating production and distribution decisions and their solution methodologies allow to tackle problem of reasonable size (up to 200 customers and 20 time periods) in short computational time. Moreover, their metaheuristic (MA|PM) has proved to be more efficient to tackle coordination of decisions at production and distribution level than the other heuristics that they have proposed.

Bard et al. [5] developed, based on the same MIP formulation, a solution methodology based on tabu search algorithm followed by a path relinking method to improve the final solution found. The novelty of the method is the elaboration of an allocation model used to find good starting feasible solutions for the tabu search procedure. Results were promising with improvements ranging from 10 to 20% compared to the result obtained with Boudia et al. [11] GRASP solution methodology but at a computational time cost. Some authors have investigated exact solution methodologies. This was the case of Bard et al. [4, 6] who investigated a heuristic implementation of a branch-and-price algorithm. Computational tests show that they were able to solve instances with 50 customers and 8 time periods within 1 hour. This performance cannot be achieved by CPLEX or standard branch-and-price alone.

Finally, Ruokoski et al. [66] proposed to solve the production-routing problem optimally by using a mixed integer linear programming formulation and several strong reformulations. Compared to the other authors, they consider the restricted environment of a single uncapacitated vehicle. In addition, they proposed two families of valid inequalities: 2-matching and generalized comb inequalities. Those reformulations combined with the valid inequalities are embedded in a branch-and-cut procedure and used to solve this coordination problem. Computational results
show that their method allows to solve instances of up to 80 customers and 8 time periods within a two-hour time limit. They compare their methodology with traditional decomposition methods and a heuristic algorithm.

Our contribution differs from previous work because we analyze the integration of tactical/operational production and distribution decisions in a particular business environment where reusable resources are shared between the two departments. Our aim is to analyze how the integration of those decisions helps in managing those reusable resources.

4.4 Problem description and mathematical formulation

In this section, we propose a definition of reusable resources in order to highlight the uniqueness of our analysis. Then we describe the assumptions we make on the environment and we propose a mathematical formulation of the problem as a mixed-integer program.

4.4.1 Definition of reusable resources

Resources include “assets, capabilities, organizational processes, firm attributes, information, knowledge, etc. controlled by a firm which enables it to conceive and implement strategies that improve its efficiency and effectiveness” [24].

In general, resources can be classified as tangible resources, such as inventory, manufacturing resources (machinery, installation, plant, equipment, etc.), logistics and transportation; or intangible resources such as information, technological innovation, human resources, intellectual property, development of new production processes and models, customer relationships, relationship between supply chain members, etc.

We are interested in managing tangible reusable resources in the supply chain that are shared among independent decision makers in order to improve its overall efficiency and effectiveness. Examples of tangible reusable resources can be gas cylinders, pallets, returnable bottles, trestle, etc.

Independent decision makers can be of two types: inter firm partners or intra firm partners. In the first case, inter firm relationships can exist in a horizontal supply chain such as one company supplying components to another or in a vertical supply chain with retailers, distributors, and manufacturers. In the second
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case, intra firm partners can be different business units like production and distribution departments or procurement and production departments. Depending on the different types of reusable resources to be shared, decision areas range from the operational decisions, for example dealing with inventory, capacity allocation, transportation decisions through the tactical decisions, for example, information sharing, negotiating contracts; to decisions taken at a strategic level, for example, investment decisions, facility locations, plant capacity, etc.

In such close loop supply chain, it is important to manage properly those reusable resources in order to improve the efficiency and effectiveness across the entire supply chain. Coordination in the supply chain can improve the management of close loop supply chain. We need to assess the results of using reusable resources using cost analysis and profit evaluation. The allocation of the collaboration surplus must lead to a more profitable situation for all actors of the collaboration ex post.

Sharing of reusable resources between independent decision makers can be of great benefit especially when those resources are expensive but it can also be very negative when those resources are not shared in an effective manner. For example, let us consider a glass company selling different types of glass panes for the automotive and construction industries. Those glass panes need to be transported on specific trestles in order to guarantee the safety of the product during the transportation. Once the consumer has consumed the glass panes, the trestle can be shipped back to the factory in order to be used for the next shipment. The trestles represent the reusable resources. If the trestle is not available at the production site when needed then the distribution cannot take place and it results in delay and poor service level. A possibility would be to have continuously a large quantity of those trestles in stock which leads to high holding cost. If there is a good coordination between the production and the distribution of glass panes then there will be fluidity in the supply chain with a reasonable amount of those reusable resources in the system.

Therefore, in this work we want to put forward, the importance of collaboration between decision makers who share reusable resources between them. To our knowledge, integration of production and distribution decisions has only been analyzed in term of impact of integration on production and distribution costs. Our aim is to analyze the impact of integration of production and distribution decisions on reusable resources management and close loop supply chain.
4.4 Problem description and mathematical formulation

4.4.2 Problem description and assumptions

We have one manufacturing site which is delivering finished products to several customers. We are in a multi-item, multi-period, multi-vehicle, capacitated environment. There exists two types of flow: the flow of finished products and the flow of empty reusable resources. The finished product cannot be produced without the use of a reusable resource. They have a relationship of one for one: each time a finished product is produced, a reusable resource is used. In addition, the transportation of a finished product cannot be achieved without the use of a reusable resource. For example, in the case of returnable bottles, the bottle is the reusable resource between the production and the distribution department and the content of the bottle (beer, soft, etc.) is the finished product. The production and the transportation of the content cannot be achieved without the use of an appropriate bottle. We suppose several types of reusable resource with each the same physical attributes (same volume, same weight, same format, etc.). For the routing decisions, we have several types of vehicles with given capacities and setup costs. The problem of concern is of the type Vehicle Routing Problems with Backhauls (VRPB). More precisely, we are concerned by problems where each customer is visited once and is associated with a pick up and a delivery point [61]. Therefore, each time a customer is visited, finished products with reusable resources are dropped and reusable resources are picked up. In addition, the amount of reusable resources picked up at a customer site must be equal to the amount of finished product and reusable resource delivered. This assumption follows what is generally done in practice in business life in order to have a traceability of the reusable resources. Moreover, the order of a customer cannot be split across different vehicles. Each customer is defined by a demand which is deterministic and a geographical position defined by Euclidean coordinates. We have two types of inventories in the system: reusable resource inventories and finished product inventories. Those inventories are at the manufacturing site and at the various customer locations. We suppose that it takes a certain number of days for the customer to consume the finished product and therefore to make the reusable resource available. This duration is called the stock rotation. This amount of time is dependent on the customer consumption rate and on the type of finished product. The production and distribution decisions are planned over an horizon of a week.

The problem is represented graphically in Figure 4.1.
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Figure 4.1: The problem under consideration
4.4 Problem description and mathematical formulation

4.4.3 Mathematical formulation

In this section, we propose a mathematical model which integrates the production and distribution lot sizing problem with the vehicle routing problem in a multi-item, multi-period, multi-vehicle and capacitate reusable resources environment. Therefore, we take into account in our objective function traditional production lot sizing costs (setup, production and storage costs), distribution lot sizing costs (setup and storage costs) and vehicle routing costs (traveling costs based on distances). The constraints of our model include the traditional production and distribution lot sizing constraints with the particularity that the production capacity will be limited by the amount of reusable resources available at the production site. This implies a direct link between the production and the distribution decisions. In addition, we also have the traditional vehicle routing constraints.

The indices used are $i : 1, \ldots, I$ to denote a type of product and $k, m : 0, \ldots, K$ to denote a customer with the index value 0 denoting the manufacturing site. Vehicles are identified through the index $j : 1, \ldots, J$ and the time period through the index $t : 1, \ldots, T$.

We describe hereafter the data and variables used in the model. For each element, we give the units of measure between brackets.

Data:

- $D_{i,k,t} =$ the demand of customer $k$ for product $i$ in period $t$ [units]
- $\text{length}_{k,m} =$ the distance between a customer $k$ and a customer $m$ [km]
- $\delta_{i,k} =$ the stock rotation of a product $i$ for each client $k$ [days]
- $CV_j =$ the capacity of vehicle $j$ [units]
- $PC =$ production cost [€/unit]
- $SC =$ production setup cost [€/setup]
- $TC =$ cost of transportation [€/km]
- $SCT_j =$ transportation setup cost for each vehicle $j$ [€/setup]
- $HFC =$ holding cost of finished product at the customer site [€/unit]
- $HPC =$ holding cost of reusable resource at the customer site [€/unit]
- $HFU =$ holding cost of finished product at the manufacturing unit [€/unit]
- $HPU =$ holding cost of reusable resource at the manufacturing unit [€/unit]
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Variables:

\[ y_{i,j} = \begin{cases} 1 & \text{if the production of product } i \text{ is started in period } t \\ 0 & \text{otherwise} \end{cases} \]

\[ p_{k,j,t} = \begin{cases} 1 & \text{if the client } k \text{ is served by the vehicle } j \text{ in period } t \\ 0 & \text{otherwise} \end{cases} \]

\[ l_{j,t} = \begin{cases} 1 & \text{if vehicle } j \text{ is used in period } t \\ 0 & \text{otherwise} \end{cases} \]

\[ f_{j,k,m,t} = \begin{cases} 1 & \text{if vehicle } j \text{ travels from customer } k \text{ to customer } m \text{ in period } t \\ 0 & \text{otherwise} \end{cases} \]

