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Inference in stochastic frontier analysis with dependent error terms

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Abstract

Stochastic frontier analysis (SFA) is often used to estimate technical efficiency of entities such as firms, countries or municipalities. A potential dependence between the two components of the error term can be taken into account by a copula function. While estimation of the model is straightforward using the Corrected Ordinary Least Squares (COLS) and Maximum Likelihood (ML) methods, an open issue concerns the inference of the technical efficiencies. We propose a parametric bootstrap algorithm which is an extension of an algorithm proposed by Simar and Wilson [18] to the dependence case. This allows us to estimate the efficiency percentile confidence intervals. We apply the model to the estimation of technical efficiencies of moroccan municipalities.

Keywords: Bootstrap, Copulas, Efficiency, Inference, Stochastic frontier analysis

1. Introduction

Efficiency analysis has often been carried out using nonparametric frontier models such as the Data Envelopment Analysis (DEA) or the Free Disposal Hull (FDH). An alternative approach is to use Stochastic Frontier Analysis (SFA), which includes an error term such that deviations from the frontier can be purely random without necessarily indicating inefficency. SFA can be formulated both in a parametric or nonparametric framework, but

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the parametric SFA has certainly been predominant in the literature and in applications. The basic idea of all approaches is the comparison between the Decision Making Units (DMU, firms for example) in order to know how inputs are used to produce outputs and the comparison is based on the Technical Efficiency (TE) score achieved by each unit. By definition, technical efficiency reflects the ability of the firm to obtain maximal output from a given set of inputs.

The nonparametric frontier approach using DEA or FDH requires minimal assumptions regarding the structure of the production and does not impose restrictions on the functional form relating inputs and outputs. It does not account for noise in the data, so it implicitly assumes that every deviation from the frontier is considered as inefficiency.

However, in the parametric SFA, assumptions have to be made both about the functional form and the distribution of the two types of error, namely, the symmetric stochastic error term and the divergence of observations from the efficient frontier. This stochastic frontier approach in the efficiency analysis was simultaneously and independently introduced by Aigner, Lovell and Schmidt [2] and by Meeusen and Van den Broeck [13]. Later, several extensions have been proposed by, for example, Agahi, Zarafshani and Behjat [1], Greene [9, 8], Kumbhakar and Knox Lovell [12], Simar and Wilson [18] and Smith [19]. A FRONTIER software was developed by Coelli [6] in order to estimate the stochastic frontier production and the cost function in the case where the two components of the error term are independent. This software is now also available in the statistical computation environment R, see [16]. As a consequence of its increasing computational availability, stochastic frontier analysis has been widely applied in several areas.

Recently, Smith [19] has proposed an SFA model allowing for dependence between the two error components. The dependence can be explicitly modelled using copula functions, while maintaining typical assumptions about the marginal distribution of the error terms. Estimation of the model using the Corrected Ordinary Least Squares (COLS) and Maximum Likelihood (ML) methods is straightforward but can be computationally challenging. Furthermore, inference about the technical efficiencies is not standard. In this paper, we propose a bootstrap procedure, which is an extension of an algorithm proposed by Simar and Wilson [18] to the copula case. This allows to obtain not only point estimates, but also confidence intervals for the estimated technical efficiencies.

We apply the model to the estimation of technical efficiencies of moroccan municipalities, defining operating receipts as input and financial autonomy as output. The model is estimated with alternative distributions for the one-sided error term, as well as alternative copulas. The best model is selected using classical information criteria. The obtained bootstrap confidence intervals for the technical efficiency estimates are narrow, confirming the adequacy of our methodology and the interpretation of the results. We find that, contrary to common understanding, no municipality in the central regions of the country is close to the frontier.

The remainder of the paper is organized as follows. The second section gives an overview of parametric SFA and its history, the third section presents the model with dependent error terms and explains the estimation and inference using the bootstrap. The fourth section presents the application of the proposed methodology, and finally the conclusions in Section 5 will summarize the analysis.

2. Parametric stochastic frontier models

Classical parametric stochastic frontier models assume that there is a production function f that converts $X \in \mathbb{R}_+^p$, a vector of inputs of dimension p, into a scalar output $Y \in \mathbb{R}_+$. Supposing that one has n observations of (X_i, Y_i) the model can be written for the i-th DMU as

$$y_i = f(x_i, \beta) + \varepsilon_i, \qquad i = 1, \cdots, n \tag{1}$$

where $y_i = \log(Y_i)$, $x_i = \log(X_i)$, β is a vector of parameters of dimension $l+1$ to be estimated, and ε_i is a stochastic error term. The function $f(x_i, \beta)$ is interpreted as the production frontier.

The stochastic term ε_i contains information about both the noise and the inefficiency. It can be decomposed into a technical inefficiency and a noise term, which can be estimated. In particular, a typical specification is given by

$$\varepsilon_i = v_i - u_i, \tag{2}$$

where v is a Gaussian error term, $(v_i \sim N(0, \sigma_V^2))$, and u is a stochastic error term with non-negative support ($u_i \geq 0$, a.s.).

Note that the stochastic component v_i that describes random noise affecting the production process is not directly attributable to the producer or the underlying technology. The noise may come from weather changes, economic adversities, etc. The other component, u_i, measures technical inefficiency in the sense that it measures the shortfall of output y_i from its maximal possible value given by the stochastic frontier $(f(x_i, \beta) + v_i)$ and it is equal to zero for a technically efficient decision unit. Then, the one-sided error term $u_i \geq 0$ allows the distinction between DMU (e.g. firms) that are on the frontier ($u_i = 0$) and others that are below the frontier ($u_i > 0$).

The stochastic model then permits to estimate β and its standard errors and, consequently, to make statistical tests of hypotheses. However, one of the criticisms of this model is that there is no *a priori* justification for the selection of the distributional form for u_i. Several choices have been made in the literature, see e.g. the overview of Kumbhakar and Knox Lovell [12], for example, the exponential, the half-normal, the truncated normal or the Gamma distribution. Furthermore, in order to decompose the error term ε into its two components, one has to make assumptions on their dependence. Classical SFA assumes that they are independent. Let us first recall this approach, see e.g. Jondrow, Knox Lovell, Materov and Schmidt [11].

