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Summary

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Keywords : Specific assets, contract design, asymmetric information, rent capture, Bayesian belief

JEL Classification: C44, C73, D86

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DYNAMIC SUPPLY CHAIN COORDINATION GAMES WITH REPEATED BARGAINING

Xavier BRUSSET[1] and Per J. AGRELL[2]

January 2010

Abstract

Coordination of supply chains may require investments in relationship-specific assets (RSA), information or human resources from all or a subset of the partners. These investments are typically partially non-verifiable, being based on internal resources or opportunity costs. We extend previous work on the topic of coordination of complementary relationship-specific "selfish" investments by considering a context of repeated single-period bargaining under double asymmetric information and outside options. Our model is a dyad in which a supplier offers a single-price single-period contract for a good to a downstream manufacturer (or retailer) who can accept or turn to some third party before investing in the corresponding RSA. We show results when the supplier uses repeated updating to estimate the manufacturer's investment cost, determining a non-decreasing sequence of offers to incite the coordination investment under asymmetric information on cost and reservation utility for the manufacturer. The discussion of the results suggests that the model may explain some differences observed between collaborative theory and practice, namely asymmetry of information, over-investments and delayed agreement because of holdup risk. The work is illustrated with numerical examples.

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1 Introduction

A supply chain is a network of connected and interdependent organizations mutually and co-operatively working together to control, manage and improve the flow of materials and information from suppliers to end users Aitken (1998). There is consensus both in academic and in applied circles that supply chain optimization involves a relatively important emphasis on intra-functional and inter-organizational collaboration, leading to coordination of processes, orders and information in areas such as customer service, production planning, logistics, and capacity utilization. Supply chain coordination has also been of very high importance for supply chain managers in practice CSC (2009) for the last ten years. Still, Kampstra et al. (2006) notice that progress towards deeper collaboration and coordination with upstream suppliers and downstream customers is slow and frequently disappointing in practice. The authors cite, among several reasons for failure, the lack of trust, fear of external competition, missing infrastructure and financial barriers for the sharing of resources and gains. Real supply chain collaboration is documented as a rather slow process built on gradually increased trust if the prerequisites for success, i.e. an adequately designed and financed plan for how the coordination instruments should be developed, deployed and monitored, are present. When it works, the partners invest time, capital and human resources into adjusting their operations to their supply chain partners' corresponding processes, e.g. by changing and coordinating product and packaging dimensions, IT standards, EAN or RFID codes, product catalogues, product development platforms, production planning frequency, detail and systems. In particular the infrastructure standard investments that Kampstra et al. (2006) identify as lagging or missing are relationship-specific, i.e. the adjustment to a given customer's IT standards has little or no value outside of that supply chain.

We identify three essential characteristics for these investments that influence the ability of the supply chain to design coordinating contracts. First, the specificity of the coordination investment, that gives rational reason to fear hold-up (i.e. post-contractual opportunistic action) from the

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The scientific responsibility is assumed by the authors.

supplier. Holdups are widely discussed in the academic literature since Williamson (1975, 1985). Classical examples of holdups include the specialized dies used by Fisher Body to stamp out auto bodies for GM cars Klein et al. (1978). A more recent example could arguably be the situation in which Ryanair has found itself when Boeing decided to increase the cost of long-term service contracts O'Doherty (2009). Since we are interested in the investment incentives, what matters is not the frequency of the outcome but the potential exposure to it.

Second, the investments of this type are intrinsically consisting of two types of cost; verifiable and non-verifiable. The projects often mobilize substantial share of non-verifiable cost, such as internal allocation of resources, opportunity costs for time etc and projects that are coordinated with other internal projects. An additional difficulty in the verifiability lies in the non-uniqueness of the supply chain affiliation, i.e. the same partner may be participating in different roles in several supply chains, potentially in downstream competition. Thus, even in the cases when investments are well-defined and verifiable, the potential cost sharing on multiple projects make "open-book" procedures ineffective in determining actual costs.

Third, the coordination investments are empirically (Cf. Kampstra et al., 2006) subject to continuous and repeated financial negotiations within the supply chain, often over several product generations.

In the particular case of logistics, Knemeyer et al. (2003) has surveyed the outsourcing practice and shown that it involves investments in specific assets and non-retrievable commitments of resources. Sucky (2007) point to the trend towards outsourcing logistic activities as support to the argument that large firms are focusing their activities on their perceived core competencies. In this paper, we address the question on how supply chains may contractually coordinate their relationship-specific investments by a stylized dynamic dyadic model of a supply chain that takes into account the three main features above; i.e hold-up incentives from asset specificity, asymmetric information on costs and opportunity costs from participation in other supply chains and repeated bargaining on coordination investments during a finite horizon. Whereas the literature has suggested a range of remedies to the hold-up problem, such as in Hart and Moore (1990) and references below, most work address the problem from one or two of these perspectives. Our contribution is thus both positive and normative. From a positive viewpoint, the findings for the investment delays and distortions in our model correspond to anecdotal evidence stemming from the stated managerial importance of the action, the available results from the mechanism design literature and the high failure rate of the actual investments. From a decision-making stance, the Bayesian updating mechanism proposed for the coordinator may be used under more general settings to inform sequential bidding procedures.

To structure the presentation, we first present the centralized benchmark under full information. This case of common information is not only the reference point for efficiency estimations, but may also exist in vertically integrated organizations where the supplier is e.g. a production division and the manufacturer a distribution organization. We then investigate the case where information about relation-specific investment cost is private to the manufacturer, the supplier only has some prior belief on the costs. We show how the supplier is able to extract extra rent from the manufacturer even when she is not fully informed about the manufacturer's other outside opportunities or investment cost.

Our results indicate that the supplier will submit a sequence of bids which leads to a positive expected delay in acceptance by the manufacturer. It is also shown, contrary to the centralized benchmark, that the asymmetric information induces over-investments by the manufacturer so as to create options, lowering both the probability of profitable interaction and the overall supply chain rents. In the following section, we give some elements of related literature on the subject. The third section describes the model in which two distinct cases are investigated. We present in section 3.3 the full information case and section 3.4 covers the case where the supplier is unaware of the investment costs that the manufacturer faces. To enable the reader to grasp some of the significant points, a numerical instance helps to position the different tradeoffs in Section 4. We conclude in Section 5.

2 Literature review

The holdup problem under incomplete contracting and asymmetric information has attracted considerable academic attention in economics, marketing and supply chain management. The properties of hold-up, asymmetry of information, renegotiation, incompleteness of contracts, switching costs and lock-ins have been investigated by authors such as Philippe Aghion, Drew Fudenberg, Oliver Hart, Paul Klemperer, John Moore and Jean Tirole among others through a range of interesting models. The stream in marketing literature on consumer switching costs and lock-in by competing firms is particularly abundant. The models explored as presented in the literature review in Farrell and Klemperer (2007) and references therein (Klemperer, 1987a,b, 1995, are the most relevant) are often restricted to full-information, two-period settings with endogenous downstream prices for various market organizations. Within supply chain management, several models explore the influence of a supplier's offers on the buyer's decision Sucky (2006, 2004), and how the supplier can tailor his offers to obtain information private to the buyer Li et al. (2009). In the case where the downstream partner Cho and Gerchak (2005) or partners Plambeck and Taylor (2007) is (are) endowed with operating costs and the possibility to invest, provide several coordination mechanisms for the decentralized chain.