- \( x_{i,t} = \) amount of finished product \( i \) produced in period \( t \) [units]
- \( f_{u,i,t} = \) amount of finished product \( i \) available in stock at the manufacturing unit at the end of period \( t \) [units]
- \( pu_{i,t} = \) amount of reusable resource of product \( i \) available in stock at the manufacturing unit at the end of period \( t \) [units]
- \( z_{i,k,j,t} = \) amount of finished product \( i \) delivered to customer \( k \) by the vehicle \( j \) in period \( t \) [units]
- \( w_{i,k,j,t} = \) amount of reusable resource of product \( i \) transported from customer \( k \) by the vehicle \( j \) in period \( t \) to the manufacturing unit [units]
- \( fc_{i,k,t} = \) amount of finished product \( i \) available in the stock of client \( k \) in period \( t \) [units]
- \( pc_{i,k,t} = \) amount of reusable resource of product \( i \) available in the stock of client \( k \) in period \( t \) [units]
The objective function is as follows:

\[
\min \sum_{i=1}^{I} \sum_{t=1}^{T} PC \times x_{i,t} + \sum_{i=1}^{I} \sum_{t=1}^{T} SC \times y_{i,t} \\
+ \sum_{i=1}^{I} \sum_{t=1}^{T} HF \times f_{ui,t} + \sum_{i=1}^{I} \sum_{t=1}^{T} HP \times pu_{i,t} \\
+ \sum_{j=1}^{J} \sum_{k=0}^{K} \sum_{m=0}^{K} \sum_{t=1}^{T} TC \times \text{length}_{k,m} \times f_{j,k,m,t} \\
+ \sum_{j=1}^{J} \sum_{t=1}^{T} SC_{j} \times l_{j,t} \\
+ \sum_{i=1}^{I} \sum_{k=1}^{K} \sum_{t=1}^{T} HF \times fc_{i,k,t} \\
+ \sum_{i=1}^{I} \sum_{k=1}^{K} \sum_{t=1}^{T} HP \times pc_{i,k,t}
\]

The objective function is composed of different terms of cost related to the production lot sizing problem and to the transportation problem. The lot-sizing decisions costs concern production cost, production setup cost (4.1) and holding cost at the manufacturing site (4.2). The transportation decision costs (4.3) concern the cost of transportation of reusable resources from customers to the manufacturing unit and the cost of transportation of finished products from the manufacturing unit to the customers. An additional transportation setup cost is incurred each time a vehicle is leaving the manufacturing site (4.4). There is also holding cost of reusable resources and finished product at the different customer sites (4.5),(4.6).
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Constraints:

\begin{align*}
  x_{i,t} + fu_{i,t-1} & = \sum_{k=1}^{K} \sum_{j=1}^{J} z_{i,k,j,t} + fu_{i,t} & \forall i, t \tag{4.7} \\
  x_{i,t} & \leq \left( \sum_{k=1}^{K} \sum_{n=1}^{T} D_{i,k,n} \right) \times y_{i,t} & \forall i, t \tag{4.8} \\
  pu_{i,t-1} + \sum_{k=1}^{K} \sum_{j=1}^{J} w_{i,k,j,t-1} & = pu_{i,t} + x_{i,t} & \forall i, t \tag{4.9} \\
  \sum_{j=1}^{J} z_{i,k,j,t} + fc_{i,k,t-1} & = D_{i,k,t} + fc_{i,k,t} & \forall i, k, t \tag{4.10} \\
  pc_{i,k,t-1} + D_{i,k,t-1} & = pc_{i,k,t} + \sum_{j=1}^{J} w_{i,k,j,t} & \forall i, k, t \tag{4.11} \\
  \sum_{i=1}^{I} \sum_{k=1}^{K} z_{i,k,j,t} & \leq CV_{j} & \forall j, t \tag{4.12} \\
  \sum_{i=1}^{I} \sum_{k=1}^{K} z_{i,k,j,t} & \leq CV_{j} \times p_{k,j,t} & \forall k, j, t \tag{4.13} \\
  \sum_{i=1}^{I} \sum_{j=1}^{J} w_{i,k,j,t} & = \sum_{i=1}^{I} \sum_{j=1}^{J} z_{i,k,j,t} & \forall i, k, t \tag{4.14} \\
  \sum_{j=1}^{J} p_{k,j,t} & \leq 1 & \forall k, t \tag{4.15} \\
  p_{k,j,t} & \leq l_{j,t} & \forall k, j, t \tag{4.16} \\
  \sum_{m=0}^{K} f_{j,k,m,t} - \sum_{m=0}^{K} f_{j,m,k,t} & = 0 & \forall k, t, j \tag{4.17} \\
  \sum_{m=1}^{K} f_{j,0,m,t} & = l_{j,t} & \forall j, t \tag{4.18} \\
  \sum_{m=0}^{K} f_{j,k,m,t} & = p_{k,j,t} & \forall j, k, t \tag{4.19} \\
  \sum_{k,m \in S, k \neq m} f_{j,k,m,t} & \leq |S| - 1 & \forall j, t, S \subseteq \{1, \ldots, K\} \tag{4.20} \\
  x_{i,t}, fu_{i,t}, pu_{i,t} & \geq 0 & \forall i, t \tag{4.21} \\
  z_{i,k,j,t}, w_{i,k,j,t} & \geq 0 & \forall i, k, j, t \tag{4.22} \\
  fc_{i,k,t}, pc_{i,k,t} & \geq 0 & \forall i, k, t \tag{4.23} \\
  y_{i,t}, p_{k,j,t}, l_{j,t}, f_{i,k,m,t} & \in \{0, 1\} & \forall i, k, m, j, t \tag{4.24}
\end{align*}

Constraints (4.7) and (4.9) are flow balance constraints at the manufacturing site for the storage of finished product and at the manufacturing site for the storage of reusable resources. Note that the latter also limits the capacity at the manufacturing site because the production can only take place if reusable resources are
available in stock. Constraints (4.10) are the flow balance constraints of finished products at the client site while constraints (4.11) are the flow balance constraints of reusable resources at the client site. Constraints (4.8) are production setup constraints. Constraints (4.12)-(4.20) are transportation constraints. Constraints (4.12) are the vehicle capacity constraints for finished products. Constraints (4.13) assign vehicles to clients. Constraints (4.14) ensure that the collecting of reusable resources at each customer site equals the quantity of finished products delivered. Constraints (4.15) force each client to be served by one and only one vehicle: orders can not be divided and delivered by different vehicles. Constraints (4.16) define the setup variable for each period and for each vehicle. Constraints (4.17)-(4.20) model the vehicle routing part of the problem.

4.5 Valid inequalities

Our aim in this section is to tighten the initial formulation presented in Section 4.4.3 by adding valid inequalities.

Hereafter, we present three high level relaxations of our model in order to derive valid inequalities which are going to be added a priori to the formulation (before the optimization starts). Those relaxations are lot-sizing relaxations [62].

The first valid inequality is defined by the sub-model composed of constraint (4.7), (4.8) and (4.10). By combining the first two constraints, we obtain the following relaxation for each product and for each period:

$$x_{i,t} + f_{u_{i,t-1}} + \sum_{k=1}^{K} f_{c_{i,k,t-1}} = \sum_{k=1}^{K} D_{i,k,t} + f_{u_{i,t}} + \sum_{k=1}^{K} f_{c_{i,k,t}} \quad \forall i, t$$  (4.25)

For ease of presentation, we use the concept of echelon stock [62] and define variable $e_{i,t}$ which replace variables $f_{u_{i,t}} + \sum_{k=1}^{K} f_{c_{i,k,t}}$. Constraints (4.25) become:

$$x_{i,t} + e_{i,t-1} = \sum_{k=1}^{K} D_{i,k,t} + e_{i,t} \quad \forall i, t$$  (4.26)

Consequently, constraints (4.26) and the production setup constraints (4.8) define an uncapacitated single item lot sizing relaxation. Therefore, we can deduce the following valid inequality:
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\[ e_{i,t-1} \geq K \sum_{h=1}^{K} \sum_{m=1}^{n} D_{i,k,m} \times (1 - \sum_{n=1}^{m} y_{i,n}) \quad \forall i,t \in T, o \in T, o \geq t \quad (4.27) \]

The second valid inequality is based on the sub-model composed of constraints (4.10) and (4.13). By defining the following new variables and parameters:

\[ CV' = \sum_{j=1}^{J} CV_j \quad z'_{k,t} = \sum_{i=1}^{I} \sum_{j=1}^{J} z_{i,k,j,t}, \quad D'_{k,t} = \sum_{i=1}^{I} D_{i,k,t}, \quad f'_{c_{i,k,t}}, \quad f'_{c_{i,k,t}}, \]

we can derive a relaxation of constraints (4.10) for each customer at each period:

\[ z'_{k,t} + f'_{c_{i,k,t}} - 1 = D'_{k,t} + f'_{c_{i,k,t}} \quad \forall k,t \quad (4.28) \]

As \( \sum_{j=1}^{J} p_{k,j,t} \leq 1 \quad \forall k,t \), we can rewrite constraint (4.13) as:

\[ z'_{k,t} \leq CV' \sum_{j=1}^{J} p_{k,j,t} \quad \forall k,t \quad (4.29) \]