2.1. Classical SFA with independence

The parameters of the model described by (1) and (2) can be estimated using, for instance, the maximum likelihood method and ε_i can be predicted by $\hat{\varepsilon}_i = y_i - f\left(x_i, \hat{\beta}\right)$, which contains information on u_i. Jondrow, Knox Lovell, Materov and Schmidt [11] propose a decomposition by considering the expected value of u, conditional on $\varepsilon = v - u$. They proceed by considering the conditional distribution of u_i given ε_i. Either the mean or the mode of this distribution can be used as a predictor of u_i.

In the Normal-half-normal case, $v_i \sim N(0, \sigma_V^2)$, u has a half-normal distribution ($u_i \sim N^+(0, \sigma_U^2)$), and v and u are supposed to be independent. Based on these assumptions, one can derive analytical expressions for the marginal distribution of ε and the conditional distribution of u given ε, see Jondrow, Knox Lovell, Materov and Schmidt [11].

2.2. SFA with dependent error components

Consider in this section the simplest cross-section case with n independent DMUs. The most general way to introduce dependence is to use copula functions, which in the present context has been proposed recently by Smith [19]. Appendix A gives a definition and some properties of copulas, and provides frequently used examples of parametric copula functions.

Let us consider the normal, half-normal production frontier model with Cobb-Douglas production function. Thus, let $v \sim i.i.d.N(0,\sigma_V^2)$ and $u \geq 0$, $u \sim i.i.d.N^{+}(0,\sigma_U^2)$ with $E(u) = \sigma_U\sqrt{2/\pi}$ and $Var(u) = ((\pi-2)/\pi)\,\sigma_U^2$. We know also that $\varepsilon = v - u$, and hence $Var(\varepsilon) = Var(u) + Var(v) - 2Cov(u,v)$. Therefore, a positive correlation between u and v ($i.e.$ $Cov(u,v) > 0$) reduces the variance of ε, and a negative correlation increases it.

The joint density of u and v when they are dependent is expressed for all $u \geq 0$ and $v \in \mathbb{R}^n$ by

$$g_\theta(u,v) \quad = \quad f_1(u) f_2(v) c_\theta(F_1(u), F_2(v))$$

In the following we give two examples. Consider first the Gaussian copula (see Appendix A.1), for which the joint density of u and v can be derived as

$$\begin{aligned} g_\theta(u,v) &= \frac{1}{\pi\sigma_U\sigma_V}\exp\left\{-\frac{1}{2\sigma_U^2}u^2 - \frac{1}{2\sigma_V^2}v^2\right\} \\ &= .\left[\frac{\phi_{2,\theta}\left(\Phi^{-1}(F_1(u)),\Phi^{-1}(F_2(v))\right)}{\phi\left(\Phi^{-1}(F_1(u))\right).\phi\left(\Phi^{-1}(F_2(v))\right)}\right], \end{aligned}$$

where $\theta \in [-1,1]$ is the parameter of the copula, $F_1(u) = 2\Phi(u/\sigma_U)$ and $F_2(v) = \Phi(v/\sigma_V)$.[1]

For the case of an FGM copula (see Appendix A.2), the joint density becomes

$$\begin{aligned} g_\theta(u,v) = {} & \frac{1}{\pi\sigma_U\sigma_V}\exp\left\{-\frac{1}{2\sigma_U^2}u^2 - \frac{1}{2\sigma_V^2}v^2\right\} \\ & .\left[1+\theta-4\theta\Phi\left(\frac{u}{\sigma_U}\right)-2\theta\Phi\left(\frac{v}{\sigma_V}\right)+8\theta\Phi\left(\frac{u}{\sigma_U}\right)\Phi\left(\frac{v}{\sigma_V}\right)\right], \end{aligned}$$

where $\theta \in [-1,1]$. The joint density of u and ε is obtained by replacing v in $g_\theta(u,v)$ by $v = \varepsilon + u$.

To obtain the density of ε , the joint density of u and ε is then integrated by the variable u,

$$g_\theta(\varepsilon) \quad = \quad \int_0^{+\infty} g_\theta(u,\varepsilon)\,\mathrm{d}u$$

[1] Note that $F_1(u) = \int_{-\infty}^{u}\frac{2}{\sigma_U\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\left(\frac{t}{\sigma_U}\right)^2\right\}\mathrm{d}t = \int_{-\infty}^{u/\sigma_U}\frac{2}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}z^2\right\}\mathrm{d}z = 2\Phi\left(\frac{u}{\sigma_U}\right)$.

$$\begin{aligned} &= \int_0^{+\infty} f_1(u) f_2(\varepsilon+u) c_\theta\left(F_1(u), F_2(\varepsilon+u)\right) \mathrm{d}u \\ &= E_U\left(f_2(\varepsilon+u) c_\theta\left(F_1(u), F_2(\varepsilon+u)\right)\right) \end{aligned} \tag{3}$$

If no analytical solution of the integral is available, one can approximate it numerically via simulation by drawing a large number m of random variables U from the marginal distribution of u, and calculate for any value ε,

$$g_\theta\left(\varepsilon_i\right) \cong \frac{1}{m} \sum_{j=1}^{m} f_2(\varepsilon_i + u_j) c_\theta\left(F_1(u_j), F_2(\varepsilon_i + u_j)\right)$$

Replacing ε by $y - f\left(x, \beta\right)$ in the expression of $g_\theta\left(\varepsilon\right)$ gives the density of y. Assuming independence across DMUs, the log likelihood function is given by

$$l\left(\vartheta\right) = \sum_{i=1}^{n} \log g_\theta\left(\varepsilon_i\right) = \sum_{i=1}^{n} \log g_\theta\left(y_i - f\left(x_i, \beta\right)\right), \tag{4}$$

where $\vartheta = (\sigma_U, \sigma_V, \theta, \beta)'$, and the ML estimator of ϑ is defined as

$$\hat{\vartheta}_{ML} = \arg\max_{\vartheta \in \Theta} l\left(\vartheta\right)$$

Maximization of the log likelihood function is typically done using numerical techniques, as analytical solutions are rarely available.

