Consider a sequence of polynomials \( (P_n) \) satisfying the (non-hermitian) complex orthogonality

\[
\int_{\Gamma} z^j P_n(z) e^{-nV(z)} \, dz = 0, \quad j = 0, \ldots, n - 1,
\]

where \( V \) is a fixed polynomial and the integration is on an unbounded simple contour \( \Gamma \) in \( \mathbb{C} \) ending up at \( \infty \) in both directions and such that \( \text{Re} \, V(z) \to +\infty \), as \( z \to \infty \) in \( \Gamma \).

If the polynomial \( V \) is real and \( \Gamma = \mathbb{R} \), the zeroes of the \( P_n \)'s are also real and their limiting distribution can be characterized in terms of an equilibrium problem with external field on the real line. In contrast, if \( V \) is no longer real we have a lot of freedom in choosing the contour \( \Gamma \), and this freedom is reflected in the behavior of the zeroes of the polynomials \( P_n \)'s.

Gonchar and Rakhmanov \cite{1} characterized the limiting distribution of these zeroes, conditioned to the existence of a curve \( \Gamma \) with a certain symmetry property - the so called \( S \)-property - over which we can compute the integrals above.

Based on recent works \cite{2,3}, we will discuss the existence of this curve \( \Gamma \) and its characterization.

This is a joint work with Arno Kuijlaars.

**References**