Therefore, constraints (4.28) and (4.29) define a capacitated lot sizing relaxation for each customer at each period. The following valid inequality can be derived:

\[ \sum_{i=1}^{I} f'_{c_{i,k,t-1}} \geq \sum_{i=1}^{I} \sum_{m=1}^{n} D_{i,k,m} \times (1 - \sum_{n=1}^{m} p_{k,j,n}) \quad \forall k,t \in T, o \in T, o \geq t \quad (4.30) \]

The third inequality is derived from the sub-model composed of constraints (4.11), (4.13) and (4.14). We define the following additional new variables:

\[ p_{c_{i,k,t}} = \sum_{i=1}^{I} p'_{c_{i,k,t}}, \quad w'_{i,k,t} = \sum_{j=1}^{J} w'_{i,k,j,t} \]

With the use of those new variables, a relaxation of the sub-model (4.11) and (4.14) can be derived for each customer and for each period:

\[ p_{c_{i,k,t}} + w'_{i,k,t} = p'_{c_{i,k,t}} + w'_{i,k,t} \quad \forall k,t \]

\[ w'_{i,k,t} = z'_{k,t} \quad \forall k,t \]

Combining the first two constraints, we get:

\[ p_{c_{i,k,t}} + w'_{i,k,t} = p'_{c_{i,k,t}} + z'_{k,t} \quad \forall k,t \quad (4.31) \]
Constraints (4.31) and (4.29) define a capacitated lot sizing relaxation for each customer at each period. Therefore, we can derive the following valid inequality:

\[
\sum_{i=1}^{I} \sum_{m=b_{i,k}} \sum_{o=m+1}^{o} D_{i,k,m} - \delta_{i,k} \times (1 - \sum_{n=m}^{T} \sum_{j=1}^{J} p_{k,j,n}) \quad \forall k \in K, t, o \in T, o \geq m, o \leq t
\]

(4.32)

Due to the limited number of those valid inequalities, we will add them a priori to our initial formulation (before the optimization starts) in order to tighten the solution space and not by the mean of a separation algorithm [62].

Computational tests on a reduced size data set \(^1\) show that, by including these valid inequalities, the gap at the root node can be reduced by 26% and that the total branch-and-bound computational time by 45%. Those computational experiments are obtained with commercial MIP solver XPRESS-MP and with TSP solver Concorde [1, 2].

### 4.6 Heuristic approaches

The model presented in Section 4.4.3 is a linear mixed integer model, multi-item, multi-period. Our aim is to solve this model at the operational level. Therefore, we want a good solution in a short computational time. Even though the valid inequalities added to the model allow to tighten the model and consequently improve the lower bound, a branch-and-bound approach does not allow to fulfill this requirement [71] (this is also confirmed by our computational results). Consequently, our aim is to find a procedure to solve the problem heuristically.

We propose in this section three different heuristics. Two of them are based on a decomposition approach of the global model into sub-models whereas the third one is a more integrated heuristic. The latter does not rely on a decomposition approach but tries to solve the global model as a whole. The two first decomposition heuristics mimic what is done currently in businesses in order to tackle the coordination between the production and the distribution decisions.

The main advantage of using a decomposition approach to solve this global model is to reduce the complexity of the model and therefore to enable the solution of larger instances. Nevertheless, this simplification implies that the coordination between production and distribution decisions is reduced and that infeasibility problems can occur when the capacity of reusable resources is limited. In order

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\(^1\)The dataset used is the same as the one used in section 4.7.2
to avoid infeasibility problems resulting from our decomposition approach, we introduce an additional variable \((pu_{addi})\) which represents the additional reusable resource of type \(i\) needed in the system. This additional reusable resources can be rented at an expensive cost in the beginning of the planning horizon.

Our integrated heuristic allows to estimate the advantage of using a more integrated approach in solving our global model, with the drawback of being more computationally challenging.

### 4.6.1 Sequential production-transportation heuristic

This heuristic is based on a decomposition procedure where the global production and transportation model is divided in an uncapacitated lot sizing model and a distribution model for the reusable resources and the finished product. This methodology mimics what is commonly done in business in order to reduce the complexity of the production and distribution model. Those production and transportation models are solved sequentially: first the production model is solved then the transportation model is solved based on the solution of the production planning model. Consequently, the lot sizing model is solved without taking into account the reusable resources capacity restriction involved by the distribution decisions. Once the production planning decisions are fixed, the transportation model is solved based on the production planning restrictions. Those transportation decisions are approximated by a generalized assignment heuristic [33] where a general assignment problem is solved followed by a traveling salesman problem. Those two sub-models are presented in the following section.

### Production planning decisions

The production planning decisions consist in choosing, for the manufacturing sites, the production level, setup run and the level of finished product stocks for each product and at each time period. This is formulated as an uncapacitated lot sizing model where the demand for product \(i\) of client \(k\) in period \(t\) takes the initial stock available at customer site into account.

The production planning model is as followed and corresponds to (4.7)-(4.8):
4.6 Heuristic approaches

\[
\min \sum_{i=1}^{I} \sum_{t=1}^{T} PC \cdot x_{i,t} + \sum_{i=1}^{I} \sum_{t=1}^{T} SC \cdot y_{i,t} + \sum_{i=1}^{I} \sum_{t=1}^{T} HFU \cdot f_{u_{i,t}}
\]

subject to

\[
x_{i,t} + f_{u_{i,t-1}} = \sum_{k=1}^{K} D'_{i,k,t} + f_{u_{i,t}} \quad \forall i, t \quad (4.34)
\]

\[
x_{i,t} \leq \left( \sum_{k=1}^{K} \sum_{n=1}^{T} D'_{i,k,n} \right) \times y_{i,t} \quad \forall i, t \quad (4.35)
\]

\[
x_{i,t}, f_{u_{i,t}} \geq 0 \quad \forall i, t \quad (4.36)
\]

\[
y_{i,t} \in \{0, 1\} \quad \forall i, k, m, j, t \quad (4.37)
\]

where

\[
D'_{i,k,t} = \max \left( 0, \min \left( \sum_{e=1}^{t} D_{i,k,e} - f_{c_{i,k}}, 0 \right) \right) \quad \forall i, k, t
\]

Each single item sub-problem has Wagner-Whitin costs [62] as the production cost does not change from one period to the other and can be solved in \( O(T) \) time.

Transportation decisions for reusable resources and finished product

The transportation decisions consist in choosing the amount of finished product and of reusable resources to deliver and pick up at each customer site in each period in order to satisfy the demand of customers and to ensure enough capacity at the manufacturing site (enough reusable resources). Those decisions have to be supported by computing the optimal route for each vehicle at each period. This transportation model is therefore composed of distribution decisions and of vehicle routing decisions where the variables \( x_{i,t} \) are fixed at their values \( \bar{x}_{i,t} \) computed at the previous step. We have to solve the global model (4.1)-(4.24) with the valid inequalities (4.27), (4.30) and (4.32) where we fix the production level (4.7) and (4.9) and the setup variables (4.27) in constraints (4.27) as follows:
An Integrated model for Production and Distribution decision with reusable resources management

\[ x_{i,t} + f_{i,t-1} = \sum_{k=1}^{K} \sum_{j=1}^{J} z_{i,k,j,t} + f_{i,t} \quad \forall i, t \quad (4.38) \]

\[ p_{i,0} + pu_{i,0} + \sum_{k=1}^{K} \sum_{j=1}^{J} w_{i,k,j,0} = p_{i,1} + x_{i,1} \quad \forall i \quad (4.39) \]

\[ p_{i,t-1} + \sum_{k=1}^{K} \sum_{j=1}^{J} w_{i,k,j,t-1} = p_{i,t} + x_{i,t} \quad \forall i, t > 1 \quad (4.40) \]

\[ e_{i,t-1} \geq \sum_{k=1}^{K} \sum_{n=1}^{n} D_{i,k,m} \times (1 - \sum_{n=1}^{n} y_{i,n}) \quad \forall i, t \in T, o \in T, o \geq t \quad (4.41) \]

The Generalized Assignment Heuristic

Our transportation model, presented in section 4.6.1, is complex to solve due to the routing decisions (4.17)-(4.20). Therefore, in order to solve this transportation model for the reusable resources and the finished product, we adapt the heuristic developed by Fisher and Jaikumar [33] for the Vehicle Routing Problem (VRP), to our particular situation. This heuristic, according to results reported by Gendreau et al. [39], is very efficient for the VRP.

The version of VRP which is tackled by Fisher and Jaikumar [33] considers a set of customers with known demand levels and a set of vehicles with fixed capacities. Those vehicles must be loaded at a depot, visit customers and return to the depot. Decisions such as which vehicle will serve which demand with which route in order to minimize delivery cost are answered. In order to solve the VRP, they propose to use a Generalized Assignment Heuristic (GAH) composed of two phases: First a Generalized Assignment Problem (GAP) is solved to determine the assignment of customers to vehicles based on an approximation of the traveling costs. Secondly, a Traveling Salesman Problem (TSP) is solved to determine the optimal route for each vehicle. Hereafter, we give more details on each of the GAH phase.