Based on ML parameter estimates, one can address the issue of estimating technical efficiencies, defined as the expectation of efficiency conditional on observed residuals, see Battese and Coelli [3]. Hence, technical efficiency of DMUs, which depends on the parameter $\vartheta = (\sigma_U, \sigma_V, \theta, \beta)$ and on the observed input x and output y, is defined by

$$TE_\vartheta = E\Big(\exp\left\{-U\right\} \mid \varepsilon \Big)$$

Using the marginal distribution of ε in (3), and the joint density of u and ε we can calculate

$$\begin{aligned} TE_\vartheta &= \int_{\mathbb{R}^+} \exp\left\{-u\right\} g_\theta\left(u \mid \varepsilon\right) \mathrm{d}u \\ &= \frac{1}{g_\theta\left(\varepsilon\right)} \int_{\mathbb{R}^+} \exp\left\{-u\right\} g_\theta\left(u, \varepsilon\right) \mathrm{d}u \\ &= \frac{E_U\Big(\exp\left\{-u\right\} . f_2\left(u+\varepsilon\right) . c_\theta\Big(F_1\left(u\right), F_2\left(u+\varepsilon\right)\Big)\Big)}{E_U\Big(f_2\left(u+\varepsilon\right) . c_\theta\Big(F_1\left(u\right), F_2\left(u+\varepsilon\right)\Big)\Big)} \end{aligned} \tag{5}$$

For a given ϑ, this expression can again be approximated via simulation by drawing a large number of random variables U and approximate the expectations appearing in numerator and denominator by the corresponding simulation means. Replacing ϑ by its ML estimator provides the ML estimator of TE_{ϑ}.

While point estimation of TE_{ϑ} is straightforward, although it may be computationally demanding, it is less obvious how to obtain interval estimates and how to do inference. We next describe an algorithm for obtaining confidence intervals for the technical efficiencies.

2.3. Bootstrap confidence intervals for technical efficiencies

We now propose a statistical inference procedure to construct confidence intervals for technical efficiencies in the SFA model with dependence. We use an extension of the bootstrap procedure described in Simar and Wilson [18]. In particular, step 2 of algorithm#3 of Simar and Wilson [18] is modified to take into account the dependence between u_i^* and v_i^* using the Clayton copula. The various steps of the algorithm are as follows:

Step 1. Estimate $\vartheta = (\sigma_U, \sigma_V, \theta, \beta_0, \beta_1)$ according to (4), using $(x_i, y_i)\,, i = 1, \ldots, n$ and using numerical optimization procedures to get $\widehat{\vartheta} = \left(\widehat{\sigma}_U, \widehat{\sigma}_V, \widehat{\theta}, \widehat{\beta}_0, \widehat{\beta}_1\right)$ and to compute $TE_{\hat{\vartheta}}$.

Step 2. For $i = 1, \ldots, n$, draw $u_i^* \sim N^+\left(0, \hat{\sigma}_U^2\right)$ and $v_i^* \sim N\left(0, \hat{\sigma}_V^2\right)$ such that their dependence is given by the Clayton copula, and then compute $y_i^* = f\left(x_i, \hat{\beta}\right) + v_i^* - u_i^*$.

There are several procedures to generate the pair (u_i^*, v_i^*) with dependence given by the Clayton copula, we mention one of them which uses the conditional distribution approach described in Nelsen [15], page 41 and denoted $c_{w_1}(w_2)$,

$$\begin{aligned} c_{w_1}(w_2) &= P\left(W_2 \leq w_2 \mid W_1 = w_1\right) \\ &= \lim_{\Delta w_1 \to 0} \frac{C\left(w_1 + \Delta w_1, w_2\right) - C\left(w_1, w_2\right)}{\Delta w_1} = \frac{\partial C\left(w_1, w_2\right)}{\partial w_1} \end{aligned} \tag{6}$$

The four steps of this procedure are:

a) Draw two independent uniform random variables (w_{1i}, t_{2i}) such that $w_{1i} \sim U(0,1)$ and $t_{2i} \sim U(0,1)$.

b) Set $w_{2i} = \left[w_{1i}^{-\hat{\theta}}\left(t_{2i}^{-\hat{\theta}/(1+\hat{\theta})} - 1\right) + 1\right]^{-1/\hat{\theta}}$.

c) Set $u_i^* = F_1^{-1}(w_{1i})$ and $v_i^* = F_2^{-1}(w_{2i})$, where F_1 and F_2 are the cumulative distribution function of the $N^+(0, \hat{\sigma}_U^2)$ and $N(0, \hat{\sigma}_V^2)$ respectively.

d) Repeat steps a) to c) to generate n pairs (u_i^*, v_i^*).

Step 3. Using the pseudo-data $\mathscr{S}_{b,n}^* = \{(x_i, y_i^*)\}_{i=1}^n$, compute a bootstrap estimate $\hat{\vartheta}_b^* = \arg\max_{\vartheta \in \Theta} l\left(\vartheta \mid \mathscr{S}_{b,n}^*\right)$ after replacing y_i by y_i^* in (4) and then compute a bootstrap estimate $\widehat{TE}_b^*$ using (5) after replacing ε by $\varepsilon_b^* = y - f(x, \hat{\beta}_b^*)$, where x and y represent the observed data.

Step 4. Repeat steps 2 and 3 B times to obtain estimates $\mathscr{B}^* = \{\hat{\vartheta}_b^*\}_{b=1}^B$. Then, use $\mathscr{B}^*$ to obtain the set of B bootstrap estimates of technical efficiency, $\mathscr{E}^* = \{\widehat{TE}_b^*\}_{b=1}^B$. For each individual i (row i of the $\mathscr{E}^*$ matrix, denoted by $\mathscr{E}_i^*$), $i = 1, \ldots, n$, compute the $\left(\frac{\alpha}{2}\right)$ and the $\left(1 - \frac{\alpha}{2}\right)$ quantiles for $\mathscr{E}_i^*$ by considering its B components. The $100 \times (1-\alpha)$ percentile bootstrap confidence interval of the statistic of interest TE is obtained by the probability $P\left((\mathscr{E}_i^*)_{\frac{\alpha}{2}} < TE_i < (\mathscr{E}_i^*)_{1-\frac{\alpha}{2}}\right) = 1 - \alpha$. Hence, using the $100 \times \left(\frac{\alpha}{2}\right)$ and $100 \times \left(1 - \frac{\alpha}{2}\right)$ percentiles, we define the lower and the upper bounds of the confidence interval as $TE_i \in \left[(\mathscr{E}_i^*)_{\frac{\alpha}{2}}, (\mathscr{E}_i^*)_{1-\frac{\alpha}{2}}\right]$.

The proposed algorithm may be computationally intensive but it is straightforward to apply and to implement. Furthermore, bootstrap techniques have the advantage of taking implicitly the estimation uncertainty of the parameters into account.