The starting point for the present model is the standard contracting model with complete but unverifiable information and "ex ante renegotiation" Maskin (1999); Maskin and Moore (1999); Watson (2006). Our model is an application of Game Theory in supply chain management, as characterized in Cachon and Netessine (2004) and Leng and Parlar (2005) and references therein. Some additional structure is added to the stylized problem by (i) considering a finite multi-period horizon, (ii) considering private information on both the coordination investment and the manufacturer's reservation utility, (iii) by excluding contractual commitment. These assumptions match firms involved in B-to-B transactions, long-term relationships, asymmetric information of the buyer's willingness to pay and sophisticated multi-period contract renegotiation. In the present model, additionally to the contract payments made, we investigate the cases where the seller offers a side-payment to the buyer. This type of side-payment is not of the supply chain coordinating type as investigated and discussed in Leng and Zhu (2009) but rather of the rent expropriation one. Cvsa and Gilbert (2002) look into the purchasing incentives offered by a monopolistic supplier to competing buyers to induce strategic commitment and hence early order-quantity decisions in the face of demand uncertainty. With this incentive, the whole supply chain gains without compromising any of the supply chain members' profits.

The current model draws on a static model under commitment as in Brusset (2009). The present setting resembles the sequential bargaining or renegotiation of rental price with one buyer under asymmetric information about his willingness-to-pay seen in Fudenberg and Tirole (1983) which characterizes the set of equilibria of two-period bargaining games when the seller and buyer each have two potential types (two-sided incomplete information), when the seller makes the offers and when the players alternate making offers. The single-buyer interpretation when the buyer is willing to trade and profitable mutual interaction is given has been looked into by Fudenberg et al. (1985) which demonstrate that a perfect Bayesian equilibrium exists when the buyer's type follows a smooth bounded density.

Segal and Whinston (2002) provide an excellent survey over mechanism design with renegotiation in settings like the current one, i.e. with hold-up risk and asymmetric information on "selfish investments". Tirole (1986) and Edlin and Reichelstein (1996) deal with investment in cost reduction by the seller which results in an advantage over the uninformed buyer. In the present model, it is the buyer who realizes a selfish investment and the seller who is not informed of the investment cost ("selfish investments" as in Hart and Moore, 1999). In Tirole (1986), the seller obtains an information rent whereas in our model the buyer does not disclose the investment cost to the seller so as to mitigate the hold-up risk in future periods.

In the games theoretic literature, Watson and Wignall (2009) builds upon the standard contracting model using unverifiable rather than incomplete information, verifiable and unverifiable actions and with and without renegotiation. In Miyagawa et al. (2008), the folk theorem in repeated games where a player can buy information is presented. Segal and Whinston (2000) investigates the use of renegotiable exclusivity contracts with the seller investing in RSA and the buyer being able to buy from a third party. In González (2004), the agent faces a hold-up situation while making a cost-reducing specific investment unobservable by the principal. To escape the hold-up, the agent randomizes the investment whereas the principal offers screening contracts. In Battaglini (2007), the model characterizes the optimal renegotiation-proof contract in a dynamic principal-agent model in which the agent can change types stochastically over time by means of contract menus offered by the principal.

We consider that the supplier is a Bayesian rational player who is able to sequentially update her belief of the manufacturer's cost and outside option using responses to past offers. This mechanism is generally called Bayesian updating with cutoff. The updating of beliefs using cutoffs has been studied in Hart and Tirole (1988) but that mechanism cannot be applied *ex abrupto* here because of the difference in the buyer's and seller's motivations and utilities. In Cachon and Lariviere (1999), another Bayesian game based on exchange only of offers and responses (no signaling) is the capacity allocation problem facing a single supplier with multiple retailers enjoying private demand information.

In this model the new prior, based upon the updating of the posterior, is defined using a simplicity criterion, or "Simplicity Postulate" as defined by Harold Jeffreys in the sense that the prior must be built with as few parameters as possible. To do so in this paper, the posterior distribution on the parameter has the same distributional form as the prior and updating these parameters can be done easily Porteus (2002).

3 Model

Two players, a manufacturer (he) and a supplier (she) may engage in mutually beneficial trade over a finite horizon of $i \in \{1, \ldots, n\}$ periods. The manufacturer can trade with the supplier or with an outside option. If he trades with the supplier, he invests A (strictly positive, irreversible). He invests a (strictly positive, irreversible) if choosing to trade with the outside option. The effect of the respective investments is lasting, such that trade is enabled in any successive period following the corresponding investment. Note that this setting means that the successive periods are interrelated and not exchangeable as in many multi-period with renegotiation game theoretic settings Watson (2006). The models assume that the value of trade is always higher than the cost. The manufacturer minimizes his expected cost C of trading either with the supplier or the outside option. The supplier can trade costlessly with the manufacturer but is required to sink a specific investment a_s to initiate trade with a third party. The supplier's objective is to maximize her partial profit function Π_s[1].

In time (see figure 1), the supplier, as Stackelberg leader, offers a contract. If the manufacturer accepts, he invests in the required specific assets A, unless not done in any prior period. Services are performed and payout takes place[2]. If the manufacturer rejects the offer, both turn to outside options: incurring the corresponding enabling investments a and a_s, respectively, unless already sunk (this option is not represented in figure 1).

At each period i, the supplier is offering single-period take-it-or-leave-it wholesale price contracts U^i with $i \in \{1, \ldots, n\}$. The strictly positive contract (i.e. service fee) which the manufacturer may sign with some third party is labeled u and shall be considered to be time invariant. The supplier can also sign a strictly positive time-invariant contract u_s with a third party. We call δ_m^i the manufacturer's participation decision variable in period $i \in \{0, 1\}$. Table 1 recapitulates the notation relative to this paper.

The investments made by the manufacturer are unobservable by the supplier. We shall study two information scenarios. In §3.3, the supplier is informed of the cost to the manufacturer of the investment he realizes. In §3.4, the supplier does not possess this information and so relies on his beliefs.

[1] The third parties for the supplier and the manufacturer are not strategic players and can be seen as recourse to an uncoordinated market.

[2] Since there is no new information revealed, compliance conditions are irrelevant here.

Table 1: Table of notations

Type	Notation	Definition
Manufacturer	A	Specific asset investment with the supplier
	a	Specific asset investment with outside option
	u	contract available to manufacturer from third party
	U^i	contract offered in period i by supplier to manufacturer
	$C^i(.)$	manufacturer's cost function in period i
	δ_m^i	binary decision variable, 1 when agreeing with supplier
	v_m^i	manufacturer trading strategy vectors in state i
	R_m^i	manufacturer traading strategy set in state i
Supplier	a_s	specific investment by supplier in outside option
	u_s	outside option contract available in period i to supplier
	$\Pi_s^i(.)$	supplier's profit function in period i
Supplier beliefs	$f_Z(A)$,	pdf of belief about $Z = u + a - A$
	$F_Z(Z)$	cdf of belief about $Z = u + a - A$
	$f_A(A)$	pdf of belief of A
	$F_A(A)$	cdf of belief of A
	$f_a(a)$	pdf and cdf of supplier's belief of a
	$F_a(a)$	cdf of supplier's belief of a
	p_1	parameter set in period 1 of belief distribution
	ω	increment to p_1
	$f_{Ai}(A^i)$	pdf of supplier's revised belief of A in period i
	$F_{Ai}(A^i)$	cdf of supplier's revised belief of A in period i
	L^i	$F_{Ai}(A^i)\big((n-i+1)u - A^i\big)+$ $\overline{F_{Ai}}(A_i)\big(u_s + L^{i+1}\big)$
	v_s^i	supplier strategy vectors in state i
	R_s^i	supplier strategy set in state i