1. The Generalized Assignment Problem

As stated above, in the first part of Fisher and Jaikumar heuristic [33], a GAP is solved: customers are assigned to vehicle based on an approximation of the traveling costs. This approximation is based on the definition of a seed location for each vehicle. The seed location corresponds more or less to the area of operation of the vehicle. The cost of assigning a customer to a vehicle is based on the distance between the customer and the seed location for that vehicle (Euclidean distance). Fisher and Jaikumar [33] determine
the seed location so that the total demand of the customers in the region covered by a vehicle corresponds, more or less, to the vehicle capacity.

Our problem differs from Fisher and Jaikumar’s basic vehicle routing problem because the quantity transported by the vehicle is not given. The choice of the seed locations is a decision to optimize. Therefore, we have adapted their heuristic to our particular problem.

We have decided to fix the possible location of the seeds depending only on the location of the customers. Consequently, a grid is formed over the space delimited by the customers. Each intersection in the grid is a potential seed location. A first decision to optimize is to decide which of those seeds will be used by which vehicle. Then a second decision is to assign each customer to a vehicle. The additional transportation cost incurred if a customer is allocated to a vehicle can be estimated. This Generalized Assignment Problem is solved using a MIP solver with a formulation discussed below.

2. The Traveling Salesman Problem

Once the Generalized Assignment Problem is solved, we know the optimal allocation of customers to vehicles and of vehicles to seeds. Based on this optimal allocation, a traveling salesman problem is solved for each vehicle at each period. Those TSPs are solved using the publicly available TSP solver Concorde [1, 2].

The transportation problem presented in Section 4.6.1 is solved using the adapted general assignment heuristic presented above. For this purpose, we define new variables for the seed locations which are referred by index \( s : 1, \ldots, S \), and new transportation costs.

The new variables are:

\[
\beta_{k,s,j,t} = \begin{cases} 
1 & \text{if customer } k \text{ is assigned to seed } s \text{ and vehicle } j \text{ in period } t \\
0 & \text{otherwise}
\end{cases}
\]

\[
q_{j,s,t} = \begin{cases} 
1 & \text{if vehicle } j \text{ is assigned to seed } s \text{ in period } t \\
0 & \text{otherwise}
\end{cases}
\]

The approximate transportation cost for the GAP formulation corresponds to the additional cost incurred when an extra customer is assigned to the route of a vehicle traveling from the depot location 0 to a seed \( s \). Therefore we can define
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$\text{length}_{k,s}$ as the additional distance traveled by a vehicle if an extra customer $k$ is serviced by this vehicle.

$\text{length}_{k,s} = \text{length}_{0,k} + \text{length}_{k,s} - \text{length}_{0,S}$

The new problem is:

$$
\min \sum_{k=1}^{K} \sum_{s=1}^{S} \sum_{j=1}^{J} \sum_{t=1}^{T} TC * \text{length}_{k,s} * \beta_{s,k,j,t} \\
+ (4.2) + (4.4) + (4.5) + (4.6) \\
+ 2 * TC * \sum_{s=1}^{S} \sum_{j=1}^{J} \sum_{t=1}^{T} (\text{length}_{0,s} * q_{j,s,t})
$$

subject to

1. $(4.41), (4.30), (4.32), (4.38), (4.40), (4.10), (4.11), (4.12),$ $(4.13), (4.14), (4.15), (4.16), (4.22), (4.23)$
2. $p_{k,j,t} = \sum_{s=1}^{S} \beta_{k,s,j,t}$ \quad $\forall k, j, t$
3. $\sum_{s=1}^{S} \sum_{j=1}^{J} \beta_{k,s,j,t} \leq 1$ \quad $\forall k, t$
4. $\sum_{s=1}^{S} \sum_{j=1}^{J} \beta_{k,s,j,t} \leq l_{j,t}$ \quad $\forall k, j, t$
5. $q_{j,s,t} \geq \beta_{h,s,j,t}$ \quad $\forall k, s, j, t$
6. $fu_{i,t}, pu_{i,t} \geq 0$ \quad $\forall i, t$
7. $\beta_{s,k,t}, q_{j,s,t}, p_{k,j,t}, l_{j,t} \in \{0, 1\}$ \quad $\forall i, s, k, m, j, t$

By solving this model, we obtain the optimal distribution planning and the optimal allocation of customers to vehicles and of vehicles to seeds. The routes are calculated by solving a traveling salesman problem at each period for each vehicle based on the allocation found with the GAP.

4.6.2 Sequential transportation-production heuristic

This heuristic is based on a decomposition of the global model in three sub-models which are solved sequentially. First, a transportation model for the finished products is solved. Then a production model is formulated and solved based on the solution of the finished product transportation model. Finally a distribution model for the reusable resources is formulated and solved by taking into account the constraints resulting from the production planning and the finished product transportation model. Note that we could have solved those three sub-models in the
reverse way with exactly same results: First the transportation model for the reusable resources then the production model and lastly the transportation model for the finished product. This is due to the fact that the two transportation models (reusable resources and finished product) are perfectly symmetric.

The transportation decisions for finished product

The transportation decisions for the finished product determine the quantity of finished product as well as the routes of the various vehicles needed to satisfy the demand of the various customers. The model to solve is:

\[
\begin{align*}
\min & \quad \sum_{j=1}^{J} \sum_{t=1}^{T} SCT_j \cdot l_{j,t} \\
& + \sum_{i=1}^{I} \sum_{k=1}^{K} \sum_{t=1}^{T} HFC \cdot f_{c_{i,k,t}} \\
& + \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{m=1}^{K} \sum_{t=1}^{T} TC \cdot length_{k,m} \cdot f_{j,k,m,t} \\
\text{subject to} & \\
& (4.10), (4.12), (4.13), (4.15), (4.16), (4.17), (4.18), (4.19), (4.20), (4.30) \\
& z_{i,k,j,t} \geq 0 \quad \forall i, k, j, t \\
& f_{c_{i,k,t}} \geq 0 \quad \forall i, k, t \\
& p_{i,k,t} l_{j,t} f_{j,k,m,t} \in \{0, 1\} \quad \forall i, k, m, j, t
\end{align*}
\]

In order to solve this transportation model, we apply the same heuristic as the one used for the sequential production-transportation heuristic: we formulate a general assignment problem followed by a TSP for each vehicle at each period.

The distribution planning obtained for the finished product \((z_{i,k,j,t})\) is recorded and will be used as data in the production planning sub-problem.

The production planning

The production planning sub-problem computes the quantity to produce at each period as well as the setup run in order to satisfy the distribution planning decisions.

The production planning problem optimizes (4.33) under the constraints:
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\[ x_{i,t} + f u_{i,t-1} = \sum_{k=1}^{K} \sum_{j=1}^{J} x_{i,k,j,t} + f u_{i,t} \quad \forall i, t \quad (4.42) \]

\[ x_{i,t} \leq M \times y_{i,t} \quad \forall i, t \quad (4.43) \]

\[ x_{i,t}, f u_{i,t} \geq 0 \quad \forall i, t \quad (4.44) \]

\[ y_{i,t} \in \{0, 1\} \quad \forall i, k, m, j, t \quad (4.45) \]

with \( M \) representing a sufficiently large integer number.

This uncapacitated lot sizing model is solved using the same methodology as the one explained in Section 4.6.1.

The transportation decisions for the reusable resources

At this decision level, it is necessary to determine the quantity of reusable resources to collect at the various customer sites in order to satisfy the production planning decisions and the finished product transportation decisions.

As the quantity of reusable resources to pick up at the customer site is restricted by the amount of finished product delivered (by constraint (4.14)), the transportation model for the reusable resources reduces to a feasibility problem where a combination of linear inequalities have to be solved.

\[ p_{c_{i,k,t-1}} + D_{i,k,t-1} - \delta_{i,k} = p_{c_{i,k,t}} + \sum_{j=1}^{J} w_{i,k,j,t} \quad \forall i, k, t \quad (4.46) \]

\[ p_{u_{i,t-1}} + \sum_{k=1}^{K} \sum_{j=1}^{J} w_{i,k,j,t-1} = p_{u_{i,t}} + x_{i,t} \quad \forall i, t \quad (4.47) \]

\[ \sum_{i=1}^{I} \sum_{j=1}^{J} w_{i,k,j,t} = \sum_{i=1}^{I} \sum_{j=1}^{J} z_{i,k,j,t} \quad \forall k, t \quad (4.48) \]

\[ p_{u_{i,t}} \geq 0 \quad \forall i, t \quad (4.49) \]

\[ p_{c_{i,k,t}} \geq 0 \quad \forall i, k, t \quad (4.50) \]

\[ w_{i,k,j,t} \geq 0 \quad \forall i, k, j, t \quad (4.51) \]

\[ p_{c_{i,k,t}} \geq 0 \quad \forall i, k, t \quad (4.52) \]
4.6 Heuristic approaches

4.6.3 An integrated production and distribution heuristic

In this section, we present a heuristic which is not based on a decomposition approach of the global model in sub-models but which directly solve the global model in which a relaxed routing model is used. This heuristic gives a higher level of integration. It allows to take transportation considerations into account in the production planning problem.

Our integrated model (see Section 4.4.3) is difficult to solve due to the transportation decisions. Therefore, we approximate our transportation decision in the integrated model by using the adapted Fisher and Jaikumar heuristic for the VRP (see Section 4.6.1). As we are using the same approximation for the vehicle routing decisions as in the two decomposition heuristics, we are able to compare the performance of this heuristic with the two previous ones. This allows to highlight the value of integration.

