Efficiency Analysis Application to financing of municipalities in Morocco

Rachida El Mehdi
Institut de Statistique,UCL

Promoter: Professor Ch. Hafner

Efficiency analysis

- Basic idea
  - Comparison between the Decision Making Units DMU (firms, for example) in order to know how the inputs are used to produce outputs.

Efficiency analysis

- Farrell efficiency (Production)

$$\theta = \frac{OB}{OR}$$

- Shephard efficiency

$$\delta = \frac{1}{\theta}$$

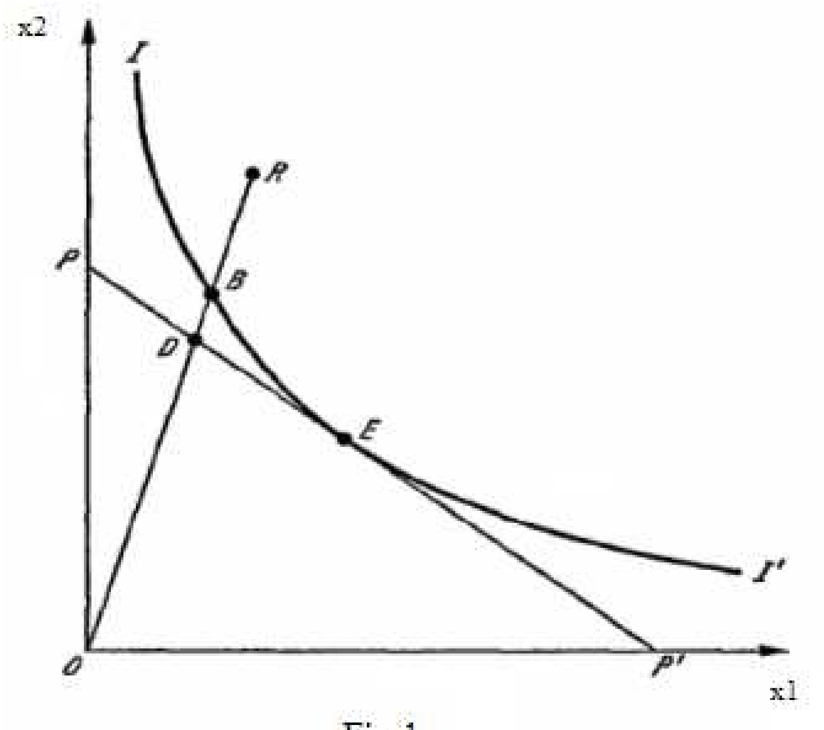


Fig.1

Efficiency analysis

Nonparametric approach

- Data Envelopment Analysis (DEA)
  - Strong disposability of inputs and outputs;
  - Variable returns to scale;
  - Convexity of the production set.

- Free Disposal Hull (FDH)
  - No convexity hypothesis on the production set is required

Efficiency analysis

Nonparametric approach

- Criticism
  - Approach does not account for noise in the data
    - Every deviation from frontier is considered as inefficiency

Data Envelopment Analysis

- Mathematical formulation in the input orientation at a point (x_0, y_0)

Primal (Envelopment)

$$
\begin{aligned}
& Min\,\theta \\
& s.t. \\
& \begin{cases}
\sum_{i=1}^{n} \lambda_i y_{ki} \geq y_{k0} & k = 1,\ldots,r \\
\sum_{i=1}^{n} \lambda_i x_{ji} \leq \theta x_{j0}\,, & j = 1,\ldots,m \\
\sum_{i=1}^{n} \lambda_i = 1 & \\
\theta \text{ free} \quad , \quad \lambda_i \geq 0, & \forall i = 1,\ldots,n
\end{cases}
\end{aligned}
$$

Dual (Multiplier)

$$
\begin{aligned}
& Max \quad \sum_{k=1}^{r} u_k y_{k0} + u^* \\
& s.t. \begin{cases}
\sum_{j=1}^{m} v_j x_{j0} \leq 1 \\
\sum_{k=1}^{r} u_k y_{ki} - \sum_{j=1}^{m} v_j x_{ji} + u^* \leq 0 \quad , \quad i = 1,\ldots,n \\
u^* \text{ free}, \quad u_k, v_j \geq 0, \quad \forall k, j
\end{cases}
\end{aligned}
$$