We investigate the performance of the proposed method in a small Monte Carlo study. We use the same model and parameters as in Simar and Wilson (2010), i.e. $\beta_0 = \log(20)$, $\beta_1 = 0.8$, $\lambda = \sigma_u/\sigma_v = 2$, and quantiles of ε given by $\varepsilon_{(q)}$, where $q \in \mathcal{Q}$ and $\mathcal{Q} = \{0.1, 0.3, 0.5, 0.7, 0.9\}$. The input value is fixed at $x_0 = 60$, and five log-output values are given by $y_{(q)} = \exp(\beta_1 + \beta_2 x_0 + \varepsilon_{(q)})$, corresponding to the five quantiles of $\varepsilon_{(q)}$. The dependence parameter θ of the Clayton copula is fixed at 0.1. We use $n = 100$ as sample size, $M = 120$ Monte Carlo trials, and $B = 100$ bootstrap replications. The estimated coverages of the bootstrap confidence intervals are given in Table 1. The results confirm those of Simar and Wilson (2010) for the independence case, but suggest that the over-coverage is slightly larger under dependence across the five quantiles.

Table 1: Estimated coverages of $E[e^{-U}|\theta, x_0 = 60, y_0 = y_{(q)}]$ by bootstrap confidence intervals

quantile	$1-\alpha$		
	0.9	0.95	0.99
0.1	0.96	0.97	1
0.3	0.95	0.97	1
0.5	0.95	0.96	0.99
0.7	0.95	0.97	0.99
0.9	0.95	0.97	1

3. An analysis of the efficiency of Moroccan municipalities

In this application, 1298 Moroccan rural municipalities (DMUs) are considered to produce one output which is the financial autonomy using operating receipts as input for the budgetary year 1998/1999. The operating receipts include ten receipts which are the urban tax, the tax on the collection of waste, the tax of the licence, the forest domain product, the taxes and assimilated taxes, the product of services, the product and the income of goods, the concessions, the subsidies and competition and finally the order receipts. We chose to use the aggregate measure of operating receipts as the single input because it is a meaningful variable and because it allows us to reduce the dimensional complexity. Some preliminary analysis with a multiple input framework did not change the main conclusions drawn from the aggregate input case.

Furthermore, financial autonomy is defined as the ratio of the own receipts and the operating expenses. As for the own receipts, they include all operating receipts except the subsidies and competition. After the decentralization of the Moroccan administration, this kind of data is not available after 1999. Thus, our data consists of pairs (X_i, Y_i) where X_i represents the single input expressed by the operating receipts of the DMU_i used to produce the output Y_i, i.e. the financial autonomy of the same DMU_i.

Municipalities are clustered in provinces and regions, and we could have added a hierarchical structure to the model, but did not pursue this direction for simplicity. In the interpretation of the results, we will come back to this point and try to interpret the estimated DMU efficiencies with respect to their geographical and political situation.

To represent the production technology, we consider the frequently used

Cobb-Douglas and translog production functions, see e.g. Christensen, Jorgenson and Lau [5] for a general definition of the translog production function. The Cobb-Douglas function is nested in the translog one, such that it can be tested. In our case with just one input variable, the test reduces to testing a linear model against a quadratic one.

Our distributional assumptions about the error terms are as follows. For the random noise term v we assume a normal distribution, while either a half-normal (HN) or truncated normal (TN) distribution are adopted for the inefficiency component u. Again, the HN model is nested in the TN model, such that it can be tested easily. The general model with translog production function and TN distribution for u then reads

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + v_i - u_i, \qquad i = 1, ..., n \tag{7}$$

where

$$v_i \sim N\left(0, \sigma_V^2\right), \qquad u_i \sim N^+\left(\mu, \sigma_U^2\right).$$

The special case Cobb-Douglas is attained by restricting $\beta_2 = 0$, and HN by restricting $\mu = 0$. The stochastic frontier model in (7) with independent u and v is estimated by Maximum Likelihood, where initial values are set to the Ordinary Least Squares (OLS) estimates. The OLS estimators of β_1 and β_2 are unbiased, and the intercept can be bias-adjusted using the Corrected Ordinary Least Square (COLS) method, see e.g. Greene [10].

Assuming independence of u and v, the results summarized in Table 2 reveal that models with the Translog function have slightly higher log-Likelihood values compared with the corresponding Cobb-Douglas models. Using a likelihood ratio test statistic, the null hypothesis $\beta_2 = 0$ can not be rejected with a p-value equal to 0.4130 in the half-normal case, and 0.6079 in the truncated normal case. Therefore, we pursue our analysis accepting the Cobb-Douglas specification. In addition, testing $\mu = 0$ in the truncated normal model does not lead to a rejection at a level of 5%, which means that we accept the half-normal specification. Thus, the Normal-HN model with Cobb-Douglas production function is considered and its estimates will be chosen as initial values in the case of dependence between u and v. They are reported in Table 3.

As it is not possible to directly estimate the error components u and v, but only their difference, $v - u$, we can not directly test the independence between them. However, it is possible to generalize the preferred model

Table 2: Technical Efficiency and Log-Likelihood values in the independence case

Name(Pop. Order)	CD-HN	TL-HN	CD-TN	TL-TN
Tagante(244)	0.919	0.917	0.916	0.915
Tidili Mesfioua(1239)	0.881	0.880	0.887	0.888
Agafay(667)	0.848	0.850	0.861	0.863
Timzguida-Ouftas(185)	0.831	0.834	0.848	0.850
Adaghas(102)	0.819	0.819	0.839	0.840
Bouhmame(1252)	0.814	0.817	0.835	0.837
Ida ou Guelloul(313)	0.809	0.811	0.831	0.834
Taghazout(234)	0.809	0.811	0.830	0.833
Mouarid(299)	0.807	0.808	0.830	0.832
Ait Aissi Ihahane(257)	0.805	0.807	0.828	0.830
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Jdiriya(46)	0.021	0.021	0.021	0.021
Tifariti(69)	0.021	0.020	0.020	0.021
Haouza(77)	0.021	0.020	0.020	0.020
log-likelihood	-1771.50	-1771.165	-1764.131	-1764.117
mean	0.478	0.475	0.558	0.560

HN: half-normal; TN: truncated normal; CD: Cobb Douglas; TL: translog

Table 3: Maximum likelihood estimates in the independence case for the CD-HN model

	Estimate	Std. Error	t value	p.value
β_0	-12.0559	0.6036	-19.975	$< 2.2e-16$
β_1	1.1149	0.0410	27.196	$< 2.2e-16$
σ^2	1.8322	0.1184	15.474	$< 2.2e-16$
γ	0.7796	0.0321	24.323	$< 2.2e-16$
λ	1.8808			
σ_U	1.1952			
σ_V	0.6355			

λ, σ_U, σ_V computed from $\lambda = \frac{\sigma_U}{\sigma_V}$, $\sigma^2 = \sigma_U^2 + \sigma_V^2$ and $\gamma = \frac{\lambda^2}{1+\lambda^2}$

under independence, i.e. CD-HN, to allow for dependence. In particular, introducing a copula which nests the product copula, i.e. independence, as a special case allows to test the null hypothesis of independence using a likelihood ratio test, or use a model selection criterion such as AIC or BIC to distinguish between the two models. In our case, it will turn out that copula models significantly outperform the model assuming independence, which indicates that the assumption of independence is too restrictive. Ignoring dependence between the error components may lead to biased estimates of β, σ_U and σ_V. In the following, we therefore consider various copula models for the joint distribution of u and v.

