

The financing of first pillar pension

Aurélie Miller (with Pierre Devolder)

Institut des Sciences Actuarielles

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Context and motivation

- Financing of pensions (first pillar) in a stochastic framework
- Two extreme concepts

	Pay-As-You-Go (unfunded)	Funded pension
Mechanism	Active people finance retired people's pensions	Contributions are invested on financial market for one's own future pension
Annual rate of return	Growth rate of the salary mass	Financial Return

- Two ways of constructing the equilibrium of the scheme

	Defined Contributions (DC)	Defined Benefits (DB)
Mechanism	You know what you're paying	You know what you'll receive at pension

Context and motivation (2)

- Ex. Belgian state pensions = PAYG, DB (salaries, length, status)
- Demographics :
 - Fact : ↓ fertility rates (less children) and ↑ life expectancy (older people)
 - Consequences :
 - ageing populations (dependency ratio : from 40,5% in 2000 to 44,7% in 2010 and 68,5% in 2030)
 - ↓ growth rate of the salary mass
 - pure PAYG systems are not viable in the long term
 - One of the most important macro economic issue for the next years.
- Solutions ?

Notional Accounts

- Classification

	PAYG	Funding
DB	Classical social security	Classical employee benefit plan
DC	NDC plans	Pension saving accounts

- PAYG scheme
- Defined contribution
 - Individual fictive account
 - Notional (fake) interest rate
 - At pension, the capital is converted into an annuity (constant or not)

Notional Accounts (2)

- Three parameters
 - $R(t)$, the annual notional rate of return
 - $a(t)$, the annuity divisor (accumulated notional capital \rightarrow annual pension)
 - $RP(t)$, the growth rate of the pensions

$$P_1(t) = \frac{1}{a(t)} \cdot \sum_{l=t-n}^{t-1} \left[c(l) \cdot \prod_{i=l+1}^t R(i) \right]$$

$$P_j(t) = P_{j-1}(t-1) \cdot RP(t)$$

- Ex. Sweden : $R(t)$ = growth of average nominal wage
 - Source of instability
 - Constant readjustments
 - Natural choice for these three parameters which guarantees automatic financial stability (ability of a pension plan to adjust to financial and demographic shocks without legislative intervention)

Notional Accounts (3)

- Case 1. Similar pension for everyone at time t

$$RP(t) = \exp(\gamma_t) \cdot \frac{DR(t-1)}{DR(t)}$$

	Choice for $R(i)$	Implication for $a(t)$
1	Total growth of the wages mass : $\frac{Y(i).s(i)}{Y(i-1).s(i-1)}$	$\sum_{k=1}^n \frac{O(t)}{Y(t-n+k-1)}$
2	Same value as $RP(i) = \exp(\gamma_i) \cdot \frac{DR(i-1)}{DR(i)}$	$\sum_{k=1}^n DR(t-n+k-1)$

- Case 2. No simplification

$$RP(t) = \frac{C(t) - O_1(t).P_1(t)}{O_2(t).P_1(t-1) + \sum_{j=3}^m \left\{ O_j(t).P_1(t-j+1) \cdot \prod_{k=t-j+2}^{t-1} RP(k) \right\}}$$

Notional Accounts (4)

- Application : Modeling in progress ...

- Demographics (cfr Lee)

- Stochastic version for projecting the Leslie Matrix

$$\mathbf{N}_t = \mathbf{X}_t \mathbf{N}_{t-1} + \mathbf{I}_t$$

- with behind

- a Lee Carter model for the mortality rates $\ln \hat{\mu}_x(t) = \alpha_x + \beta_x \kappa_t + \varepsilon_{xt}$

- a constrained Lee Carter model for fertility rates $\hat{f}_x(t) = \gamma_x + \delta_x \cdot \lambda_t + \varepsilon'_{xt}$

- Financial (Hull and White with one factor)

$$dr(t) = (\theta(t) - a \cdot r(t)) \cdot dt + \sigma_r \cdot dW_r(t)$$

- Results : soon !

Mixing PAYG and funding

	Pay-As-You-Go (unfunded)	Funded pension
Mechanism	Active people finance retired people's pensions	Contributions are invested on financial market for one's own future pension
Annual rate of return	Growth rate of the salary mass	Financial Return

- Idea : PAYG and funding schemes are dealing with different risks and have therefore specific advantages and drawbacks
 - What about mixing them ?
 - We therefore seek for an equilibrium between those two “extreme” pension schemes

Mixing PAYG and funding (2)

- Main questions
 - Can we obtain a theoretical justification for diversification between pay as you go and funding ?
 - What is the optimal level of diversification ?
 - Influence of the correlation between demography and financial markets?
- Hypothesis
 - Macro economic point of view
 - OLG two periods' set up
 - x_0 : affiliation age to the pension plan (working period)
 - x_r : retirement age
 - Defined contributions