Data Envelopment Analysis

- At the optimum

$$\hat{\theta}(x_0, y_0) = \sum_{k=1}^{r} \hat{u}_k y_{k0} + \hat{u}^*$$

- Efficiency scores estimators are often over-estimated
  - Use the bootstrap procedure to correct the bias

Data Envelopment Analysis

- Bootstrapping efficiency
  - First used by L. Simar
  - Pseudo estimates of efficiency scores

$$\left\{\hat{\theta}_b^*(x, y)\right\}_{b=1}^{B}$$

    - These pseudo estimates give an approximation of the unknown sampling distribution of the efficiency scores

Data Envelopment Analysis

- Bootstrap correcting bias for efficiency scores
  - Bias

$$\hat{b}ias^{*}\left(\hat{\theta}(x,y)\right) \approx \frac{1}{B}\sum_{b=1}^{B}\hat{\theta}_{b}^{*}(x,y) - \hat{\theta}(x,y)$$

  - Standard deviation

$$\hat{s}td^{*}\left(\hat{\theta}(x,y)\right) \approx \frac{1}{B}\sum_{b=1}^{B}\hat{\theta}_{b}^{*,2}(x,y) - \left(\frac{1}{B}\sum_{b=1}^{B}\hat{\theta}_{b}^{*}(x,y)\right)^{2}$$

  - Bias corrected estimator

$$\tilde{\theta}(x,y) = \hat{\theta}(x,y) - \hat{b}ias^{*}\left(\hat{\theta}(x,y)\right)$$

Data Envelopment Analysis

- The bootstrap approximation of the confidence interval for $\theta(x, y)$ is given by

$$P\left(\hat{\theta}(x, y) - \hat{a}_{1-\frac{\alpha}{2}} \leq \theta(x, y) \leq \hat{\theta}(x, y) - \hat{a}_{\frac{\alpha}{2}}\right) \approx 1 - \alpha$$

$$\hat{a}_{1-\frac{\alpha}{2}} = \hat{c}_{1-\frac{\alpha}{2}} - \hat{\theta}(x, y)$$

and $\hat{c}_{1-\frac{\alpha}{2}}$ is the $\left(1-\frac{\alpha}{2}\right)$ quantile of empirical distribution of the estimators $\left\{\hat{\theta}_b^*(x, y)\right\}_{b=1}^{B}$.

Free Disposal Hull (FDH)

- Formulation

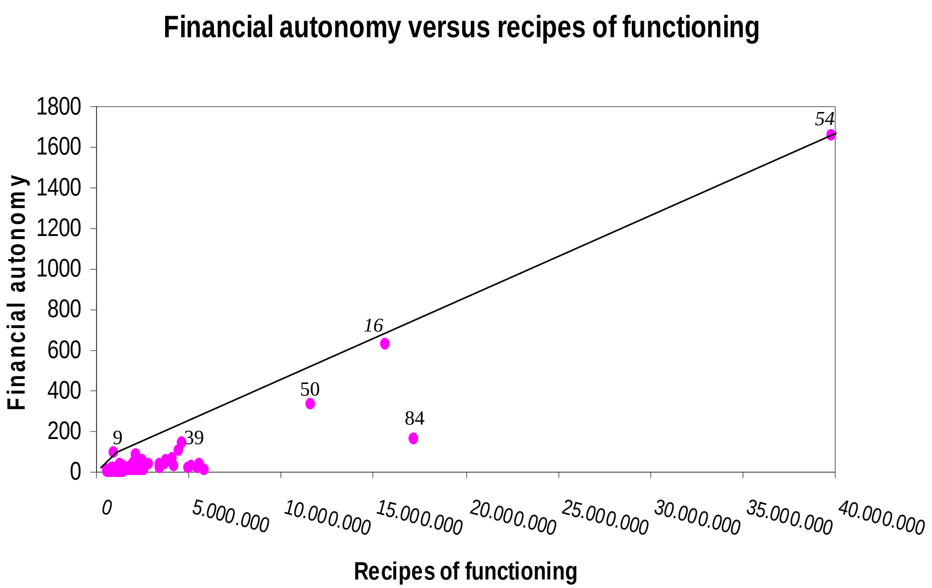
$$
\begin{aligned}
&Min\ \theta \\
&s.t. \\
&\begin{cases}
\sum_{i=1}^{n} \lambda_i y_{ki} \geq y_0 \quad k = 1,\ldots, r \\
\sum_{i=1}^{n} \lambda_i x_{ji} \leq \theta x_0\ , \quad j = 1,\ldots, m \\
\sum_{i=1}^{n} \lambda_i = 1 \\
\theta\ free \quad , \quad \lambda_i \in \{0,1\}, \quad \forall i = 1,\ldots, n
\end{cases}
\end{aligned}
\quad \Rightarrow \quad \hat{\theta}_{FDH}(x_0, y_0)
$$