The integrated GPA-Production planning model can be formulated as:

\[
\min \ (4.1) + (4.2) + (4.4) + (4.5) + (4.6) \\
\quad + \sum_{k=1}^{K} \sum_{s=1}^{S} \sum_{j=1}^{J} \sum_{t=1}^{T} TC \cdot \text{length}_{k,s} \cdot \beta_{s,k,j,t} \\
\quad + 2 \cdot TC \cdot \sum_{j=1}^{J} \sum_{s=1}^{S} \sum_{t=1}^{T} \text{length}_{0,s} \cdot q_{j,s,t} \\
\text{subject to} \\
(4.7), (4.8), (4.10), (4.11), (4.12), (4.13), (4.14), (4.15), (4.16), (4.21), (4.22), (4.23), (4.27), (4.30), (4.32) \\
p_{u,i,t-1} + puaddi + \sum_{k=1}^{K} \sum_{j=1}^{J} w_{i,k,j,t-1} = pu_{i,t} + x_{i,t} \quad \forall i, t \\
p_{k,j,t} = \sum_{s=1}^{S} \beta_{s,k,j,t} \quad \forall k, j, t \\
\sum_{s=1}^{S} \beta_{k,s,j,t} \leq l_{j,t} \quad \forall k, j, t \\
\sum_{s=1}^{S} \sum_{j=1}^{J} \beta_{s,k,j,t} \leq 1 \quad \forall k, t \\
q_{j,s,t} \geq \beta_{k,s,j,t} \quad \forall j, k, s, t \\
y_{i,t}, p_{k,j,t}, l_{j,t}, q_{j,s,t}, \beta_{s,k,j,t} \in \{0, 1\} \quad \forall i, k, s, j, t \\
\]

Once this integrated GAP-production problem is solved, we know which customer is served by which vehicle, the production level and setup run for each product at each period, and the distribution planning. The routing of the dif-
An Integrated model for Production and Distribution decision with reusable resources management

Different vehicles is performed in a second phase by solving a Traveling Salesman Problem at each period for each vehicle. This TSP is solved using TSP solver Concorde [1, 2].

4.7 Computational experiments

In this section, we highlight the advantages and disadvantages of our three heuristics. In order to achieve this objective, we perform three experiments.

The first test consists in analyzing the quality of the solution obtained with our three heuristics. Therefore, we compare the performance of our heuristics against an optimal solution approach in different reusable resources capacity situations. Our aim is to analyze the “decomposition cost”: the increase in production and distribution costs implied by using a less coordinated solution methodology. This test is performed on an instance of reduced size (25 customers, 2 types of product, 5 vehicles, 5 time periods).

The last two computational tests are performed on 16 instances of larger size composed of 100 customers, 5 types of product, 7 vehicles and 5 time periods. Those instances differ according to the demand and customer location variability and mean.

The aim of the second experiment is twofold: analyze the performance of our heuristics in terms of computational time and analyze the impact of demand and customer location fluctuation and mean on the heuristics performance. This test is achieved in situation of scarce and excess reusable resource capacity.

The last computational analysis details the impact of variation in production and distribution costs on the performance of the heuristics. The goal is to analyze the impact of relative changes on the performance of our heuristic. This test is achieved with excess reusable resource capacity.

Our computational experiments are obtained with commercial MIP solver XPRESS-MP and with TSP solver Concorde [1, 2].

4.7.1 Problem instances

The geographical position of customers is considered as following a normal distribution whereas the demand of customers follows a Gamma distribution.

The size of the different instances used varies according to the computational tests performed. For the first experiment, we use an instance of reduced size composed of 25 customers, 2 types of product, 5 vehicles and 5 time periods. For
4.7 Computational experiments

For the two other tests, we have used 16 different instances composed each of 100 customers, 5 products, 7 vehicles and 5 time periods. Those instances differ on the type of demand (low/high variance, low/high mean) and on the geographical position of the customer (low/high variance, low/high mean).

The various experiments are realized in situation where the amount of reusable resources available in the system varies (scarce/excess capacity). More details on the dataset are given in the appendix B.

4.7.2 Computational results

Quality of the heuristic solutions

Our aim in this section is to evaluate the performance of our three heuristics compared to an optimal solution approach. By optimal solution approach, we mean the solution obtained by solving our integrated model with a branch-and-cut technique where subtour elimination constraints are added as cuts at each integer node of the branch-and-bound tree. Therefore, we have tested the three heuristics and the optimal solution approach in situations of scarce and excess reusable resources capacity. The production and distribution objective costs were fixed so that the importance of production costs compared to distribution costs were more or less the same. Due to the complexity of solving the global model to optimality [71], the various tests were performed on an instance of reduced size (25 customers, 2 types of products, 5 vehicles). Table 4.1 and Table 4.2 reports results in term of percentage of variation of cost (lot sizing and transportation costs) between the heuristics and the optimal solution procedure as well as the additional amount (expressed in percentage) of reusable resources needed in the system.

Two of the heuristics are based on a decomposition approach. This leads to a risk of infeasibility when there are not enough reusable resources in the system (see

<table>
<thead>
<tr>
<th></th>
<th>Int. GPA-Prod. plan.</th>
<th>Seq. transp.-prod. heur.</th>
<th>Seq. prod.-transp. heur.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lot sizing cost</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Transportation cost</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Total cost</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Table 4.1: Performance test: excess reusable resources capacity
An Integrated model for Production and Distribution decision with reusable resources management

<table>
<thead>
<tr>
<th></th>
<th>Int. GPA-Prod. plan.</th>
<th>Seq. transp.-prod. heur.</th>
<th>Seq. prod.-transp. heur.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lot sizing cost</td>
<td>0%</td>
<td>47.15%</td>
<td>47.15%</td>
</tr>
<tr>
<td>Transportation cost</td>
<td>0.01%</td>
<td>76.23%</td>
<td>76.23%</td>
</tr>
<tr>
<td>Total cost</td>
<td>0.01%</td>
<td>75.54%</td>
<td>75.54%</td>
</tr>
<tr>
<td>% of additional reusable resources</td>
<td>0 %</td>
<td>4.50%</td>
<td>4.50%</td>
</tr>
</tbody>
</table>

Table 4.2: Performance test: scarce reusable resources capacity

Table 4.2). To avoid infeasibility problems, businesses need to rent extra capacity at an expensive cost. This explains the poor performance of those heuristics in that case.

In the case of the production-transportation heuristic, the stock minimal property of the production model [62] implies that production takes place as late as possible (no production capacity limit). This production planning is optimal for the production lot sizing decisions but can be infeasible when reusable resources limit are considered. This leads to more reusable resources in the system. The additional amount of reusable resources needed is determined when solving the transportation problem. For the transportation decisions, there is a coordination between the pick up of reusable resources and delivery of finished products. This allows to optimize the transportation decisions.

For the transportation-production heuristics, the production decisions are taken considering the transportation decisions for the finished product as fixed. Nevertheless, the production planning does not consider the reusable resources decisions which can lead to infeasibility problems. In this heuristic, we have two types of infeasibility that can occur. As production level and transportation quantities have been fixed before the reusable resources decision, reusable resources can be scarce at the production site as well as at the customer site (see Section 4.6.2).

Both of our decomposition heuristics give roughly the same result. This can be explained by the assumptions made on the model as well as the structure of the different sub-models. Regarding the production-transportation heuristic, we know that the production will take place as late as possible (see Section 4.6.1 for more details). Therefore, when the transportation sub-model is solved (with the production variables fixed), the transportation of finished goods will be performed as late as possible. As there is no transportation lead time, no demand backlogging and no
transportation capacity limit, the production and transportation of finished goods
will be done simultaneously. Concerning the transportation-production heuristic,
the same reasoning can be applied. The structure of the transportation sub-model
implies that the amount transported takes place as late as possible. Therefore,
when the production level is calculated with the Wagner-Whitin production sub-
model (with the amount transported fixed by the previous sub-model), we produce
exactly what needs to be sent. Note that additional feasibility problem could have
appeared if the transportation lead time and/or transportation capacity was con-
straining.

The integrated GAP-production heuristic does not have these infeasibility
problems. This leads to better performance compared to our decomposition heuris-
tics. We can also observe that the performance of our integrated heuristic is very
close to the performance of the optimal solution approach.

Performance of the heuristics

In this section, we report results of tests of our three heuristics on 16 instances
of larger size composed of 100 customers, 5 types of product, 7 vehicles and 5
time periods. The three heuristics were run for 30 minutes at most and with
different reusable resource capacity limits (excess/scarc capacity limit). We report
results, in Tables 4.3 and 4.4, in terms of percentage of variation in production
and distribution cost between the decomposition heuristics and the integrated
heuristic. Table 4.5 shows the amount of additional reusable resources (expressed
in percentage) needed in the system when the reusable resource capacity is scarce.

The optimal solution approach was unable to even provide a feasible solution
to the global production and distribution model for any instance in the given time.

Cost analysis

• Unlimited reusable resources capacity

In the case of unlimited reusable resources capacity in the system, the three
heuristics give the same lot sizing and transportation cost for the 16 in-
stances. As there is enough reusable resources in the system, the impact
of no coordination between the production and distribution department is
absorbed by the excess amount of reusable resources in stock.