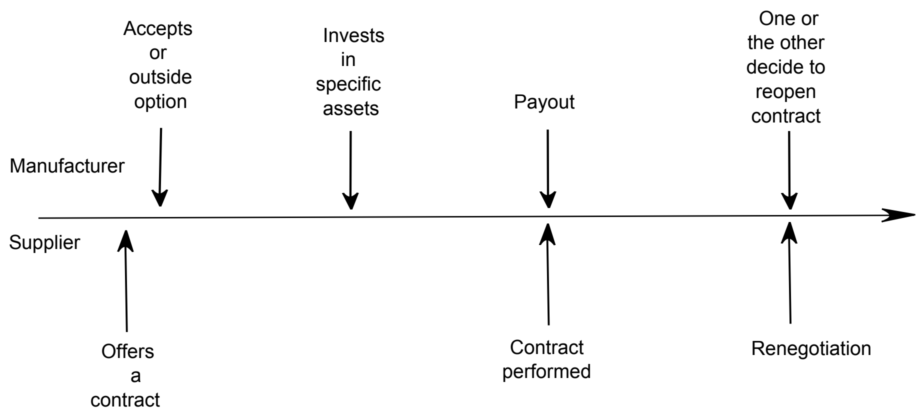

Figure 1: Timeline of events when manufacturer and supplier agree on a contract and to a new relationship. If the manufacturer in one turn of the game does not agree to a contract, trade with third parties occur (not represented).

3.1 Updating process for the supplier

We assume that the supplier is a Bayesian rationalist who builds her assumptions from her experience at the start of the first period. Using these priors, she calculates her most profitable estimate of the variables involved and makes the corresponding offer to the manufacturer. Unless the manufacturer agrees to the first contract offered, in the following period the supplier must update her belief about these unknowns in a *dynamic updating process* Selten (1975). This belief is based upon a distribution of the unknowns which can be described by continuous distribution functions with increasing failure rates (IFR) as defined in Barlow and Proschan (1965). These distributions are characterized by a single parameter which is a function of the period and a prior estimate of this parameter in period 1. Given the complexity of determining a dynamically optimal updating policy under a given set of priors, we opt for a pragmatic approach that provides a fixed increment in the updating. The increment is a parameter in the model, although theoretically it could be a function of past responses, remaining horizon and updated priors. We have

$$p^n = p^1 + n\omega, \tag{1}$$

where ω is the increment between periods. If we consider that p^1 is the mean of the distribution, ω is the scaling coefficient. Each period's prior should be updated using the posterior from the preceding period. Since the manufacturer refused the earlier period's offer, then the prior was too low and the parameter should be increased by this posterior to define the new parameter. However, given that we shall proceed in a backward recursion to solve the evaluation of the unknowns in previous periods, applying H. Jeffreys' *Simplicity Postulate* and following the method provided in Porteus (2002), we set ω as fixed in all periods and will start with the last period.

The distribution can thus be scaled by a parameter using a function of the belief about the unknown to enhance the likeliness of the manufacturer's acceptance but without compromising the supplier's profit. We have

$$F^i(.|p^i) = F^{i-1}(.|p^{i-1}) \tag{2}$$

with F^i as the distribution function of the belief about the unknown in period i. In the following we shall indicate each unknown's distribution function only by its period superscript.

3.2 Coordinated benchmark

Consider the integration of the two firms under a common ownership. The per-period benefit of service is positive. The problem is a cost minimization one. Denote the joint cost function $V(.)$ The integrated firm has two options: (i) performing the services internally by the supplier, or (ii) relying on external service provision from period 1. Naturally, the option of changing regime has no value to the integrated firm as the outside options have constant prices. $V^* = AV' = a + a_s + n^*(u - u_s)$. By inspection, the shifting policy dominates iff $a + a_s + n(u - u_s) < A$. We limit our interest to the "mutual beneficial trade" case, defined by the following two conditions:

$$\begin{aligned} nu + a - A &\geq nu_s - a_s. && \text{Condition 1} \\ u &\geq u_s, && \text{Condition 2} \end{aligned} \tag{3}$$

If the second condition is violated, there is no marginal room for beneficial trade. The first condition provides a basis for evaluation of any switching policy, say in period τ. The coordination loss equals then:

$$V(\tau) - V^* = A + a + a_s + \tau(u - u_s) - A = a + a_s + \tau(u - u_s). \tag{4}$$

Using condition 2 in (4), we can then state that the marginal cost of delay is positive and that there are two sources of coordination losses: overinvestment in relationship assets $a + a_s$ and inferior trade conditions $u - u_s$.

Table 2: The manufacturer's investment in RSA according to his decisions in first (horizontal) and second (vertical) period.

		δ_m^1	
δ_m^2	1	$A+a$	A
	0	a	$A+a$
		0	1

Table 3: The supplier's investment in RSA according to the manufacturer's decisions in first (horizontal) and second (vertical) period.

		δ_m^1	
δ_m^2	1	a_s	0
	0	a_s	a_s
		0	1

These and other aspects will be studied in depth in the asymmetric information scenario in §3.4.

3.3 Full information and renegotiation

In the full information case, the supplier presents offers using full information about the manufacturer's investment costs A and a as well as his outside options u. The corresponding relationship specific investments which he incurs are presented in table 3 for a two-period example.

We enumerate the manufacturer's options in the n period game starting with the decisions to be taken at the leaves of the decision tree.

The game is logically decomposed in three states depending on the enabling investment(s) by the manufacturer: (S) exclusive investment for the supplier, (ST) investments for both the supplier and the third party, (T) investment only in third party trade. Calculations of the manufacturer's optimal contract for each state are in Appendix §6.1.

Using the mutual beneficial trade conditions (3) in State S , we present in figure 2 a graph of the acceptation area. The area in which the supplier will offer mutually interesting terms to the manufacturer is when the cost for the RSA that the manufacturer has to invest in to work with the supplier is lower than what both would have to invest separately if they turned to their respective outside options.

We present the result of the common information game as a Proposition.

Proposition 1 *Full information*

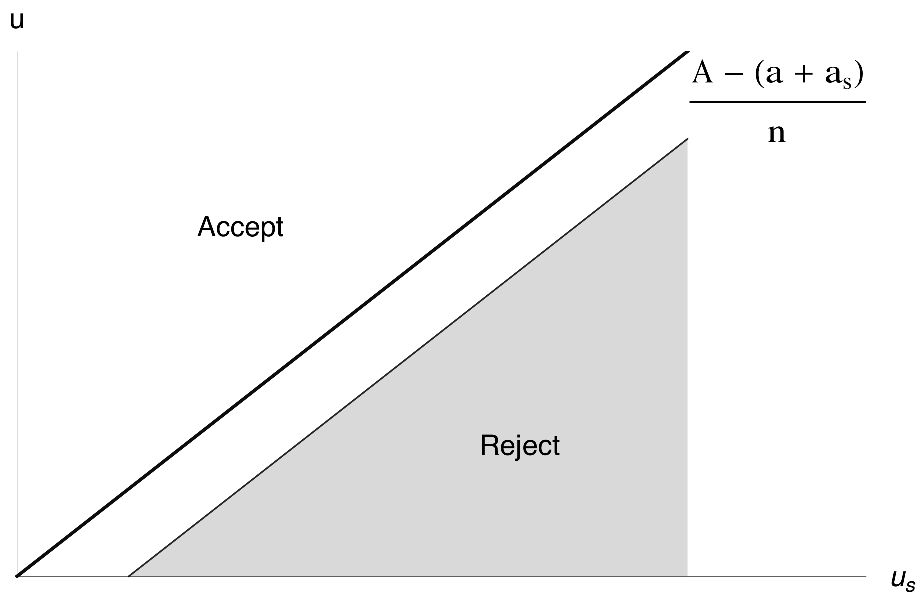

Figure 2: The acceptance or rejection areas in which supplier and manufacturer over n periods in terms of u, u_s.