Mixing PAYG and Funding - Deterministic

- Pay as you go : $L(x_0; t) \cdot \Pi \cdot S(t) = L(x_r; t) \cdot P(t)$
 - Replacement rate : $RR(t) = \Pi \cdot \frac{1}{p(x_0; 1)} \cdot (1 + d)$
- Funding : $L(x_0; t - 1) \cdot \Pi \cdot S(t - 1) \cdot (1 + i) = L(x_r; t) \cdot P(t)$
 - Replacement rate : $RR(t) = \Pi \cdot \frac{1}{p(x_0; 1)} \cdot \frac{1 + i}{1 + g}$
- With :
 - $L(t; x)$: the number of people aged x at time t
 - i : rate of return of the financial investment
 - g : rate of increase of the mean salary
 - d : demographic rate of increase of the active population
- Decision : Samuelson rule : $1 + d > \frac{1+i}{1+g} \rightarrow \text{PAYG}$

Mixing PAYG and Funding - Stochastic

- Deterministic case :
 - No diversification
 - Only rates of return matters as no risk
- Now :
 - Stochastic processes
 - The cotisation is split in two parts :
 - funding part (a)
 - PAYG part (1-a)
 - Replacement rate for one surviving person aged x_r at time t :

$$RR(t) = \Pi \cdot \frac{1}{p(x_0; 1)} \cdot \left[a \cdot \frac{1+i}{1+g} + (1-a) \cdot (1+d) \right]$$

- Process of interest : $X = a.F + (1-a).D$
 - Hyp : longevity risk is independant of the others
 - All other processes may be correlated

Mixing PAYG and Funding – Stochastic (2)

- Aim : determine the the optimal part which should be invested in the contribution system (a) in order to optimize the pension benefit received at retirement time.
- Porfolio Theory (risk return approach)

- Choice of an utility function :

$$U[X] = E[X] - \frac{\gamma}{2} \cdot V[X]$$

- Solution

$$a^{SOL} = \frac{V[D] - Cov[F; D]}{V[F - D]} + \frac{1}{\gamma} \cdot \frac{E[F - D]}{V[F - D]} = a^* + \frac{1}{\gamma} \cdot \Delta$$

- First part : same for everyone (minimum variance)
- Second part : linked to
 - the coefficient of risk aversion of the individual
 - link between the financial and demographic expectations and risks

Mixing PAYG and Funding – Stochastic (3)

- Example: lognormal model

$$dD = \mu_d D \cdot dt + \sigma_d \cdot D \cdot \omega_d(t)$$

$$dG = \mu_g G \cdot dt + \sigma_g \cdot G \cdot \omega_g(t)$$

$$dI = \mu_i I \cdot dt + \sigma_i \cdot I \cdot \omega_i(t)$$

➤ Correlated brownian motions

➤ $a^{SOL} = a^* + \frac{1}{\gamma} \cdot \Delta$

$$a^* = \frac{e^{2\mu_d + \sigma_d^2} \cdot (e^{\sigma_d^2} - 1) - e^{(\mu_f + \mu_d) + \frac{1}{2}(\sigma_f^2 + \sigma_d^2)} \cdot (e^{\sigma_f \sigma_d} - 1)}{e^{2\mu_f + \sigma_f^2} \cdot (e^{\sigma_f^2} - 1) + e^{2\mu_d + \sigma_d^2} \cdot (e^{\sigma_d^2} - 1) - 2 \cdot e^{(\mu_f + \mu_d) + \frac{1}{2}(\sigma_f^2 + \sigma_d^2)} \cdot (e^{\sigma_f \sigma_d} - 1)}$$

$$\Delta = \frac{e^{\mu_f + \frac{1}{2}\sigma_f^2} - e^{\mu_d + \frac{1}{2}\sigma_d^2}}{e^{2\mu_f + \sigma_f^2} \cdot (e^{\sigma_f^2} - 1) + e^{2\mu_d + \sigma_d^2} \cdot (e^{\sigma_d^2} - 1) - 2 \cdot e^{(\mu_f + \mu_d) + \frac{1}{2}(\sigma_f^2 + \sigma_d^2)} \cdot (e^{\sigma_f \sigma_d} - 1)}$$

Mixing PAYG and Funding – Stochastic (4)

- Diversification effect :

- If $0 < a < 1$

- a^* depends on the $\text{Corr}(F;D)$ correlation between the risks

- $\text{Corr} = 0$ (independence case) \rightarrow diversification $a^* = \frac{V[D] - \text{Cov}[F;D]}{V[F-D]}$
 - $\text{Corr} < 0 \rightarrow$ diversification is guaranteed
 - $\text{Corr} > 0$ (most probable) \rightarrow we cannot say anything

- Six cases :

Case	$E[F] > E[D] \Leftrightarrow \Delta > 0$	$E[F] < E[D] \Leftrightarrow \Delta < 0$
$a^* < 0$	[1] $a_{OPT} \in [0; 1]$	[2] $a_{OPT} = 0$
$a^* > 1$	[3] $a_{OPT} = 1$	[4] $a_{OPT} \in [0; 1]$
$a^* \in [0; 1]$	[5] $a_{OPT} \in [a^*; 1]$	[6] $a_{OPT} \in [0; a^*]$

- No diversification cases (2 and 3) : only funding or only PAYG (extreme, kind of « arbitrage » cases)
 - Every strategy cases: cases (1 and 4)
 - In between cases (5 and 6)

Thank You !