• Limited reusable resources capacity
In the case of limited reusable resource capacity (see Tables 4.3, 4.4 and 4.5), we observe the same behavior as in the first test. The two decomposition heuristics perform badly due to feasibility problems. In this scenario, extra capacity has to be rented at an expensive cost which leads to poor results.

From Tables 4.3, 4.4 and 4.5, we observe that integration of decisions allows to gain in production as well as in transportation cost. Moreover, the value of integration is greater when the demand of customers has low variability (data sets 9 to 16 (see the appendix for more details)) and the demand’s mean is low. Nevertheless, the demand’s mean do not seem to have an impact as strong as the variability of the demand. In addition, the variability and mean of customer locations does not seem to have an impact on the value of integration. In conclusion, the combination of low demand variability and mean allows to benefit the most from integration of production and distribution decisions.

**Computational time analysis** The three heuristics were run for a maximum of 1800 seconds on the 16 instances in both limited and unlimited reusable resource capacity situations. Our aim here is to compare the time performance of our heuristics. As the most time consuming phase of our heuristics was, in each case, the sub-problem using the general assignment heuristic methodology, we report only the time performance of this phase. Indeed, the sub-problem using the general assignment heuristic is accounting for 98% of the computational time. Therefore, we present performance graphs (see Figures 4.2 and 4.3) that report, for each solution method, the gap between each integer solution found in 1800 seconds and the lower bound for the corresponding heuristic expressed in percentage. As the sub-problem using the general assignment heuristic differs for each of our heuristics, the lower bound used to calculate our gap is different. Therefore, the performance graphs can only be used to analyze the time performance of our heuristics and not the quality of the solution obtained (which was the aim of the previous computational experiments). Note that the x axes is represented in log scale.

In the case of unlimited reusable resources in the system, we observe that the production-transportation heuristic is less time consuming than the two other heuristics. First of all, the production-transportation heuristic finds less integer solutions than the two previous heuristics and secondly the “best” integer solution, which is the same for the three heuristics, is found earlier than for the other
### Table 4.3: Lot sizing Cost analysis: scarce reusable resources capacity-results are reported in terms of percentage of variation in production and distribution cost between the decomposition heuristics and the integrated heuristic.

<table>
<thead>
<tr>
<th>Instances</th>
<th>Seq. transp.-prod. heur.</th>
<th>Seq. prod.-transp. heur.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lot sizing cost</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>28.34%</td>
<td>28.39%</td>
</tr>
<tr>
<td>2</td>
<td>28.61%</td>
<td>28.61%</td>
</tr>
<tr>
<td>3</td>
<td>27.41%</td>
<td>28.33%</td>
</tr>
<tr>
<td>4</td>
<td>28.25%</td>
<td>28.52%</td>
</tr>
<tr>
<td>5</td>
<td>18.12%</td>
<td>18.16%</td>
</tr>
<tr>
<td>6</td>
<td>18.07%</td>
<td>18.27%</td>
</tr>
<tr>
<td>7</td>
<td>15.24%</td>
<td>18.16%</td>
</tr>
<tr>
<td>8</td>
<td>18.31%</td>
<td>18.33%</td>
</tr>
<tr>
<td>9</td>
<td>42.24%</td>
<td>42.25%</td>
</tr>
<tr>
<td>10</td>
<td>42.24%</td>
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<tr>
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<tr>
<td>15</td>
<td>46.23%</td>
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<tr>
<td>16</td>
<td>46.31%</td>
<td>46.31%</td>
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An Integrated model for Production and Distribution decision with reusable resources management

<table>
<thead>
<tr>
<th>Instances</th>
<th>Seq. transp.- prod. heur.</th>
<th>Seq. prod.- transp. heur.</th>
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<tr>
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<td>50.31%</td>
<td>50.37%</td>
</tr>
<tr>
<td>2</td>
<td>50.38%</td>
<td>50.38%</td>
</tr>
<tr>
<td>3</td>
<td>49.92%</td>
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<td>5</td>
<td>33.57%</td>
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</tr>
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<td>6</td>
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<td>10</td>
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</tr>
<tr>
<td>16</td>
<td>75.33%</td>
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</table>

Table 4.4: Transportation Cost analysis: scarce reusable resources capacity-results are reported in terms of percentage of variation in production and distribution cost between the decomposition heuristics and the integrated heuristic.
### Table 4.5: Percentage of additional reusable resources needed in the system when the capacity of reusable resources is scarce.

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<thead>
<tr>
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<th>Seq. transp.-prod. heur.</th>
<th>Seq. prod.-transp. heur.</th>
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<tbody>
<tr>
<td>% of additional reusable resources</td>
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<tr>
<td>1</td>
<td>1.81%</td>
<td>1.81%</td>
</tr>
<tr>
<td>2</td>
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<td>1.81%</td>
</tr>
<tr>
<td>3</td>
<td>1.77%</td>
<td>1.81%</td>
</tr>
<tr>
<td>4</td>
<td>1.81%</td>
<td>1.81%</td>
</tr>
<tr>
<td>5</td>
<td>0.88%</td>
<td>0.88%</td>
</tr>
<tr>
<td>6</td>
<td>0.89%</td>
<td>0.88%</td>
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<tr>
<td>7</td>
<td>0.89%</td>
<td>0.88%</td>
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<tr>
<td>8</td>
<td>0.89%</td>
<td>0.88%</td>
</tr>
<tr>
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<td>5.18%</td>
</tr>
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<tr>
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<td>4.32%</td>
</tr>
</tbody>
</table>
An Integrated model for Production and Distribution decision with reusable resources management

Figure 4.2: Performance profile in the case of unlimited reusable resource capacity - The gap represents the difference between each integer solution found in 1800 seconds and the lower bound for the corresponding heuristic expressed in percentage.

Heuristics. Figure 4.2 presents a typical performance graph of the three heuristics in the case of unlimited reusable resources capacity.

In the case of limited reusable resources, the transportation-production heuristic is the more time consuming while giving worse results than the integrated heuristic and slightly better result than the production-transportation heuristic. Figure 4.3 presents a typical performance graph of the three heuristics in the case of limited reusable resources capacity.

Even though the transportation-production heuristic performs a little better than the production-transportation heuristic, the time performance is a lot worse and does not justify the gain obtained (on average a 0.53% improvement in total cost with the transportation-production heuristic compared with the production-transportation heuristic). The production-transportation heuristic is less time consuming than the integrated heuristic but gives worse result in terms of total production and distribution costs. Nevertheless, the first integer solution for our integrated heuristic is found in maximum 52 seconds and is better than the production-transportation heuristic solution because it does not use extra reusable resources capacity.
4.7 Computational experiments

Figure 4.3: Performance profile in the case of limited reusable resource capacity - The gap represents the difference between each integer solution found in 1800 seconds and the lower bound for the corresponding heuristic expressed in percentage

Sensitivity analysis

We have performed a sensitivity analysis to assess the impact of the production and distribution objective function costs on the performance of the three heuristics. We have used the same instances as the one in the previous test with excess reusable resources.

Our aim in those tests is to consider changes in distribution costs relatively to production costs. Those changes can easily occur in business whenever fuel price, taxes, etc. change.

For the transportation-production heuristic, there is no impact on the production and distribution planning (routing, distribution and production lot sizing decisions) when there are changes in production and distribution objective function costs. This is due to the fact that the objective function of each sub-model is composed only of production or distribution costs.

In the case of the production-transportation heuristic, the production and distribution planning are invariant to changes in the production and distribution objective function costs. The production model considers only production costs which explains why there is no impact on the production planning. Even though the transportation problem contains production cost (storage cost at the manufac-
An Integrated model for Production and Distribution decision with reusable resources management

turing site), it is preferable to produce and directly send products to customers. This can be explained by the structure of our model. It would have been interesting to store finished products at the production facility site if there was a production capacity limit. In our model, we have omitted this limit which implies that whatever relative difference between production and distribution cost, it is never interesting to store at the manufacturing site.

For the integrated heuristic, changes in production and distribution costs have an impact on the production and distribution planning. When distribution costs are greater than production costs, the production planning includes more setups. This is due to the fact that as storage cost is important at the customer site, the demand is satisfied as much as possible on time whereas when distribution costs are less than production costs, the demand is satisfied on stock. In the later case, there will be less production setup and more storage at the customer site. Nevertheless, the impact of those changes on the performance (better production and distribution costs) of the integrated heuristic compared to the decomposition heuristic are negligible.

4.8 Managerial insight

Our computational experiments allow us to derive some managerial rules. First of all, the integration of production and distribution decisions is interesting for the management of reusable resources only if there is scarce reusable resource capacity. In the case of unlimited reusable resource capacity, a traditional production-distribution decomposition method can be used to manage reusable resources. In case of limited reusable resource capacity, companies can benefit, in cost and time, from integration. In addition, businesses facing demand of low variability and mean benefit the most.