Given mutual interaction potential for manufacturer and supplier (3)*, the following optimal contract is a (weak) Nash Equilibrium (NE)*

$$\begin{cases} U^1 = u - A, \\ U^i = u, \qquad \forall i, 2 \leq i < n, \\ U^n = u + a. \end{cases} \tag{5}$$

Because this equilibrium is weak, the supplier is exposed to considerable risk for costly mis-coordination by the manufacturer. This can be reworded in the following remark.

Remark 1 *The NE in Proposiion 1 is not trembling hand perfect.*

The proofs are provided in §6.2 of the Appendix 6.

The remark has practical implications: when the manufacturer changes to his outside option, the cost is the same to him but not the profit to the supplier. We note (although not developed here in a supplier-driven bargaining) that this condition naturally would be exploited by the manufacturer in an open bargaining process.

We now turn our attention to the case of asymmetric information.

3.4 Asymmetric information and renegotiation

We assume that the relationship-specific costs A and a are now the manufacturer's private information. To make offers, the supplier relies on a priori information on the distribution for A as supported by $[\underline{A}, \overline{A}]$, following a cumulative density distribution F_A, and for a as supported by $[\underline{a}, \overline{a}]$ and following a cumulative density distribution F_a. We assume that these distributions have *increasing failure rates* (IFR). The supplier updates her belief about A and a, respectively, as new information on rejection potentially arrives in the dynamic game. Note that the supplier still observes the cost u to the manufacturer (it could be the market price for similar product or service).

The Nash Equilibirum in the full-information case in (3.3) no longer holds. If $A^{1*} < A$ or if $a^* > a$, the manufacturer will reject the supplier's offer.

We spell out a new proposition applicable to the present scenario using the results from the calculations relegated to the Appendix in §6.3.

Proposition 2 *Asymmetric information*
Given asymmetric information about investment specific costs A and a and

mutual interaction potential, the contracts offered are

$$\begin{cases} U^1 = u - A^{1*}, & \\ U^i = u, & 1 < i < n, \\ U^n = u + a^*. & \end{cases} \tag{6}$$

If $A^{1*} < A$, the supplier offers in subsequent periods i contracts such that

$$\begin{cases} U^i = u - A^{i*}, & i \leq \max_j(n-j), \\ U^t = u, & i < t \leq n, \end{cases} \qquad \text{iff } \delta_m^i = 1, \tag{7}$$

with

$$\begin{cases} A^{i*} + \dfrac{F_{Ai}(A^{i*})}{f_{Ai}(A^{(i*})} = (n-i+1)u - u_s - L^{i+1}, \\ \quad L^i = F_{Ai}(A^i)\big((n-i+1)u - A^i\big) + \\ \qquad \overline{F_{Ai}}(A_i)\big(u_s + L^{i+1}\big) \\ \quad n - j \geq \dfrac{A^{j*}}{u - u_s}, & \text{(PC)} \\ \quad A^{i*} > A^{(i-1)*}. & \text{(IC)} \end{cases} \tag{8}$$

We can also look at the rent which accrues to the supply chain composed of the supplier and the manufacturer over the n periods and formulate the following third proposition, the proof of which has been relegated to §6.4 in the appendix..

Proposition 3 *Given asymmetric information about investment specific costs A and a and mutual interaction potential over the n periods, there is a risk that interaction between the supplier and manufacturer will not start in the first period, reducing the supply chain rent. There is also a risk of over-investment equal to $a + a_s$ by both supplier and manufacturer in the event that the manufacturer decides to fall back on his outside option in any period.*

We now illustrate these propositions.

4 Numerical illustration

A manufacturer and a supplier interact over $n = 20$ periods. Let

$$\begin{array}{llll} A = 50, & u = 100, & a = 60, & u_s = 80, \\ a_s = 55, & p_1 = 30, & \omega = 1. & \end{array} \tag{9}$$

4.1 Scenario of full information

In the case of common information the offers are straightforward and, according to Proposition 1, yield the following multi-period contract

$$\begin{cases} U^1 = 100 - 50, & \\ U^i = 100, & \forall i, 2 \leq i < 20, \\ U^{20} = 100 + 60. & \end{cases} \tag{10}$$

The manufacturer pays over 20 periods 2060 (including $A = 50$), exactly the same cost than his outside option (2060). The supplier earns a larger profit than with her outside option (1545). The chain has no coordination loss.

4.2 Scenario of asymmetric information

When information about investment in RSA is asymmetric, the supplier must establish the thresholds for A and a which will maximize her profits.

We implement here the updating procedure described in §3.4 for the belief about A. In the first period, the supplier must set up her prior belief about A, define the seed p^n and the step ω by which this parameter will decrease between period 20 and period 1. We set $\omega = 1$. For example, when we start with a normal distribution with $p^{20} = 50$ in period 20, the threshold $A^{20*} = 17.2$. In period 1, from (1), we write the mean of the distribution as $p^1 = p^{20} - 20 \times 1 = 30$ and $A^{1*} = 49$. In this illustration, we shall present the results for several values of the seed p^n but a fixed increment ω. Let A follow a normal distribution function such that $\mathcal{N}_A(p^n, 10)$ truncated at 0. Initially, the supplier sets the belief about a to follow a normal distribution function $\mathcal{N}_a(60, 10)$ truncated at 0. We apply the results assembled in Proposition 2. We begin with a^* so as to later find A_{s1^*}. Solving using (45), we obtain $a^* = 42.22$.

4.3 Updating mechanism

In our first approach, we evaluate A^{j*} using (8) and updating the distribution functions with $\omega = 1$ and $p^{20} = 50$ so that $p^1 = 30$. This yields the results listed in column 3 of table 4.

The first two period estimates do not allow the supplier to win over the manufacturer's agreement: she has to wait till the third period to obtain an agreement by the manufacturer. However, in the meantime, both have had to invest in their outside option's specific relationship assets a and a_s. This illustrates Proposition 3 presented above. Hence, the supplier is no longer able to extract the amount a from the manufacturer in the last period.

In the third column of this table, the supplier's profit is to be compared to what she would have obtained from using her outside option from the start (first column). In the same way, the manufacturer's cost in this setup is to be compared to the cost incurred if he had turned to his outside option from the outset. The supplier is much better off, whereas the manufacturer bears essentially the same cost.

4.4 The supplier starts with different beliefs

Table 4 presents the result for p^{20} ranging between 50 and 160 in steps of 10, enlarging the previous numerical example to different seeds p^{20} which represent different a priori beliefs in period 1. In the last three lines are recorded the contract in the first period, the profit to the supplier given the acceptance by the manufacturer of the relevant contracts and the cost to the manufacturer.