The maximization of the log-likelihood function in (4) often requires numerical derivatives. We use the MLE function from the stats4 package in the R software. The estimates under independence reported in the Table 3 are used as initial values. Several optimization methods as a variant of a simulated annealing method (SANN) given in Bélisle [4] and the Nelder-Mead method given in Nelder and Mead [14] are offered in the R package, but the selected one is the Nelder-Mead method.

As pointed out by Ritter and Simar [17] and Simar and Wilson [18], the estimation with location parameter μ in the TN model can be numerically difficult and may require very large sample sizes. The reason is that, for moderate sample sizes, the likelihood function is flat with respect to μ, such that a practical identification issue arises, although asymptotically the model is well identified. An alternative is to set this parameter to a predetermined value. In our case, we have experimented with several values and have chosen to set $\mu = -1$ for the models with truncated normal distribution, as it gave in most cases the best fit. Technical efficiencies are estimated according to (5) for ten models using the Cobb-Douglas function, the normal distribution for the noise term v, the half-normal and the truncated normal distributions with $\mu = -1$ for the inefficiency error u and using five copulas. These copulas are the Ali-Mikhail-Haq (AMH), Clayton, Fairlie-Gumbel-Morgenstern (FGM), Frank and Gaussian copulas.

Table 4: Technical Efficiency and Log-Likelihood values with dependence

N	HN-AMH	HN-FGM	HN-Clay	HN-Frank	HN-Gauss	TN-AMH	TN-FGM	TN-Clay	TN-Frank	TN-Gauss
244	0.835	0.801	0.748	0.903	0.788	0.838	0.849	0.779	0.874	0.742
1239	0.751	0.722	0.647	0.850	0.752	0.758	0.779	0.695	0.813	0.716
667	0.686	0.674	0.577	0.803	0.723	0.695	0.727	0.633	0.762	0.695
185	0.658	0.660	0.550	0.779	0.709	0.667	0.708	0.607	0.739	0.684
102	0.640	0.654	0.533	0.763	0.701	0.651	0.701	0.594	0.726	0.675
1252	0.633	0.653	0.527	0.757	0.695	0.642	0.697	0.586	0.718	0.673
313	0.627	0.651	0.522	0.751	0.692	0.636	0.694	0.581	0.714	0.669
234	0.625	0.651	0.520	0.749	0.690	0.633	0.693	0.578	0.711	0.669
299	0.623	0.651	0.519	0.748	0.690	0.633	0.694	0.579	0.712	0.668
...	...	...	...	...	...	...	...	...	...	...
46	0.006	0.014	0.004	0.012	0.011	0.007	0.008	0.005	0.018	0.005
77	0.006	0.014	0.004	0.012	0.011	0.007	0.008	0.005	0.018	0.005
69	0.006	0.013	0.004	0.011	0.011	0.007	0.008	0.005	0.017	0.004
Mean	0.382	0.426	0.342	0.421	0.423	0.459	0.454	0.394	0.471	0.407
Median	0.452	0.462	0.435	0.452	0.460	0.535	0.500	0.487	0.516	0.455
θ	0.965	0.984	1.375	1.618	0.348	0.981	0.960	1.259	1.989	0.501
λ	2.329	1.920	2.528	2.276	1.986	2.619	2.574	2.649	2.130	2.694
logLik	-1752.811	-1761.141	-1750.948	-1759.379	-1767.649	-1755.447	-1758.865	-1756.608	-1759.623	-1763.715

$\mu = -1$ for the truncated normal distribution

Table 5: ML estimator of ϑ for the HN-Clay model

ϑ	Estimate	Std. Error	$t_{stat} = \frac{\widehat{\vartheta}_k}{SE(\widehat{\vartheta}_k)}$
β_0	-11.6612	0.0017	-6859.530
β_1	1.1290	0.0037	305.135
θ	1.3750	0.0027	509.259
σ_U	1.9167	0.0018	1064.833
σ_V	0.7583	0.0017	446.059
logLik	-1750.948		

Table 4 reports a subsample of the estimated technical efficiencies for the ten alternative models, together with the estimated θ of the corresponding copula. Furthermore, the ratio of standard deviations $\lambda = \sigma_U/\sigma_V$ is reported, which is a measure for the asymmetry of the composite error distribution. Finally, the estimated likelihood values are reported. Since all models have the same number of parameters, standard model selection criteria such as AIC or BIC are equivalent to choosing the model with the highest likelihood value. In our case, this is the model where the error term v has a Normal distribution, the inefficiency term u has a half-normal distribution and where the dependence between u and v is expressed by the Clayton copula. This preferred model will be denoted HN-Clay.

The parameter estimates of the HN-Clay model are presented in Table 5. Note that all parameters are significantly different from zero at all usual significance levels. In particular, the independence hypothesis $\theta = 0$ is clearly rejected. To interpret the association between u and v we calculate Kendall's τ which is the probability of concordance minus the probability of discordance, and is thus standardized to the interval $[-1, 1]$. For the Clayton copula, Kendall's τ is given by $\tau = \theta/\left(\theta + 2\right)$ and for the estimated $\theta = 1.375$ takes the value $\tau = 0.407$. Clearly, the probability of concordance is higher than the probability of discordance for the random variables u and v.