4.9 Conclusion

Nowadays, coordination of all functional areas is fundamental in order to improve the efficiency of the supply chain. With the development in information and technological tools, it is now possible to develop decision tools which help manager to coordinate decisions at all level in the supply chain. Our aim in this chapter is to analyze the advantage of coordination between the production and the distri-
bution department when resources are shared between those two functional areas. We focus our study on three decisions: lot-sizing decisions at the production and distribution level and vehicle routing decisions. Therefore, we develop a global production/distribution model and solve this model with three heuristics. Two of those heuristics are based on a decomposition approach of the global model in production and transportation sub-models. The third heuristic offers a higher level of integration by considering transportation decisions when solving the production problem. In all of those heuristics, the transportation decisions are approximated based on Fisher and Jaikumar’s approach. Computational tests show that the performance of the heuristics depends on the amount of reusable resources in the system, on the variability of customer demand and not on the weight of production cost against distribution cost. In addition, we show that the three heuristics allow to solve instances of larger size than an optimal solution approach. This case study allows to appreciate the advantages of intelligent decomposition methodology in another field than inventory and warehouse area. Our integrated heuristic which decomposes our global model in two sub-models has allowed to reduce the computational difficulty linked to our global model while keeping the link between production and distribution decisions strong. Those advantages have been highlighted in our computational results. In addition, our methodology has allowed to handle efficiently reusable resources which are shared between the production and the distribution departments.
An Integrated model for Production and Distribution decision with reusable resources management
Chapter 5

Conclusion

Nowadays, businesses are facing an increased competition, a merciless battle on product prices and a huge cost reduction pressure. Managers are thus looking for suitable tools that enable them to handle those new challenges.

In this thesis, the aim is to propose a methodology that helps building such kind of tools and takes coordination mechanisms into account. We claim that enormous gains in operating costs can be actually achieved by developing new management decision tools which respect the link between decisions of different nature and/or of different time horizons. Traditionally, managers made their business decisions based on hierarchically structured tools. This way of acting met the expectations of decision makers at that time. Those needs were related to the complexity of problems and the respect of the hierarchical structure of business decisions. Today with the achieved improvement in information technologies, it is possible to create decision tools able to provide answers to the new needs of managers. Those new decision tools aim at solving mangers problems in a more integrated view which allows to respect coordination mechanisms. This results in decision tools that are complex to solve and an adequate decision making process is needed.

5.1 Objectives and methodology

Based on the above facts, we have articulated our research and the structure of the thesis around four research questions.
Question 1: Integration is an approach which can be applied in various ways: problems integration, models integration and tools integration. Is it then possible to suggest a taxonomy, based on a literature review, in order to capture the exact meaning of integration and to enhance the comprehension of its benefits?

In order to answer this research question and to understand the new challenges faced by businesses, an analysis of the various streams of research on collaboration tools is realized in Chapter 1. Our aim through this analysis is to understand the various collaboration techniques in order to limit the scope of our study. We have concentrated our analysis on three fields of research: hierarchical planning, collaborative planning and supply chain integration.

From this analysis, we have concluded that decision tools based on supply chain integration are the most suitable for our objectives. Nevertheless, we could not find information on the methodology to apply to create integrated decision tools. Consequently, in Chapter 2, we detailed our proposition of the methodological process to follow to build decision tools based on supply chain integration. We make the distinction between three types of integration: model integration (Model I.), method integration (method I.) and tool integration (Tool I.). Our various types of integration has been validated on selected research articles.

As a result, we suggest to create integrated decision making tools based on a mix of model and method integration techniques. Whereas the formulation of a global model makes it possible to understand the link between issues, intelligent decomposition of the global model reduces computational complexity without destroying the link between decisions in the supply chain. Our integrated process decision making tool (Intelligent decomposition approach) is summarized in Figure 5.1 and has been tested on two case studies.

Question 2: Monolithically constructed models are difficult to solve: their optimization consumes time and computing resources. Despite those difficulties, what are the evidences in favor of integration?

Question 3: How can the traditional sequential procedure still be applied in a more integrated environment?

Question 4: How can an integrated approach handle various aspects of resources management more accurately than what is currently done in the literature?
In order to answer to those research questions, two case studies have been performed. The first case study concerns the integration of tactical inventory and warehouse decisions. First of all, the decisions considered have been structured and are as followed: the replenishment decision at the inventory management level, the allocation of products to warehousing systems and the assignment of products to storage locations at the warehousing management level. Those decisions have been translated into a global mathematical model. Through an analysis of this global mathematical model, two decomposition procedures have been realized which offer different level of coordination between those decisions. The first decomposition procedure decomposes the global model in two sub-models: Inventory sub-model and warehouse sub-model. The inventory sub-model is created by fixing the value of the warehouse variables in the global model and relaxing the warehouse capacity constraint as well as approximating the ordering cost. The second decomposition method divides the global model in the same two sub-models as in the previous methodology. Each of the sub-models contains variables related to the other sub-model: the inventory (respectively warehouse) sub-model contains the optimal value of the warehouse (respectively inventory) variables. Those two sub-models are solved iteratively and at each iteration the value of warehouse and inventory variables are updated. Each of the sub-models created are linked together and are solved using a Lagrangian combined with a Branch-and-Bound procedure. Those integrated decision making techniques are represented in Figure 5.2 and Figure 5.3. Through computational experiments, those various decomposition techniques are compared and the coordination strength is measured through a comparison of inventory and warehouse cost and factors making integration interesting are
highlighted. In addition, we analyze the critical resource impact on coordination. In this case study, the critical resource is the capacity of the warehouse. Results show that when the capacity of the warehouse is limited, coordination between inventory and warehouse decisions is needed.

The second case study concerns tactical and operational production and distribution decisions. As in the previous case study, the decisions considered have been delimited. We deal only with tactical lot-sizing decisions at the production and distribution level and operational vehicle routing problem. A global model which integrates those three decisions has been formulated and has led to the proposition of three different methodologies. Those methodologies are based on a
5.2 Main research results

In this section, our aim is to put forward the major research findings achieved in this thesis.

5.2.1 Hierarchical planning vs supply chain integration

Supply chain integration is usually opposed to hierarchical planning when, in fact, those two methodologies address issues of different time horizons and information uncertainty. Indeed, supply chain integration implies that the information on inputs used in the decision process is available. This implies that the decisions to coordinate must, more or less, be of the same time length. In this thesis, we have developed integrated decision tools that coordinate tactical warehouse decomposition of the global model in two or three sub-models depending on the level of integration achieved. Those sub-models are then connected together and solved using a Generalized Assignment Heuristic and a Branch-and-cut procedure. Those integrated decision making techniques are represented in Figure 5.4, Figure 5.5 and Figure 5.6. Computational tests are achieved in order to calculate the value of integration through an analysis of production and distribution costs. Factors in favor of supply chain integration are also highlighted. In this case study, we considered reusable resources which are shared between those two departments. Through computational tests, it has been proven that when those resources are critical, coordination is needed.
Figure 5.5: Integrated decision making techniques: an application on production and distribution decisions

Figure 5.6: Integrated decision making techniques: an application on production and distribution decisions
and inventory decisions as well as tactical-operational production and distribution decisions. Following us, it would have been difficult to integrate strategic and operational decision because those decisions concern different level of information uncertainty.

In the case of hierarchical planning, the planning can be made on a time horizon which covers strategic and operational decisions. Information at the strategic level may be uncertain and unknown. Therefore aggregate information is used at the strategic level and more precise information will be used at the operational level to take a decision. The issue with this method concerns the link to make between the aggregate and disaggregate information.

5.2.2 Integration types

In chapter 2, we have introduce various types of integration namely model integration, method integration and tool integration. In the two case studies performed in this thesis, those three integration types were used.

For the inventory and warehouse integrated decision tool, model integration is achieved through the creation of a global inventory and warehouse model. This global model turned out to be computationally complex for large industrial database. Based on the idea underlying method integration, we decompose this integrated model in sub-models such that the coordination link represented in the global model is maintained as much as possible. Various decomposition procedures have been developed and tested on three aspects: the computational complexity reduction, the coordination strength and the sensibility to warehouse and inventory costs. Tool integration is not needed in this case study in order to implement the various integrated decision tools because only one information tool (commercial MIP solver XPRESS-MP) is used.

In the production and distribution case study, as in the previous case study, a global model is created (model integration) and results in our first integrated decision tool. This integrated decision tool could solve instances of small size and is therefore not useful for industrial application. Consequently, method integration is used in order to obtain an integrated decision tool which could solve instances of industrial size. Nevertheless, method integration is applied in such a way that coordination strength is maintained as strong as possible. Finally, tool integration is needed in order to implement the integrated production and distribution decision tools. Indeed, commercial MIP solver XPRESS-MP as well as the publicly available TSP solver Concorde [1, 2] is used to implement the production and
distribution sub-models. The link between those two information tools has been realized by the mean of an Excel file.

5.2.3 Resource availability

From the computational results performed in each of our case studies, we could observe some similar conclusions concerning the advantages of supply chain integration. In each of our applications, the integrated decision tools are created based on a decomposition procedure of a global integrated model. This decomposition procedure is a relaxation procedure where the coordination of decisions is reduced. Various decomposition techniques are achieved in each of the case studies that offer different level of relaxation and therefore of coordination of decisions. Through our computational tests, we have observed that in the case where the critical resource (warehouse capacity for the first case study or reusable resource for the second case study) is scarce then the integrated decision tool with the weakest relaxation has a negative impact on results obtained. Indeed, in the first case study, two decision tools are developed: an heuristic sequential solution and an integrated heuristic solution. The first decision tool is based on a relaxation that is weaker than the second decision tool. We have observed that in case of limited warehouse capacity, the integrated heuristic solution is offering better results than the heuristic sequential solution. In case of unlimited warehouse capacity, both our decision tools are giving the same results. In our second case study, three different decision tools are developed based on different level of relaxation. We observe in our computational tests the same results as in the first application. Our decision tool based on a strongest relaxation was the most valuable when reusable resources were scarce.