DEA /FDH

- Error
  - DEA $$\hat{\theta}_{DEA}(x,y)-\theta(x,y)=O\left(n^{-\frac{2}{r+m+1}}\right)$$
  - FDH $$\hat{\theta}_{FDH}(x,y)-\theta(x,y)=O\left(n^{-\frac{1}{r+m}}\right)$$
    - Dimension reduction improves the rate
    - DEA approach has a faster rate

Application

- DMUs : 91 municipalities
- One input : Recipe of functioning (Urban tax, tax on the collection of the waste, subsidies…)
- One output : Financial autonomy



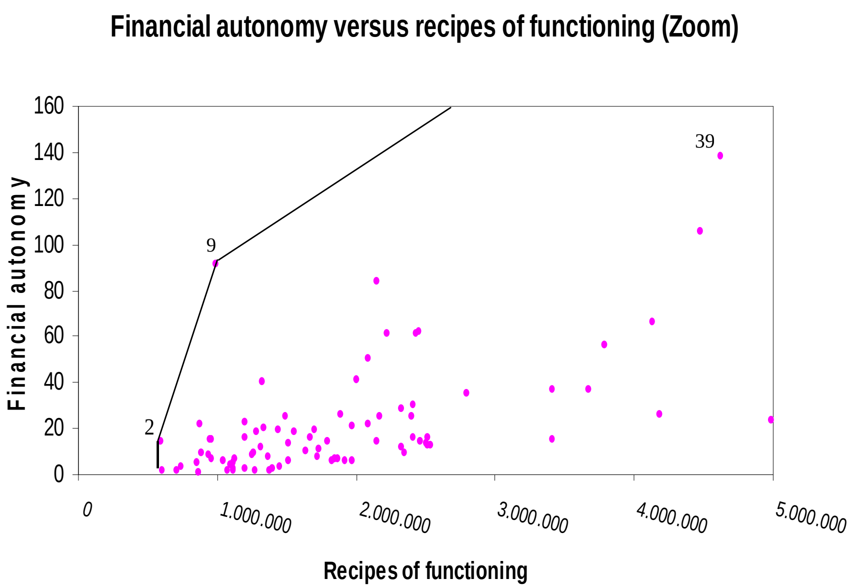


Figure 2

DEA results

- Table 1: Efficiency scores of FDH, DEA and their confidence intervals

Ord Pop	Efficiency R 1in/1out FDH	Obs Dominate Obs i FDH	Efficiency R 10input DEA (VRS	Efficiciency R 1in/1out DEA (VRS)	Bias corrected 1/bcSh R 1/1 DEA (VRS)	Bias R 1in/1out DEA (VRS)	Std R 1in/1out DEA (VRS)	Lower Bound	Upper Bound
1	0,4798	10	0,8396	0,4798	0,4511	0,0346	0,0260	0,4085	0,4777
2	1,0000	1	1,0000	1,0000	0,9217	0,0963	0,0583	0,8395	0,9878
3	0,8397	2	1,0000	0,8397	0,7955	0,0524	0,0458	0,7156	0,8375
4	0,9899	2	1,0000	0,9899	0,9381	0,0614	0,0541	0,8436	0,9877
5	0,6365	4	0,8070	0,6365	0,5983	0,0458	0,0345	0,5419	0,6337
6	0,9079	3	1,0000	0,6300	0,5818	0,0591	0,0367	0,5285	0,6234
7	0,6913	6	1,0000	0,6913	0,6551	0,0429	0,0378	0,5891	0,6897
8	0,5242	3	0,4923	0,3501	0,3209	0,0365	0,0247	0,2875	0,3459
9	1,0000	1	1,0000	1,0000	0,7117	0,7834	0,4077	0,6168	0,9360
10	0,4981	21	1,0000	0,4981	0,4719	0,0311	0,0271	0,4245	0,4968
11	0,8053	2	1,0000	0,8053	0,7625	0,0509	0,0438	0,6863	0,8031
12	1,0000	1	0,7955	0,7345	0,6779	0,0694	0,0457	0,6130	0,7265
13	0,7035	2	0,7607	0,7035	0,6649	0,0461	0,0381	0,5995	0,7014
14	0,3476	26	0,4163	0,3476	0,3275	0,0240	0,0188	0,2962	0,3463
15	0,5591	13	0,8294	0,5591	0,5297	0,0349	0,0305	0,4765	0,5577
...	...	...	...	...	...	...	...	...	...

DEA results

- Initial DEA efficiency estimators
  - All scores are well included between 0 and 1
  - Only 3 districts are efficient (RAS ASFOUR (2), LEBKHATA (9) and AIN LEHJER (54))

- Bias correction
  - No rural district is efficient
  - Only 6% of the districts are close to the efficiency frontier with a score estimated above 0.70.
  - $\tilde{\theta}_i$ are in the corresponding confidence interval,

DEA results

- Rural districts having a small population size have also a weak score of efficiency (far from the efficiency frontier)