To classify the estimation results with respect to the 1298 districts, we rank these with respect to increasing population size. Note from the results in Table 4 that the ranking of efficiency estimates is almost always the same irrespective of the model. No district is close to the frontier, the highest efficiency is attained for the Tagante (244) district, which is in the Guelmim Province (50) in the south of Morocco and which is ranked 244-th according to (increasing) population size. The following districts are Tidili Mesfioua (1239) in the Al Haouz Province (20), Agafay (667) in Agadir Idaoutanane Province (16), Timzguida-Ouftas (185) in Essaouira Province (47), Adaghas (102) in Essaouira Province (47), and Bouhmame (1252) in Safi Province (44). For the chosen HN-Clay model, the Tagante (244) district, for instance, could reach efficiency by reducing its resources by 25.2 percent. The least efficient of all districts is Tifariti (69) in the Es-Smara province(48), with receipts covering less than 1 percent of its expenses.

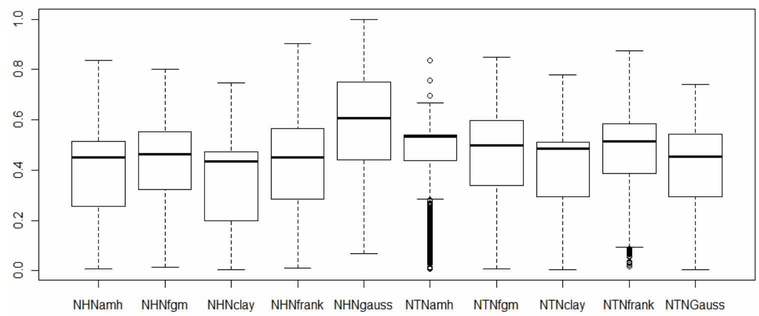
Note also that there is a very big disparity between districts of the Guelmim-Es Semara region in so far as it includes the the most efficient district as well as the three least efficient ones. However, they belong to completely different provinces which are Guelmim for the first one and Es-Semara for the three last ones and both provinces have different geographical specificities. On the other hand, among the ten most efficient districts, seven are in the Marrakech-Tensift-Al Haouz region and among these seven municipalities, five are in the Essaouira Province. Even if they are well classified, their estimates of technical efficiency remain quite far from the frontier irrespective of the model used for the dependence.

It is surprising not to find among the most efficient municipalities those of the central regions (for example the Rabat-Salé-Zemmour-Zaër or the Grand Casablanca regions) which are close to the central administration and where local council members typically have a high training level. However, in the absence of data on the geographical distance and on the training level of the local elected officials, their effects on the municipality efficiency cannot be formally tested.

Except for the models with Frank and Gaussian copula, efficiency means and medians of all models where the inefficiency term u has a half-normal distribution are lower than those using the truncated normal distribution. Note that the choice of copula affects the estimated efficiency level. The highest levels are obtained for the Frank copula, the lowest for Clayton (HN) and Gaussian (TN). Note also that the medians are higher than the means in both cases, reflecting the fact that both distributions are negatively skewed in all cases. This can be seen in Figure 1, which displays the Box-plots of estimated efficiencies for all models and which illustrate the dispersion and skewness of their distributions.

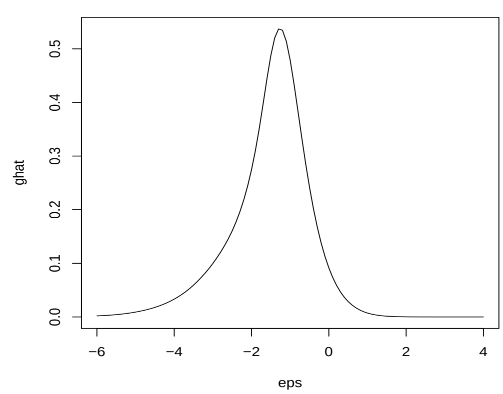
Figure 2 depicts the estimated $g_\theta(\varepsilon)$ density for the HN-Clay model. It clearly shows the skewness, which can also be expressed in terms of the estimated ratio of standard errors, $\lambda = \sigma_U/\sigma_V$, taking a value 2.528, recalling

Figure 1: Boxplot of TE for ten models with HN and TN($\mu = -1$)



that a symmetric density would be obtained for $\lambda = 0$. Note also that due to the dependence between the two error components, the mode of the density is not at zero but shifted to the left. Intuitively, introducing a dependence between u and v, which is positive in our case, tends to reduce the general level of estimated technical efficiencies, which can also be seen by comparing the results of Table 4 with those of the independence case in Table 2.

Figure 2: $g_\theta\left(\varepsilon\right)$ distribution for the N-HN-Clay Copula model



We now provide estimated 95% confidence intervals for the technical efficiencies using the bootstrap algorithm presented in Section 2.3. Table 6 gives an overview of the lower and the upper bounds of bootstrap confidence

Table 6: TE confidence intervals for the HN-Clay model

Name(Pop. Order)	Province	Region	Lower	TE	Upper
Tagante(244)	Guelmim	Guelmim-Es Smara	0.6933	0.7484	0.8753
Tidili Mesfioua(1239)	Alhouz	Marrakech-T.-H.***	0.5863	0.6468	0.8111
Agafay(667)	Marrakech M.*	Marrakech-T.-H.***	0.5183	0.5772	0.7558
Timzguida-Ouftas(185)	Essaouira	Marrakech-T.-H.***	0.4939	0.5495	0.7284
Adaghas(102)	Essaouira	Marrakech-T.-H.***	0.4813	0.5331	0.7145
Bouhmame(1252)	El Jadida	Doukkala-Abda	0.4752	0.5269	0.7039
Ida ou Guelloul(313)	Essaouira	Marrakech-T.-H.***	0.4712	0.5217	0.6985
Taghazout(234)	Agadir I. O.**	Sous-Massa-Draâ	0.4695	0.5202	0.6945
Mouarid(299)	Essaouira	Marrakech-T.-H.***	0.4693	0.5190	0.6959
Ait Aissi Ihahane(257)	Essaouira	Marrakech-T.-H.***	0.4674	0.5169	0.6925
...	...	...	...	...	...
Ain Blal(237)	Settat	Chaouia-Ouardigha	0.4388	0.4741	0.6034
Amerzgane(611)	Ouarzazate	Sous-Massa-Draâ	0.4483	0.4739	0.5377
Sidi Lahsen(773)	Taourirt	Oriental	0.4483	0.4739	0.5375
...	...	...	...	...	...
Jdiriya(46)	Es-Semara	Guelmim-Es Smara	0.0032	0.0039	0.0066
Haouza(77)	Es-Semara	Guelmim-Es Smara	0.0032	0.0038	0.0066
Tifariti(69)	Es-Semara	Guelmim-Es Smara	0.0031	0.0037	0.0063

*Marrakech M.: Marrakech Menara, **Agadir I. O.: Agadir Ida Outanane, ***Marrakech-T.H.: Marrakech-Tensift-Al Haouz.

intervals for the N-HN-Clay model with number of bootstrap replications $B = 700$. As summarized in this table, each estimated efficiency is covered by the associated confidence interval as expected, and generally the range of each confidence interval is rather small. Note that the estimated efficiencies are generally closer to the lower limit of the interval and, hence, the intervals are not symmetric around the estimated TE values, which is also as expected.