5.3 Managerial implications

We outline below the main advantages and the implications on the managerial side of our proposed integration strategy.

5.3.1 The concept of integrated decision tool is easily understandable by decision makers

As said in the introduction of this thesis, hierarchical planning method is a well known decision tool which is used often in business environment. Decision makers understand and are used to such kind of tool and may be reluctant to change it.
We think that businesses willing to install decision tools based on an integrated approach will be less reluctant because it is an understandable and intuitive tool. Explaining how such decision tools work and the importance of those decision tools is straightforward. Indeed, as in hierarchical planning, decomposition techniques are used. One of the advantage of decomposition techniques is that it makes the problem more understandable. Our decision making process uses the same idea but focuses on additional objectives. The decision makers’s problem is decomposed in sub-problems of reduced size and therefore those sub-problems are easier to understand than the original problem . In our case, instead of using the hierarchical structure and the time horizon as basis for the decomposition, we use the structure of the problem in order to decompose the decisions in a way to keep coordination in the supply chain strong.

5.3.2 The proposed integrated tools can be used as a management tool

This tool can be used as a day to day management tool. For example, the integrated production and distribution decision tool can be used on a daily basis for production planning and vehicle routing decisions. In that case, a business manager who wants to implement an integrated decision tool can easily insert it in the actual architecture of the information system of the company. Hereafter (Figure 5.7), we present an example of a possible implementation of an integrated decision tool in a traditional information system structure. To do so, we need to establish links between different modules (e.g. Methods’ library, models’ library, . . .) of different nature. The shaded part of the figure represents our integrated decision tool. Decision makers interact with the integrated tool through the user interface. Either the decision makers must choose inside a list of questions which one is the closest to their interrogations or the decision makers must type their questions and key words are selected by a program. Once those questions have been identified, an expert intervene in order to translate these in a global model and performs the decomposition procedure by the mean of a mathematical models library. Data is uploaded from the ERP database. Those sub-models are then solved by using a method taken from the methods library and implemented using a solver from the solvers library. The resulting outputs are given in various file formats: PDF file, Excel file, Text file, . . . for the convenience of the various users and the ERP database is adapted. Outputs can also be used as inputs for other information tools in the company. The expert involvement could be reduced if more
5.3.3 Some managerial rules can be derived from the use of an integrated tool

This integrated decision tool can be used to point out the factors in favor of coordination. Indeed, companies can simulate a general setting (e.g. numbers of clients, number of different products, homogeneous vehicles (even if it is not the case)...) and analyze the impacts of the characteristics of the setting on the importance of the coordination mechanism. In our first case study, we have seen that when the capacity of the warehouse is limited then coordination mechanisms are important. We have also analyzed the impact of changes in warehouse and inventory costs on coordination. In our second case study, limited reusable resources made coordination important and the relative importance of production against distribution.
5.4 Research limits

Hereafter, we list the main limits regarding the thesis’s methodology and concerning the integrated tools developed in this thesis.

5.4.1 Limited number of case studies

Our methodology has been tested on two case studies and has proven to be interesting in terms of value of coordination and time complexity. In order to be able to generalize our results, our integrated decision making process should be applied in other environments where different type of decisions are integrated at different levels and regarding different actors/firms.

5.4.2 The importance of inputs and outputs in our methodology

In order to apply our methodology in an effective manner, it is assumed that the actors involved in the collaboration totally share the inputs needed for the collaboration. We consider in our work the problem linked with inputs sharing mechanisms and did not analyze the importance of those inputs on the collaboration gain. We also did not tackle the issue related to the sharing of the gain obtained by the collaboration between the various actors. Collaboration is going to take place only if the actors involved in the collaboration accept the surplus sharing agreement and are willing to share totally the inputs needed for the collaboration.

5.4.3 Expertise is needed to apply the decomposition scheme

A last important issue related to our integrated decision making process is that the decomposition applied to the global model can only be achieved by an expert. Indeed, the decomposition applied is realized after an analysis of the global model and therefore is dependent on the global model structure and on the environment studied. For example, in the case of the integrated warehouse and inventory model, two different transformations had to be applied to the global model in order to...
achieve the decomposition procedure. In one of the methodologies proposed, a relaxation and an approximation had to be applied to the global model and a transformation in two linear models for the other methodology. Therefore, only an expert is able to achieve such modifications in order to apply the decomposition procedure correctly. This makes the decomposition rule of the integrated process difficult to generalize.

5.5 Future work

We end this final chapter by giving some directions for future work.

5.5.1 The analysis of sharing mechanisms to distribute the collaboration gain

Following the various limits exposed here over, one possible future work would be to analyze how to include in our integrated decision making process the decision on sharing the collaboration gain. This is an important aspect of the decision making process which is not taken into account in this project and which can be determinant for the success of the collaboration.

5.5.2 A priori analysis of structural factors in favor of coordination

Integration of decisions is an important business component but is generally difficult to enforce due to the internal reorganization needed with such process. Therefore, an interesting study is to determine which structural factors (e.g. factors which depends on the environment and the structure of the business) enables to determine a priori if it is interesting to consider integrated decision tools. For example, we have shown in our two case studies that integrated decision tools are interesting when an important resource is available in limited quantity. In the case of our inventory and warehouse integrated model, we have shown that coordinating warehouse and inventory decisions is interesting when the capacity (resource) of the warehouse is limited. In the case of production and distribution decisions, we have shown that coordinating those decisions is only interesting when the reusable resource which is shared between production and distribution department is available in limited quantity. In other word, if this resource is available in excess then
the efficiency of the supply chain will not be improved with better coordination mechanisms.

5.5.3 Comparisons of integrated techniques

In this thesis, we have introduced different integrated decision making techniques (model integration, method integration, technique integration) that we used to build our integrated decision making approach. Our approach is a combination of model and method integration. It can be interesting to analyze on a same setting, the three different techniques. This will allow to see how the techniques perform on the same case study. In addition, the factors in favor of one technique more than another can be highlighted.
Appendix A

Appendix for the Integrated Model for Warehouse and Inventory Planning case study

Description of the dataset

1. Products
2. Objective function warehouse and inventory cost description
Appendix for the Integrated Model for Warehouse and Inventory Planning case study

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| reserve reception cost | 5 |
| forward reception cost | 7 |
| advance replenishment cost | 20 |
| concurrent replenishment cost | 25 |
| forward picking cost | 2 |
| reserve picking cost | 10 |
| additional capacity cost | 50 |

| inventory carrying cost | 3 |
| acquisition cost | 6 |
| shortage cost | 100 |

Sensitivity analysis : scenario description

(a) Scenario 1: Warehouse sensitivity test

The relationship between the warehouse cost coefficients can be expressed as followed:
\begin{align*}
CostF &= \beta_1 \times CostR \\
PickCostR &= \beta_2 \times PickCostF \\
CostRepC &= \beta_3 \times CostRepA
\end{align*}

We have construct 27 scenarios by varying the $\beta_1$, $\beta_2$ and $\beta_3$ from 1.5 to 2.5.

(b) Scenario 2: Inventory and warehouse sensitivity test

We have construct 4 scenarios: the inventory cost being one third, half, three quarter and 1.25 of the warehouse cost.

(c) Scenario 3: Inventory sensitivity test

We have constructed the following relationship:

\begin{align*}
CostRecp &= \beta_4 \times CostCar \\
CostShort &= \beta_5 \times CostCar
\end{align*}

We have construct 9 scenarios by varying the $\beta_4$ and $\beta_5$ from 1.5 to 2.5.
Appendix B

Appendix for An Integrated model for Production and Distribution decision in an environment of shared resources case study

Description of the dataset

- Demand of customer

  The demand of customer follows a beta distribution with parameters $\alpha$ and $\beta$. For each instances, the coefficient of variation was used as a mean to measure the variability of each data set. Data set of low/high variability have a coefficient of variance of respectively 0.5/2.

This gives the value for the beta distribution as followed:

1. low variability
   - $\alpha = 2$ and $\beta = 3$
   - $\alpha = 3$ and $\beta = 12$
   - Instances: 1 to 8
2. High variability
   \[ \alpha = 0.2 \text{ and } \beta = 4.8 \]
   \[ \alpha = 0.15 \text{ and } \beta = 1.7 \]
   Instances: 9 to 16

Concerning the mean of customer demand, the data set 5 to 12 has a mean demand three time bigger than data set 1 to 4 and 13 to 16.

• customer location

For the location of customers, we have decided to use a normal distribution. The mean and the variance of each data set has been chosen so that variability is either high or low and the mean is either high or low relative to each other.

1. \( \mu = 10 \sigma = 5 \): variability low, mean low
2. \( \mu = 20 \sigma = 10 \): variability low, mean high
3. \( \mu = 10 \sigma = 40 \): variability high, mean high
4. \( \mu = 5 \sigma = 10 \): variability high, mean low

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Bibliography