If we take the cases where $p^{20} \geq 60$ (fifth to last columns), the thresholds in period 1 A^{1*} are higher than A and hence the contracts offered would have been accepted in the *first* period. The supplier entices the manufacturer into accepting her first offer. Thereafter, the supplier can extract the last period investment cost a without the manufacturer refusing to work with her.

In figure 3, we can observe how the supply chain rent constituted by the difference between the supplier's profit and the manufacturer's cost evolves according to the seed. The impact of Proposition 3 can clearly be seen when $p^{20} \leq 50$. As this seed increases the overall rent increases quickly before reaching a plateau due to the increasing profit generated by the supplier. Note that the first contract offered when $p^{20} \geq 110$ is a *payment* by the supplier to the manufacturer! Even in these cases, the profit over the 20 periods is higher than the one obtained by turning to her outside option or by missing the manufacturer's first period agreement (see figure 3).

The attentive reader will note that the manufacturer stands to reduce his cost the higher he signals A to be. Moreover, this signaling coordinates better the supply chain. The supplier's profit is maximal when she slightly underestimates the manufacturer's investment cost A in the first period (when $p^{20} = 60$, $p^1 = 40$). She should not consider the manufacturer's signal as trustworthy. Both attitudes can be observed in practice.

The risk, cost of over-investment and potential for supply chain inefficiency are clearly apparent when the supplier underestimates the manufacturer's investment cost (first four columns).

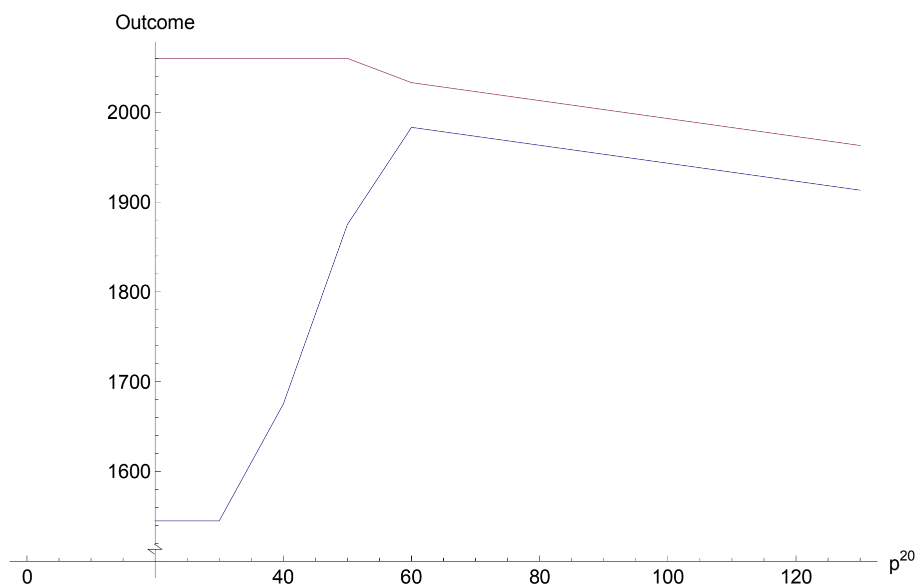

Figure 3: Evolution of the supplier's profit (bottom thin line) and manufacturer's cost according to the supplier's initial belief. When $p^{20} \geq 60$, the manufacturer accepts the supplier's first offer and is held up in the last period.

5 Conclusion

In this paper, we have presented a model of a manufacturer and a supplier when the manufacturer has yet to invest in the RSA required to work with the supplier and has private information about the cost of such investment. We characterize the NE when the supplier's Bayesian belief about the manufacturer's investment costs follow distributions which exhibit Increasing Failure Rates. Under full information, the ensuing NE are weak and not trembling hand perfect. Under asymmetric information, the NE is trembling hand perfect and the manufacturer's best course is to agree with the supplier's offers whenever it is less onerous.

We show that, when engaging with the manufacturer in a multi-period relationship, the supplier has initially to sweeten her offer and can later hold the manufacturer up by the cost of RSA with a third party. This holdup is only partly reduced when the manufacturer withholds the RSA cost information. We show that this rent extraction can only take place in the last period, an attempt at holding up the manufacturer any earlier can only be met by evasion. To protect himself, the manufacturer invests in two different RSA, thus impairing supply chain efficiency.

The supplier's strategy induces a temporal link among the negotiating periods: she is motivated to renew the relationship so as to be in a position to hold the manufacturer up in the last period. This effect contributes to explain the inertia of the partners when considering switching decisions. This effect is similar to the marketing literature lock-in effect considered in Klemperer (1995).

Finally, we show that there exist risk of supply chain rent reduction. This rent reduction is independent of the supplier's estimate of the manufacturer's costs.

We show in the numerical illustration how the manufacturer's cost is reduced when the supplier over-estimates his RSA cost. This effect can be related to common anecdotic observation of customers over-declaring the effort deployed to establish the relationship to their suppliers.

6 Appendix

6.1 Evaluation of contracts offered in each of manufacturer's states

We present here the evaluations for the offers made by the supplier according to the state the manufacturer finds himself in in the full information scenario of §3.3.

In state S, the manufacturer has only undertaken investment A; in state ST, he has incurred both investment A and investment a, whereas in state T, he has only invested a.

State S:

Manufacturer has made a single investment, A in period one.

In the last period n, her cost becomes

$$C^n = \min(U^n, u + a). \tag{11}$$

Under strict rationality, the supplier's dominant offer must thus be:

$$U^n = u + a, \tag{12}$$

In period $n - 1$, the manufacturer faces the cost function

$$C^{n-1} = \min(U^{n-1} + u + a, 2u + a), \tag{13}$$

bounding $U^{n-1} \leq u$ for mutual trade.

It follows that in any period j, $j > 1$, the manufacturer has to solve

$$C^j = \min(U^j + (n - j)u + a, (n - j + 1)u + a). \tag{14}$$

So, to ensure incentive compatibility, the supplier is limited to offering

$$U^j \leq u, \qquad \forall j,\, 2 \leq j < n. \tag{15}$$

In state S, the overall minimized cost function becomes

$$C = nu + a. \tag{16}$$

The supplier's maximal profit is

$$\Pi_s = nu + a - A. \tag{17}$$

The supplier's optimal strategy under the conditions (3) consist of the profit maximizing contracts as an n-sized vector:

$$v_m^S = ((u - A), u, \ldots, u + a). \tag{18}$$

Let us call the manufacturer's strategy when he is in state S the n-sized vector comprised of the decisions δ_m^i in response to the offers as

$$v_r^S = (\delta_m^1, \delta_m^2, \ldots, \delta_m^n), \quad \forall i, 1 \leq i \leq n,\, \delta_m^i = 1. \tag{19}$$

The strategy sets R_m^S and R_r^S for each player are reduced to just one vector in each.

State ST:

The manufacturer has invested A and a, the supplier has invested a_s. In the last period n, the manufacturer's cost function is

$$C^n = \min(U^n, u) \Rightarrow U^n \leq u \tag{20}$$

for acceptance. By extension, $U^j \leq u$, in periods j, $2 \leq j \leq n$.