It may be of further interest to discover any links of estimated technical efficiencies with observed characteristics such as the population size. For the selected model HN-Clay we use Kendall's independence test between technical efficiencies and population size, which yields a statistic $\tau = -0.0468$ and corresponding p-value equal to 0.0011. Thus we reject the null hypothesis of independence. The relation between the two variables is opposite, so that highly populated districts tend to be less efficient. This may suggest that population size influences financial autonomy, in which case it could be included in the model.

4. Conclusion

In the framework of a stochastic frontier analysis with dependence between the noise term V and inefficiency U, we introduce a bootstrap procedure to estimate confidence intervals for technical efficiencies. Applying the model to the financing of Moroccan rural districts, we find that estimated technical efficiencies allowing for dependence through copulas tend to be lower than under independence, while the ranking remained basically the same. Furthermore, the most efficient districts are in the regions of Guelmim-ES Semara, Marrakech-Tensift-El Haouz, Sous-Massa-Draâ and Doukkala-Abda and, contrary to prior expectations, no districts of the central regions is among the top classified. We find a significant negative link between estimated technical efficiencies and population size, indicating that highly populated districts tend to be less efficient. Future research may provide a detailed analysis of the socio-economic and demographic factors that could explain inefficiencies such as the geographical distance from the center and the training level of the local councils members.

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Appendix A.

In this appendix we give some definitions and properties of copula functions, which can be used to model the dependence between random variables in a general way.

Definition *An n-dimensional copula is a distribution function defined on* $[0, 1]^n$ *with standard uniform marginal distributions.*

For general properties of copulas, we refer to Nelsen [15]. The following fundamental theorem has mainly motivated the widespread use of copulas.

Sklar's theorem (1959): *Given a multidimensional distribution function F which has $F_1, ..., F_n$ as marginals, there exists a copula C of dimension n such that for all $a = (a_1, ..., a_n) \in \mathbb{R}^n$:*

$$F(a_1, ..., a_n) = C(F_1(a_1), ..., F_n(a_n)),$$

where $F_i(a_i) = w_i$ for all $i = 1, ..., n$ and $C\left(\vec{w}\right) = C(w_1, ..., w_n)$ is a joint distribution of uniform marginals. Furthermore, if marginals are continuous, the copula C is unique and for all $w \in [0, 1]^n$ we can write

$$C\left(\vec{w}\right) = F(F_1^{-1}(w_1), ..., F_n^{-1}(w_n)).$$

The function $\Pi(w_1, w_2) = w_1.w_2$ is called the product copula and has an important statistical interpretation: Let $w_1 = F_1(u)$ and $w_2 = F_2(v)$, $\Pi(w_1, w_2)$ is the copula of U and V if and only if they are independent. In the following we give expressions for some bivariate copulas and their densities. The copulas considered in this paper are the Ali-Mikhail-Haq (AMH), Clayton, Fairlie-Gumbel-Morgenstern (FGM), Frank and Gaussian copulas. They all nest the product copula (i.e. independence) as a special case. For more details, see e.g. Genest and Favre [7] and Nelsen [15].

Appendix A.1. Gaussian copula

The Gaussian copula is given by

$$C_\theta(w_1, w_2) = \Phi_{2,\theta}\left(\Phi^{-1}(w_1), \Phi^{-1}(w_2)\right) \tag{A.1}$$

and the Gaussian copula density function is obtained by differentiation w.r.t. w_1 and w_2 as

$$c_\theta(w_1, w_2) = \frac{\phi_{2,\theta}(\Phi^{-1}(w_1), \Phi^{-1}(w_2))}{\phi(\Phi^{-1}(w_1)).\phi(\Phi^{-1}(w_2))} = \frac{\phi_{2,\theta}(t_1, t_2)}{\phi(t_1).\phi(t_2)} \tag{A.2}$$

with $t_i = \Phi^{-1}(w_i)$ for all $i = 1, 2$, and where $w_1 = F_1(u)$ and $w_2 = F_2(v)$ are the marginal distributions of u and v, respectively, θ is equal to the correlation coefficient, $\phi(t_i) = \Phi'(t_i)$ denotes the standard normal probability density function (p.d.f.), Φ the cumulative distribution function (c.d.f.) of the univariate standard normal distribution, and $\Phi_{2,\theta}$ denotes the c.d.f. of a bivariate Gaussian random variable with correlation θ and whose marginals are standard normal. The function $\Phi_{2,\theta}$ does not have a closed form expression, but it can be evaluated numerically. Furthermore, being in the class of elliptical distributions, the Gaussian copula is symmetric.

Appendix A.2. FGM copula

The Farlie-Gumbel-Morgenstern copula, denoted FGM copula is the only copula which has a functional form as a second order polynomial in w_1 and w_2. This FGM copula is defined in the bivariate case as

$$C_\theta(w_1, w_2) = w_1 w_2 P_\theta(w_1, w_2), \qquad \theta \in [-1, 1] \tag{A.3}$$

where the polynomial $P_\theta(w_1, w_2) = 1 + \theta(1 - w_1)(1 - w_2)$, hence

$$C_\theta(w_1, w_2) = w_1 w_2 [1 + \theta(1 - w_1)(1 - w_2)], \qquad \theta \in [-1, 1], \tag{A.4}$$

where

$$c_\theta(w_1, w_2) = \frac{\partial^2 C_\theta(w_1, w_2)}{\partial w_1 \partial w_2} = 1 + \theta - 2\theta w_1 - 2\theta w_2 + 4 w_1 w_2, \tag{A.5}$$

C_θ and c_θ are respectively the c.d.f. and the p.d.f. copula. The product copula is obtained as a special case for $\theta = 0$. The FGM copula is symmetric (exchangeable), meaning that $C_\theta(w_1, w_2) = C_\theta(w_2, w_1)$ for all $(w_1, w_2) \in I^2$ and that (w_1, w_2) and (w_2, w_1) are identically distributed.