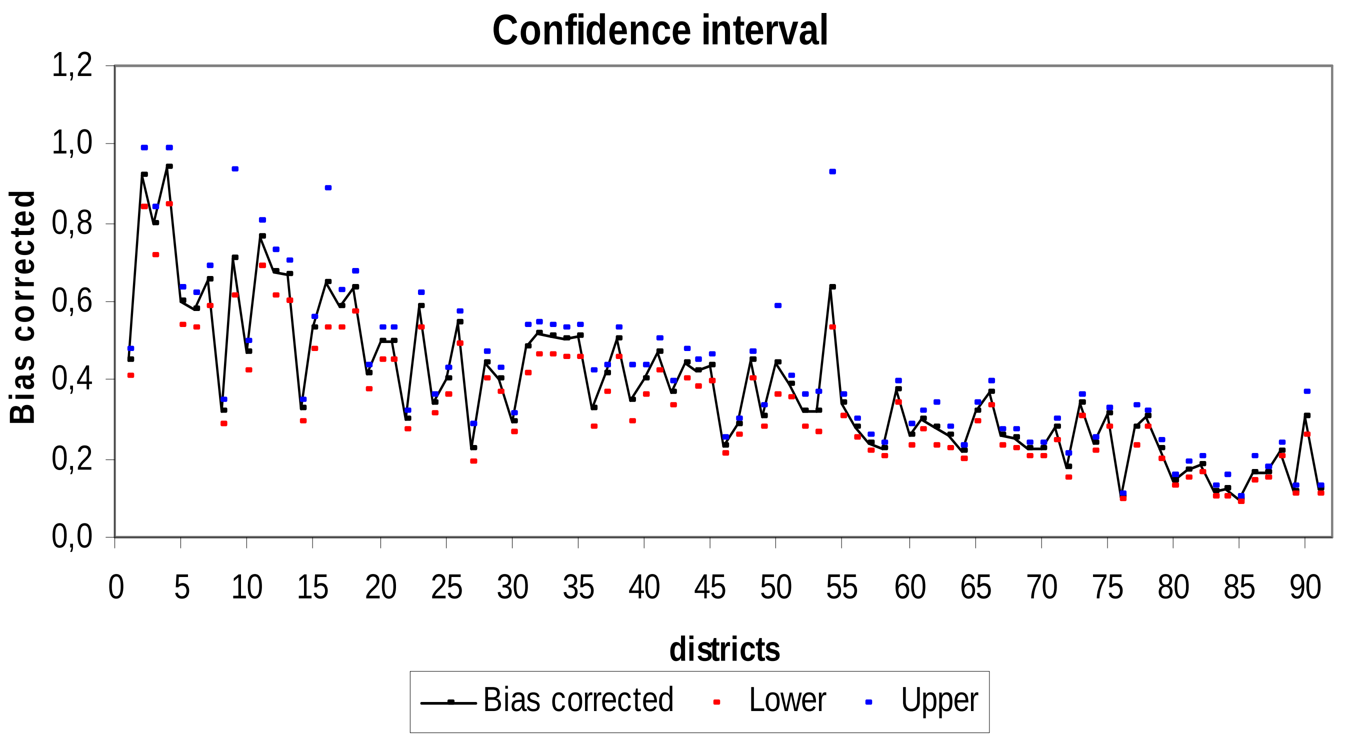


Figure 3

FDH results

- 8 districts are considered efficient (8.79%)
- Almost three times higher than DEA approach
  - Over-estimation of the number of efficient DMU

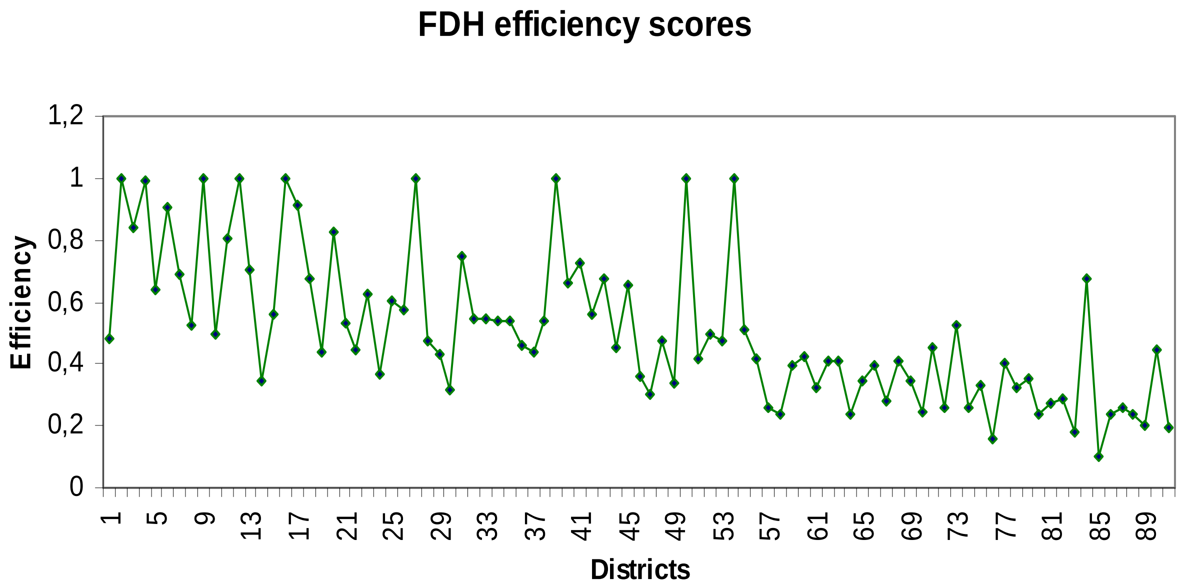


Figure 4

Effect of population size on efficiency

- Estimate the following model

$$\hat{\delta}_i = z_i \beta + \xi_i \geq 1 \qquad i = 1, 2, 3, \ldots, n$$

  - OLS and Tobit are not feasible
  - Estimation in two stages (Simar & Wilson, 2007)
    - Truncated regression
      - Population size has a weak positive relationship with the efficiency scores

$\hat{\beta}_0$: -0.55006

$\hat{\beta}_1$: 0.00033

To do

- Build the confidence intervals for the algorithm 1
- Algorithm2 (Simar & Wilson, 2007)
  - Using bias corrected estimators
- Stochastic analysis