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IMPROVING YOUR CHANCES:
AN EXTENSION OF JINDAPON AND NEILSON

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Improving your chances: An extension of Jindapon and Neilson (2007)

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Abstract

In this short note, we show that the results presented by Jindapon and Neilson (2007) for changes in risk à la Ekern (1980) can be extended to stochastic dominance changes, with appropriate additional conditions on the utility function.

Key words and phrases: Risk aversion, downside risk, prudence, temperance, risk apportionment, stochastic dominance.

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Consider two random variables X and Y valued in $[0, b]$ such that all the decision-makers whose preferences satisfy risk apportionment of degrees 1 to s consider that Y is more attractive than X, that is, the inequality $E[u(X)] \leq E[u(Y)]$ holds for all utility functions u with derivatives $u^{(1)}, u^{(2)}, \ldots, u^{(s)}$ of orders 1 to s satisfying $(-1)^{k+1}u^{(k)} \geq 0$. This is henceforth denoted as $X \preceq_s Y$. We refer the reader to Eeckhoudt and Schlesinger (2006) for more details about risk apportionment of various degrees.

Let F_X be the distribution function for such a random variable X. Starting from $F_X^{[1]} = F_X$, we define $F_X^{[2]}, F_X^{[3]}, \ldots$ recursively from repeated integrals:

$$F_X^{[k+1]}(x) = \int_0^x F_X^{[k]}(y)dy, \;\; k = 1, 2, \ldots$$

Then,

$$X \preceq_s Y \;\; \Leftrightarrow \;\; \begin{cases} F_X^{[k]}(b) \geq F_Y^{[k]}(b) \text{ for } k = 1, 2, \ldots, s-1 \\ F_X^{[s]}(t) \geq F_Y^{[s]}(t) \text{ for } t \in [0, b]. \end{cases}$$

Now, assume that such an economic agent can choose to improve his payoff distribution from F_X to $H(\cdot, t)$ defined as

$$H(x, t) = (1-t)F_X(x) + tF_Y(x), \;\; t \in [0, 1],$$

for a cost of $c(t)$ where $c(0) = 0$, $c(1) = b$ and $c^{(1)} > 0$. The agent selects t in order to maximize

$$U(t) = \int_0^b u(x - c(t))dH(x, t)$$

for some utility function u satisfying $(-1)^{k+1}u^{(k)} \geq 0$ for $k = 1, 2, \ldots, s$. Let t_u be the optimal value, fulfilling the first-order condition

$$\int_0^b u\big(x - c(t_u)\big)d\big(F_Y(x) - F_X(x)\big) - \int_0^b u^{(1)}\big(x - c(t_u)\big)c^{(1)}(t_u)dH(x, t_u) = 0.$$

Now, let us consider another agent with utility function v such that $(-1)^{k+1}v^{(k)} \geq 0$ for $k = 1, 2, \ldots, s$, facing the same situation. Let us denote his optimal value by t_v. Furthermore, assume that u is more kth degree Ross risk averse than v for $k = 1$ to s, that is,

$$(-1)^{k-1}\frac{u^{(k)}(x)}{u^{(1)}(y)} \geq (-1)^{k-1}\frac{v^{(k)}(x)}{v^{(1)}(y)} \text{ for all } x \text{ and } y \text{ in } [-b, b],$$

for $k = 1, 2, \ldots, s$. Theorem 2 in Jindapon and Neilson (2007) ensures that $t_u \geq t_v$ when u is more sth degree Ross risk averse than v and F_X has more sth degree risk in the sense of Ekern (1980). The next result shows that the equality of the first $s-1$ moments can be relaxed provided additional conditions are imposed on u and v.

Proposition. Consider $X \preceq_s Y$. If u is more kth degree Ross risk averse than v for $k = 1$ to s then $t_u \geq t_v$.

Proof. Repeated integrations by parts give

$$\int_0^b u(x-c(t_u))d(F_Y(x)-F_X(x)) = \sum_{j=1}^{s-1}(-1)^j u^{(j)}(b-c(t_u))(F_Y^{[j]}(b)-F_X^{[j]}(b)) + (-1)^s \int_0^b u^{(s)}(x-c(t_u))(F_Y^{[s]}(x)-F_X^{[s]}(x))dx.$$

Define y_u as the solution of $u^{(1)}(y_u) = \int_0^b u^{(1)}(x-c(t_u))dH(x,t_u)$ and rescale the utility v so that $v^{(1)}(y_u) = \int_0^b v^{(1)}(x-c(t_u))dH(x,t_u)$. Now, proceeding as in Jindapon and Neilson (2007), we have to show that

$$\begin{aligned}\theta &= \sum_{j=1}^{s-1}(-1)^j \frac{u^{(j)}(b-c(t_u))}{u^{(1)}(y_u)}(F_Y^{[j]}(b)-F_X^{[j]}(b)) \\ &+(-1)^s \int_0^b \frac{u^{(s)}(x-c(t_u))}{u^{(1)}(y_u)}(F_Y^{[s]}(x)-F_X^{[s]}(x))dx \\ &-\left(\sum_{j=1}^{s-1}(-1)^j \frac{v^{(j)}(b-c(t_u))}{v^{(1)}(y_u)}(F_Y^{[j]}(b)-F_X^{[j]}(b))\right. \\ &\left.-(-1)^s \int_0^b \frac{v^{(s)}(x-c(t_u))}{v^{(1)}(y_u)}(F_Y^{[s]}(x)-F_X^{[s]}(x))dx\right)\end{aligned}$$

is non-negative. This is indeed the case, since $(-1)^{j-1}\frac{u^{(j)}(b-c(t_u))}{u^{(1)}(y_u)} \geq (-1)^{j-1}\frac{v^{(j)}(b-c(t_u))}{v^{(1)}(y_u)}$ and $F_Y^{[j]}(b) \leq F_X^{[j]}(b)$ for $j = 1$ to $s-1$ and since $(-1)^{s-1}\frac{u^{(s)}(x-c(t_u))}{u^{(1)}(y_u)} \geq (-1)^{s-1}\frac{v^{(s)}(x-c(t_u))}{v^{(1)}(y_u)}$ and $F_Y^{[s]}(x) \leq F_X^{[s]}(x)$ for all x.

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