Appendix A.3. Ali-Mikhail-Haq (AMH) copula

The AMH copula can represent both positive and negative independence. The distribution and the density expressions of this copula are respectively

$$C_\theta(w_1, w_2) = \frac{w_1 w_2}{1 - \theta(1 - w_1)(1 - w_2)}, \qquad \theta \in [-1, 1] \tag{A.6}$$

$$c_\theta(w_1, w_2) = A_\theta(w_1, w_2) / B_\theta(w_1, w_2), \qquad \theta \in [-1, 1] \tag{A.7}$$

where

$$\begin{aligned} A_\theta(w_1, w_2) \;\; = \;\; & -(1 - 2\theta + \theta^2 w_1 w_2 + \theta w_1 w_2 - \theta^2 w_2 \\ & + \theta^2 + \theta w_1 + \theta w_2 - \theta^2 w_1), \end{aligned} \tag{A.8}$$

$$B_\theta(w_1, w_2) = (-1 + \theta - \theta w_1 - \theta w_2 + \theta w_1 w_2)^3. \tag{A.9}$$

If $\theta = 0$, then the two variables U and V are independent.

Appendix A.4. Clayton copula

The distribution clayton copula is defined by

$$C_\theta(w_1, w_2) = \left(w_1^{-\theta} + w_2^{-\theta} - 1\right)^{-1/\theta}, \qquad \theta > 0 \tag{A.10}$$

The density expression is

$$c_\theta(w_1, w_2) = w_1^{-1-\theta} w_2^{-1-\theta} \left(\left(w_1^{-\theta} + w_2^{-\theta} - 1\right)^{-2-1/\theta}\right)(1+\theta), \ \theta > 0. \tag{A.11}$$

As the parameter θ approaches zero, the two variables U and V become independent and the product copula is obtained as the limiting case: $\lim_{\theta\to 0} C_\theta(w_1, w_2) = \Pi(w_1, w_2)$.

Appendix A.5. Frank copula

The cdf of the Frank copula is given for all $\theta \in \mathbb{R}^*$ by

$$C_\theta(w_1, w_2) = -\frac{1}{\theta} \ln\left(1 + \frac{(\exp\{-\theta w_1\} - 1)(\exp\{-\theta w_2\} - 1)}{\exp\{-\theta\} - 1}\right), \tag{A.12}$$

and the corresponding density by

$$c_\theta(w_1, w_2) = D_\theta(w_1, w_2) / E_\theta(w_1, w_2), \qquad \theta \in \mathbb{R}^* \tag{A.13}$$

where D and E are defined as

$$D_\theta(w_1, w_2) = \exp\{(1 + w_1 + w_2)\theta\}(\exp\{\theta\} - 1)\theta, \tag{A.14}$$

$$\begin{aligned} E_\theta(w_1, w_2) &= (\exp\{\theta\} + \exp\{(w_1 + w_2)\theta\} \\ &\quad - \exp\{\theta + w_1\theta\} - \exp\{\theta + w_2\theta\})^2. \end{aligned} \tag{A.15}$$

As for the Clayton copula, if $\theta \to 0$, then the product copula is obtained as the limiting case.

[1] Agahi, H., Zarafshani, K., Behjat, A.M., 2008. The Effect of Crop Insurance on Technical Efficiency of Wheat Farmers in Kermanshah Province: A Corrected Ordinary Least Square Approach. Journal of Applied Sciences. 8, 5, 891-894.

[2] Aigner, D. J., Knox Lovell, C. A., Schmidt, P., 1977. Formulation and estimation of stochastic frontier production function models, Journal of Econometrics. 6, 21-37.

[3] Battese, G. E., Coelli, T. J., 1988. Prediction of firm level technical efficiencies with a generalized frontier production function and panel data. Journal of Econometrics. 38, 169-179.

[4] Bélisle, C. J. P., 1992. Convergence Theorems for a Class of Simulated Annealing Algorithms on Rd. Journal of Applied Probability. 29, 4, 885-895.

[5] Christensen, L. R., Jorgenson, D. W., Lau, L. J., 1971. Conjugate Duality and the Transcendental Logarithmic Production Function. Econometrica. 39, 255-256.

[6] Coelli, T., 1996. A Guide to FRONTIER, Version 4.1: A Computer Program for Stochastic Frontier Production and Cost Function Estimation. Centre for Efficiency and Productivity Analysis, CEPA Working Paper 96/07, Department of Econometrics, University of New England.

[7] Genest, Ch., Favre, A. C., 2007. Everything You Always Wanted to Know about Copula Modeling but Were Afraid to Ask. Journal of Hydrologic Engineering. 12, 4, 345-439.

[8] Greene, W., 2010. A Stochastic Frontier Model with Correction for Sample Selection. Journal of Productivity Analysis, 34, 1, 15-24.

[9] Greene, W., 1980a. Maximum likelihood estimation of econometric frontier function. Journal of Econometrics. 13, 1, 27-56.

[10] Greene, W., 1980b. On the estimation of a flexible frontier production. Journal of Econometrics. 13, 1, 101-115.

[11] Jondrow, J., Knox Lovell, C. A., Materov, I. S., Schmidt, P., 1982. On the Estimation of Technical Inefficiency in the Stochastic Frontier Production Function Model. Journal of Econometrics. 19, 233-238.

[12] Kumbhakar, S. C., Knox Lovell, C. A., 2000. Stochastic Frontier Analysis, first ed. Cambridge University Press. United Kingdom.

[13] Meeusen, W., Van Den Broeck, J., 1977. Eficiency Estimaton From Cobb-Douglas Production Funtion With Composed Error. International Economic Review, 18, 2, 435-444.

[14] Nelder, J. A., Mead, R., 1965. A simplex method for function minimization. Computer J., 7, 308-313.

[15] Nelsen, R. B., 1999. An Introduction to Copulas, first ed. Springer, New York.

[16] R Development CORE team, 2011. R: A language and environment for statistical computing. R Foundation for Statistical Computing. Vienna, Austria. http://www.R-project.org/.

[17] Ritter, Ch., Simar, L., 1997. Pitfalls of normal-gamma stochastic frontier models. Journal of Productivity Analysis, 8,167-182.

[18] Simar, L., Wilson, P. W., 2010. Inferences from Cross-Sectional, Stochastic Frontier Models. Econometric Reviews, 29, 1, 62-98.

[19] Smith, M. D., 2008. Stochastic frontier models with dependent error components. The Econometrics Journal, 11, 172-192.