The threat strategy for the manufacturer is to accept the introductory offer $u - A$ in period 1 and then reject the renewal of the contract in period 2, resulting in

$$C^2 = u + a, \tag{21}$$

whether or not the supplier offers u. The total cost for this strategy over n periods is

$$C = nu + a, \tag{22}$$

with the supplier's corresponding profit as

$$\Pi_s = (n-1)u + u_s - A - a_s \tag{23}$$

Imagine that in period k, $1 < k \leq n$, the manufacturer invests A, the profit function becomes

$$\Pi_s' = (k-1)u - A - a_s + u_s + (n-k)\max(u, u_s), \tag{24}$$

and $\Pi_s' \leq \Pi_s$.

The strategies of manufacturer and supplier are now described by the following sets of vectors

$$\begin{aligned}
R_m^{ST} &= \{v_m^{ST}, v_m^{ST} = \big((u-A), u, \ldots, u\big)\} \\
R_r^{ST} &= \{v_r^{ST}, v_r^{ST} = (\delta_m^1, \delta_m^2, \ldots, \delta_m^n), && (25)\\
&\qquad \exists i, 1 < i < n,\, \delta_m^i = 0\}. && (26)
\end{aligned}$$

STATE T:

The manufacturer has only invested a, supplier has invested a_s. In period j, the minimum acceptable offer from the supplier would be

$$U^j = u - A, \qquad \text{if } \forall i \in \{1, \ldots, j-1\}, \delta_m^i = 0, \tag{27}$$

which the supplier can repeat as long as $j \leq n - \frac{A}{u - u_s}$. At a latest period j_{max} above that limit, the supplier makes no offer that could be accepted by the manufacturer. If the manufacturer agrees in period k, $k \leq n-j$ the stream of profits to the supplier over n periods is:

$$\Pi_s = (k-1)u_s - a_s + (n-k+1)u - A, \tag{28}$$

In any case, the cost to the manufacturer remains

$$C = nu + a. \tag{29}$$

In this state, the sets of vectors representing the strategies available to the players are

$$\begin{aligned} R_m^T &= \{v_m^T, v_m^T = \big((u-A), u, \ldots, u\big)\} \\ R_r^T &= \{v_r^T, v_r^T = (0, \delta_m^2, \ldots, \delta_m^n), \\ &\quad \exists i, 2 \leq i \leq n, \delta_m^i = 1\}. \end{aligned} \tag{30}$$

6.2 Proof of Proposition 1

Proof. When comparing the strategies among the three states, we see that in the first state, the supplier exploits the incumbent's advantage of the relationship specific investment with the manufacturer's outside option only in the very last game. If she were to try doing so before, the manufacturer would simply defect. If the supplier's participation constraint

$$nu + a - A \geq nu_s - a_s, \tag{31}$$

is satisfied, the profit extracted in state 1 is larger than the ones in either other states. This justifies that she offers in all periods

$$U^i = u, \quad \forall i, 1 < i < n. \tag{32}$$

Given that the supplier is Stackelberg leader and the manufacturer is reduced to accepting or rejecting the offers and that both work in a full information environment, the NE strategy is the one presented in the case of state 1: the manufacturer agrees to the contract offered in the first period and, under sequential rationality Watson (2002), subsequently accepts all the offers made by the supplier without deviating by working with his outside option.

We will now investigate the weakness of this NE. The cartesian set $R_s \times R_m$ represents all the available strategies of both players. This set is larger than the union of the three sets defined when describing the three states in which the manufacturer finds herself. However, all the strategies which do not belong to the sets R_{m1}, R_{m2}, and R_{m3} are evidently not profit maximizing ones or do not satisfy the participation constraints as binding constraints and shall be discarded.

For the supplier, it can easily be seen that, with v_{s1} as defined in (19),

$$\forall v_m \in R_m \,\text{and}\, v_m \neq v_{s1},\; \Pi_s(v_{s1}) > \Pi_s(v_m). \tag{33}$$

For the manufacturer, evidently

$$R_m = R_{m1} \cup R_{m2} \cup R_{m3}, \tag{34}$$

so

$$\forall v_m \in R_m, C(v_m) = nu + a,\; \Rightarrow \\ \nexists\, v_m^* \in R_m \,|\, C(v_m^*) > C(v_m). \tag{35}$$

Hence, the NE is weak.

If the manufacturer chooses other responses, his strategies can be assimilated to the "Trembling Hand" Selten (1975). The three states presented above are in fact occurrences of this Trembling Hand argument: as can be seen in table 5, the manufacturer may costlessly play a different strategy which hurts the supplier. Due to the Stackelberg structure, we do not explore the other rent-appropriation possibilities and conclude that the NE is not a trembling-hand perfect equilibrium.

■

6.3 Evaluations of offers according to the manufacturer's state

We present here the calculations which enable the supplier to evaluate optimal offers for each of the manufacturer's states in the scenario of asymmetric information in §3.4. The supplier now must formulate offers as functions of thresholds A^{1*}, a^* and eventually the updated thresholds A^{i*} in periods i so that she maximizes her profit. We first evaluate the strategies open to both players before presenting the calculations of those thresholds.

6.3.1 Strategies and manufacturer's states...

STATE S
This state is attained only if the supplier's threshold $A^{1*} \geq A$ to incite an acceptance by the manufacturer. So, given the sequential rationality of

the manufacturer, he works with the supplier for this and all periods up to period $n-1$ with probability 1 as long as the reservation utility is met. In the last period, the manufacturer accepts the final hold-up only if the supplier's threshold $a^* \leq a$.

The supplier's strategy is structurally analogous to the full information case in §3.3:

$$\begin{cases} U^1 = u - A^{1*}, & \\ U^i = u, & 1 < i < n, \\ U^n = u + a^*, & \end{cases} \tag{36}$$

under the adjusted participation constraint for the supplier:

$$nu - A^{1*} + a^* \geq nu_s - a_s. \qquad \text{(PC)} \tag{37}$$

The manufacturer's cost and supplier's profit over the n periods are

$$\begin{aligned} C &= nu - A^{1*} + A + a^*, \\ \Pi_s &= nu - A^{1*} + a^*. \end{aligned} \tag{38}$$

When period n starts, the supplier must now show her offer using a^*. If $a^* > a$, the manufacturer rejects the offer, $\delta_m^n = 0$. The manufacturer's cost and supplier's profit when terminating in state S are

$$\begin{aligned} C &= nu - A^{1*} + A + a, \\ \Pi_s &= (n-1)u - A^{1*} + u_s - a_s. \end{aligned} \tag{39}$$

STATE ST

The double-investment state is attained at the earliest in period 2 if the supplier's threshold $A^{1*} \leq A$ made the manufacturer reject the initial offer and accept it in a subsequent period k, $1 < k \leq n$. The supplier has incurred a_s and the manufacturer has incurred investments $A + a$ up to this period.

The dominant strategy for state ST is trivial, there is no opportunity for hold-up in the last period since the manufacturer has a sunk investment with the third party. Thus, the resulting strategy for the supplier for all subsequent periods $j = k+1, ..., n$ is

$$U^j = u, \tag{40}$$

which follows from Condition 2.

