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Functional estimation in systems defined by differential equation using bayesian smoothing methods

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Abstract.
Differential equation models with observational errors are used in modeling complex experiments in chemical engineering, pharmacokinetics and other sciences.

Current methods for the parameter and functional estimations in models defined by a set of differential equations use minimization techniques and numerical solver. These approaches are computationally intensive and often poorly suited to statistical inference. Instability issues arise, essentially due to numerical integration (Li et al. (2005)).

Improved estimation methods of the state function and of the dynamic model parameters were proposed by Poyton et al. (2005) and Ramsay et al. (2007). These methods account for measurements errors under some continuous distribution and elude numerical integrations. Similarly to smoothing methods, that approach involves some basis function expansion and a penalty term expressed using the set of differential equations.

The methodology above may be viewed as a generalization of the P-spline theory for which a Bayesian framework was proposed by Berry et al. (2002). We aim at providing a Bayesian framework for the more general approach described by Ramsay. First, we present a brief introduction to dynamic models defined by systems of differential equations. We then propose a Bayesian smoothing method for the parameter and functional estimations of differential equation model (Campbell (2007)). Finally, we illustrate the usefulness of this Bayesian approach on simple practical settings.

Keyword.
Differential equations; Dynamic systems; Bayesian smoothing methods; B-splines; Functional estimation.

Reference.

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