Adaptive circular deconvolution by model selection under unknown error distribution

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We consider a circular deconvolution problem, where the density of a random variable X has to be estimated nonparametrically based on an iid. sample from $Y = X + \varepsilon$. Here, ε is an additive measurement error which is supposed to be independent of X. The densities f and φ of X and ε are assumed to be defined on the unit circle and to be square integrable. The objective of this paper is the construction of a fully data-driven estimation procedure when the error density φ is unknown. However, we suppose that in addition to the iid. sample from Y of size n we have at our disposal an additional iid. sample of size m independently drawn from the error distribution.

In the present work, we first derive a lower bound of the maximal estimation risk depending on both sample sizes n and m when f belongs to some weighted ellipsoid and φ is smooth in a certain sense. A typical example is given by a Sobolev ellipsoid and an ordinary or super smooth error density. We show that a regularized orthogonal series estimator of f can attain this lower bound and is therefore optimal in a minimax sense. However, this estimator requires an optimal choice of a dimension parameter depending on certain characteristics of f and φ, which are not known in practice.

The main issue addressed in our work is the adaptive choice of this dimension parameter using a model selection approach. In a first step, we develop a penalized minimum contrast estimator supposing the degree of ill-posedness of the underlying inverse problem to be known, which amounts to assuming partial knowledge of the error distribution. We show that this data-driven estimator can attain the lower risk bound in both sample sizes n and m for a wide range of weighted ellipsoids for f and smoothness conditions on φ. In particular, this estimator is minimax optimal if f belongs to certain Sobolev ellipsoids and φ is ordinary or super smooth.

Finally, by randomizing the penalty and the collection of models, we construct a penalized minimum contrast estimator which does not require any prior knowledge of the error distribution. Even when dispensing with any hypotheses on φ, this fully data-driven estimator preserves minimax optimality in most of the above-mentioned cases.

References

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