The manufacturer cost and supplier profit expressions are, respectively:

$$\begin{aligned} C &= nu - A^{k*} + A + a \\ \Pi_s &= (n-k)u - A^{k*} + ku_s - a_s. \end{aligned} \tag{41}$$

STATE T

In State T, the manufacturer has only invested in a third-party relation a, resulting from $A^{k*} < A$ in all preceding periods k. The supplier may propose a new contract A^j that, if accepted by the manufacturer, would change the state to ST. An updated offer in period j, is made if If $u > u_s$, in the second period and in all posterior ones up to period $n - j$ such that the supplier's participation constraint and the manufacturer's incentive compatibility constraint

$$\begin{aligned} j+1 &\geq \frac{A^{(n-j)*}}{u-u_s}, && \text{(PC)} \\ A^{(n-j)*} &< A^{(n-j+1)*} && \text{(IC)} \end{aligned} \tag{42}$$

are satisfied, the supplier updates[3] her threshold $A*$.

If the game terminates in state T, the manufacturer and supplier have *never* worked together: the manufacturer's overall cost and supplier's overall profit can be written

$$\begin{aligned} C &= nu + a \\ \Pi_s &= nu_s - a_s. \end{aligned} \tag{43}$$

Figure 4 represents the three possible states and the manufacturer's decisions leading to them.

6.3.2 Bid determination

We now present the evaluations of the thresholds that enable the supplier to calibrate her offers to the manufacturer. Her objective is to maximize her profit given these thresholds and possible strategies. The objective function

[3] This evaluation is presented later.

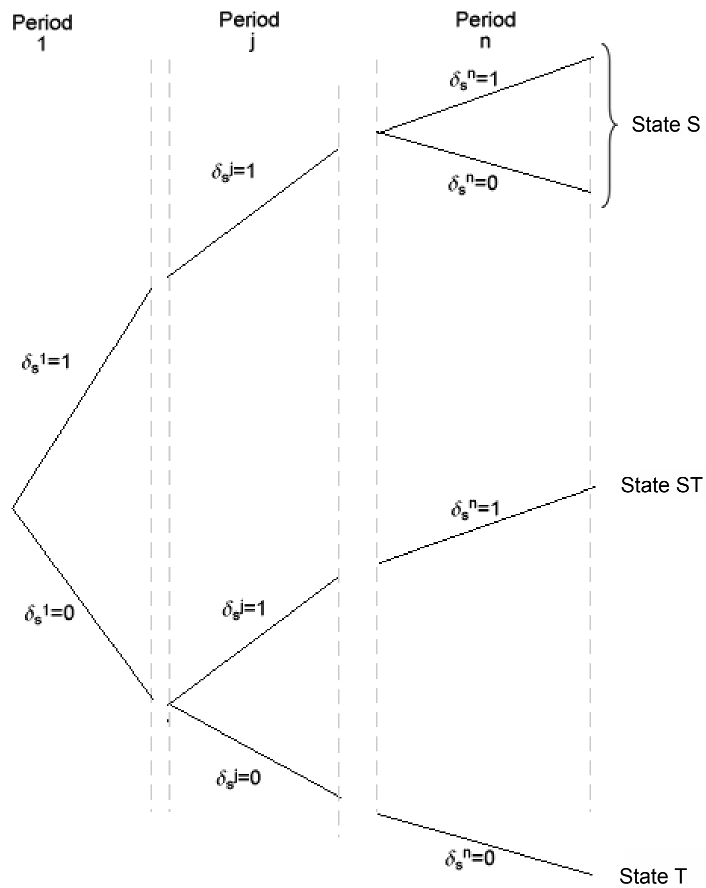

Figure 4: States of nature according to the supplier's thresholds A^{j*} and a^{*}.

for the supplier can be summarized as

$$
\begin{aligned}
&\max\Pi_s(A^j) = \\
&\quad F_{A1}(A^1)\Big[(n-1)u - A^1 + \overline{F_a}(a)\Big(u+a\Big) + \\
&\quad F_a(a)(u_s - a_s)\Big] + \\
&\quad \overline{F_{A1}}(A^1)\Bigg[u_s - a_s + F_{A2}(A^2)\big\{(n-1)u - A^2\big\} + \\
&\quad \overline{F_{A2}}(A^2)\Bigg(u_s + F_{A3}(A^3)\Big[(n-2)u - A^3\Big] + \dots \\
&\quad F_{A(n-j)}(A^{n-j})\big((j+1)u - A^{n-j}\big) + \\
&\quad u_s + \overline{F_{A(n-j)}}(A^{n-j})((j+1)u_s)\dots\Bigg)\Bigg],
\end{aligned}
\tag{44}
$$

$$
\text{s.t.} \begin{cases} (j+1)(u-u_s) \geq A^{(n-j)}, & \text{(PC)} \\ A^{(n-j)} \geq A^{(n-j+1)}, & \text{(IC)} \\ nu + a - A^1 \geq nu_s - a_s, & \text{(PC).} \end{cases}
$$

6.3.3 Belief about the manufacturer's RSA a

We first turn to the belief about the manufacturer's investment into specific assets relative to his outside option a which the supplier has to make in period n (if at all).

Assuming that f_a has an IFR and $f_a(a) \neq 0$, there exists an interior value a^* such that

$$a^* - \frac{\overline{F_a}(a^*)}{f_a(a^*)} = u_s - a_s - u. \tag{45}$$

Proof. The first differential of the profit function (44) is written:

$$\frac{\partial \Pi_s(a)}{\partial a} = F_A(A)\Big[f_a(a)\big(u_s - a_s - u - a\big) + \overline{F_a}(a)\Big]. \tag{46}$$

This leads to the F.O.C.

$$(u+a) - (u_s - a_s) = \frac{\overline{F_a}(a)}{f_a(a)}. \tag{47}$$

The S.O.C. requires that the following inequality be true

$$f_a'(a^*)\big(u_s - a_s - u - a\big) - 2f_a(a^*) < 0, \tag{48}$$

which, when replacing $u_s - a_s - u - a$ by its value in (47), means that we must have

$$-f_a'(a)\frac{\overline{F_a}(a)}{f_a(a)} - 2f_a(a) < 0. \tag{49}$$

Since $f_a(a)$ is positive for all a in the range $[\underline{a}, \overline{a}]$, we can restate this inequality as

$$f_a'(a)(F_a(a) - 1) - 2f_a^2(a) < 0. \tag{50}$$

However, we have assumed that the distribution of a is IFR which means that the failure rate $r(a) = f_a(a)/\overline{F_a}(a)$ is weakly increasing for those values of a for which $F_a(a) < 1$ Barlow and Proschan (1965). Then the first differential of the function r, which is written

$$\frac{\partial r(a)}{\partial a} = \frac{f_a'(a)(1 - F_a(a)) + f_a(a)^2}{(1 - F_a(a))^2} \tag{51}$$

must be positive or null, so that

$$\frac{\partial r(a)}{\partial a} \geq 0 \Rightarrow f_a'(a)(F_a(a) - 1) - f_a(a)^2 \leq 0. \tag{52}$$

This last condition is stronger than the one spelt in (50) because $f_a(a)^2 > 0$.
■

6.3.4 Updated belief about the manufacturer's RSA with the supplier A^{n-j}

In period $n-j$, the last period in which (42) are satisfied, the manufacturer is in state T. The supplier makes a last update of A. The first differential of her profit function in terms of A^{n-j} is written

$$\begin{aligned}\frac{\partial \Pi_s^{(n-j)}(A^{n-j})}{\partial A^{n-j}} =& f_{A(n-j)}(A^{n-j})\big((j+1)(u-u_s) - A^{n-j}\big) \\ &- F_{A(n-j)}(A^{n-j})\Big].\end{aligned} \tag{53}$$

The S.O.C. requires that

$$\begin{aligned}&f'_{A(n-j)}(A^{(n-j)*})\big((j+1)(u-u_s) - A^{(n-j)*}\big) - \\ &\qquad 2f_{A(n-j)}(A^{(n-j)*}) < 0.\end{aligned} \tag{54}$$

The proof is similar to the one presented in §**??** of the Appendix 6.

So, from (53), and since (54) is true, we can deduct the optimal threshold $A^{(n-j)*}$ in period $n-j$ as solution to

$$A^{(n-j)*} + \frac{F_{A(n-j)}(A^{(n-j)*})}{f_{A(n-j)}(A^{(n-j)*})} = (j+1)(u-u_s), \tag{55}$$

Using backward induction, we can now evaluate the threshold for A in the previous period starting with period $n-j-1$. The profit function in preceding periods i, $1 \le i \le n-j-1$, can be written as

$$L^i = F_{Ai}(A^i)\big((n-i+1)u - A^i\big) + \overline{F_{Ai}}(A_i)\big(u_s + L^{i+1}\big). \tag{56}$$

In period $n-j$, the last term of the series L is written as

$$\begin{aligned} L^{n-j} =& F_{An-j}(A^{n-j})\big((j{+}1)u - A^{n-j}\big) + \\ & \overline{F_{An-j}}(A^{n-j})\big((j{+}1)u_s\big). \end{aligned} \tag{57}$$

The optimal threshold in each period i is the result of evaluating the F.O.C. and S.O.C. of the expression L^i differentiated in A^i for $2 \le i < n-j$. Following along the lines of the proof provided in §**??** in Appendix 6, we obtain

$$A^{i*} + \frac{F_{Ai}(A^{i*})}{f_{Ai}(A^{i*})} = (n-i+1)u - u_s - L^{i+1}. \tag{58}$$

and proceeding in a bootstrapping iteration we evaluate all the preceding L^i back to L^2 and A^1:

$$A^{1*} + \frac{F_{A1}(A^{1*})}{f_{A1}(A^{1*})} = (n-2)u - u_s - L^2 \tag{59}$$

To be incorporated in contracts that can be offered to the manufacturer, the thresholds evaluated above must follow the strategy which we described when the manufacturer is in state ST or T, namely that $A^{i*} < A^{(i+1)*}$ for the first few periods.

By definition, L^i represents the expected profit to the supplier going forward in periods $i+1$ to n when she has not yet gotten an acceptance from the manufacturer. As i increases, this expected profit decreases since each period's profit is positive, even if it consists in taking her outside option. So

$$L^{i+1} < L^i. \tag{60}$$

Further, from (58), even if the distribution of the belief about A is not updated, in each period, the period's threshold is higher than the previous one's.

Moreover, since the supplier offers a sequence of initially increasing bids,

$$A^{*1} < A^{*2} < \ldots < A^{i*}. \tag{61}$$

We reach a point where A^i becomes large compared to $(n-i+1)u$ within L^i in (56). By construction, after that point A^i can no longer increase and in fact decreases as can be seen in the numerical illustration presented in §4. So the supplier cannot make an offer more enticing than the previous one to the manufacturer. Instead, the supplier turns to her outside option.

6.4 Proof of Proposition 3

Proof. We have seen in the integrated supply chain that if the mutual interaction is delayed by τ periods, then the loss in rent to the supply chain is given by the difference between the cost to the manufacturer and the profit to the supplier:

$$\begin{cases} C = a + nu + A - A^{\tau+1*} \\ \Pi_s = \tau u_s - a_s - A^{\tau+1*} + (n-\tau)u, \end{cases} \Rightarrow C - \Pi_s = \tau(u_s - u) - a_s - a - A. \tag{62}$$

Whereas, if the interaction starts in period 1, the partners objective function work out as

$$\begin{cases} C = A - A^* + nu - a^* \\ \Pi_s = nu - A^* + a^*, \end{cases} \Rightarrow C - \Pi_s = -A. \tag{63}$$

This leads to the difference between both cases of

$$\Delta = \tau(u_s - u) - a_s - a, \tag{64}$$

which is strictly negative because $u_s \leq u$ according to the PC in (3).

Note that this result is independent of the period in which the manufacturer decides to change supplier. It is also independent of the supplier's estimates of a and A. ■

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Table 4: Results for the estimates of A when the seed p^{20} of the last distribution function in period 20 increases from 50 to 160 and $\omega = 1$. The last three lines present the first accepted contract, the overall profit and cost to the supplier and manufacturer.

	Seed											
n	20	30	40	50	60	70	80	90	100	110	120	130
1	22.1	29.8	39.1	49.	$\boxed{59.}$	$\boxed{69.}$	$\boxed{79.}$	$\boxed{89.}$	$\boxed{99.}$	$\boxed{109.}$	$\boxed{119.}$	$\boxed{129.}$
2	22.8	30.6	40.1	$\boxed{50.}$	60.	70.	80.	90.	100.	110.	120.	130.
3	23.5	31.5	41.	51.	61.	71.	81.	91.	101.	111.	121.	131.
4	24.2	32.4	42.	52.	62.	72.	82.	92.	102.	112.	122.	132.
5	24.9	33.4	43.	53.	63.	73.	83.	93.	103.	113.	123.	133.
6	25.7	34.3	44.	54.	64.	74.	84.	94.	104.	114.	124.	134.
7	26.4	35.2	45.	55.	65.	75.	85.	95.	105.	115.	125.	135.
8	27.2	36.2	46.	56.	66.	76.	86.	96.	106.	116.	126.	136.
9	28.1	37.1	47.	57.	67.	77.	87.	97.	107.	117.	127.	137.
10	28.9	38.1	48.	58.	68.	78.	88.	98.	108.	118.	128.	138.
11	29.8	39.1	49.	59.	69.	79.	89.	99.	109.	119.	129.	139.
12	30.6	40.1	$\boxed{50.}$	60.	70.	80.	90.	100.	110.	120.	130.	139.9
13	31.5	41.	51.	61.	71.	81.	91.	101.	110.9	120.8	130.5	139.3
14	32.4	42.	52.	62.	72.	82.	91.9	101.8	111.4	120.1	126.6	130.3
15	33.4	43.	53.	63.	72.9	82.8	92.4	100.9	107.1	110.5	112.	114.
16	34.3	44.	53.9	63.8	73.3	81.7	87.6	90.7	92.2	94.2	95.8	96.9
17	35.1	44.6	54.1	62.4	68.	70.9	72.4	74.4	75.9	77.	77.6	78.
18	33.8	41.8	48.	51.	52.6	54.5	56.1	57.	57.7	58.1	58.4	58.6
19	23.	28.2	32.2	34.7	36.2	37.1	37.7	38.1	38.4	38.6	38.8	40.
20	13.	15.	16.3	17.2	17.7	18.1	18.4	18.6	18.9	20.	20.	20.
U^i	78	70	61	51	41	31	21	11	1	-9	-19	-29
Π_s	1545	1545	1675	1875	1983	1973	1963	1953	1943	1933	1923	1913
C	2060	2060	2060	2060	2033	2023	2013	2003	1993	1983	1973	1963

Table 5: Outcome for supplier when the manufacturer applies trembling-hand strategies.

Outcome Carrier	State
n u + a - A	State 1
$(k-1)u - A - a_s + (n-k)\max(u, u_s)$	State 2
$(k-1)u_s - a_s + (n-k+1)u - A$	State 3
$nu_s - a_s$	No contract