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Decreasing higher-order absolute risk aversion and higher-degree stochastic dominance

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DECREASING HIGHER-ORDER ABSOLUTE RISK AVERSION AND HIGHER-DEGREE STOCHASTIC DOMINANCE

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Abstract

Fishburn and Vickson (1978) showed that, when applied to random alternatives with an equal mean, 3rd-degree and DARA stochastic dominances represent equivalent rules. The present paper generalizes this result to higher degrees. Specifically, higher-degree stochastic dominance rules and common preference by all decision makers with decreasing higher-order absolute risk aversion are shown to coincide under appropriate constraints on the respective moments of the random variables to be compared.

Key words and phrases: rate of substitution, decreasing risk aversion, stochastic dominance, risk apportionment.

JEL Classification Codes: D81

1 Introduction

Fishburn and Vickson (1978) showed that for random alternatives with an equal mean, 3rd-degree stochastic dominance expresses the common preference by all the decision makers with decreasing absolute risk aversion (DARA, in short) in the expected utility setting, i.e. 3rd-degree stochastic dominance and DARA stochastic dominance represent equivalent rules. Precisely, for two random variables X and Y such that $E[X] = E[Y]$, $E[u(X)] \leq E[u(Y)]$ for every utility function $u(\cdot)$ such that the first derivative u', the second derivative u'' and the third derivative u''' fulfill $u'(x) > 0$, $u''(x) < 0$ and $u'''(x) > 0$ for all x if, and only if, $E[u(X)] \leq E[u(Y)]$ for every utility function $u(\cdot)$ such that $u'(x) > 0$, $u''(x) < 0$ and $A'_u(x) \leq 0$ for all x, where $A_u(x) = -u''(x)/u'(x)$ is the classical Arrow-Pratt absolute risk aversion measure for $u(\cdot)$. It is well known that, assuming $u' > 0$, $A'_u \leq 0$ is sufficient but not necessary for $u''' > 0$. So 3rd-degree stochastic dominance implies DARA stochastic dominance but not vice versa. That DARA stochastic dominance also implies 3rd-degree stochastic dominance when they are applied to equal-mean random alternatives is because for every utility function $u(\cdot)$ such that $u''' > 0$, one can define an auxiliary utility function $v(\cdot)$ as $v(x) = u(x) + kx$ for some non-negative real constant k. And one can easily see that $v(\cdot)$ ranks any two equal-mean random variables the same way as $u(\cdot)$, and $A'_v(x) \leq 0$ when k is sufficiently large. Therefore, if Y does not dominate X by 3rd-degree stochastic dominance, then Y would not dominate X by DARA stochastic dominance either[1].

This paper proves a result that simultaneously generalizes the Fishburn and Vickson theorem in two directions. First, maintaining the equal-mean condition, the equivalence between higher-degree stochastic dominances and their corresponding higher-degree DARA stochastic dominances is established. Second, the implications of a more general restriction on random alternatives - that they have equal first m moments, where $m \geq 1$ - are also explored. Under these moment conditions, higher-degree stochastic dominance rules can be equivalently characterized using risk apportionment notions as in Eeckhoudt and Schlesinger (2006) or imposing that higher-degree coefficients of absolute risk aversion decrease with the initial wealth level.

Higher-degree stochastic dominance rules are interesting because the higher the degree a stochastic dominance rule is of, the more random alternatives it can be used to rank. Moreover, the assumption of equal mean or equal first few moments does not seem to be overly restrictive in many situations. For example, the assumption of the existence of actuarially fair insurance ensures that insurance buyers' choice is over random alternatives with an equal mean. And decisions involving third- or even higher-degree risk increases require comparing random alternatives with equal first two or more moments[2].

In the sense that it sheds some light on the relationship between preference conditions $u''' > 0$ and $A'_u(x) \leq 0$, this paper is related to studies giving economic interpretations to these conditions and their higher-degree counterparts. Pratt's (1964) famous justification of

[1]This intuition is based on an alternative proof of the Fishburn and Vickson theorem in Liu and Meyer (2012a).

[2]For example, Menezes, Geiss and Tressler (1980) defined a third-degree or downside risk increase as a change in a random variable that moves risk from right to left while keeping the mean and variance intact. And Ekern (1980) defined an nth-degree risk increase as an nth-degree stochastically dominated change in a random variable that keeps the first $n-1$ moments the same.

$A_u(\cdot)$ as a local intensity measure of risk aversion makes $A'_u \leq 0$ a condition of risk aversion decreasing in initial wealth. On the other hand, Eeckhoudt and Schlesinger (2009) revived an interpretation of $u''' > 0$, i.e. of prudence or 3rd-order risk apportionment, as the pain from a (2nd-degree) risk increase decreasing in wealth. Similarly, Eeckhoudt et al. (2009) and Denuit and Rey (2010) provided interpretations of risk apportionment of order $n+1$ as decreasing sensitivity to a detrimental nth-degree risk increase in the sense of Ekern (1980).

The remainder of this paper is organized as follows. Section 2 relates decreasing absolute risk aversion of higher degree to risk apportionment. Section 3 recalls stochastic dominance rules, paying particular attention to the cases where the first few moments of the random variables to be compared coincide. Section 4 contains the main results, establishing that higher-order stochastic dominance rules and common preference by all decision makers with decreasing higher-order absolute risk aversion coincide under appropriate constraints on the respective moments of the random variables to be compared. Several particular cases often encountered in the literature are discussed.

All the random variables considered in this paper are valued in some interval $[a,b]$ of the real line. We denote as u', u'', and u''' the first derivative, the second derivative, and the third derivative of the utility function u. More generally, we write $u^{(n)}$ for the nth derivative of u, $n = 1, 2, 3, 4, \ldots$; the notations u', u'', and u''' and $u^{(1)}$, $u^{(2)}$, and $u^{(3)}$, respectively, can be used interchangeably.

2 Decreasing higher-order absolute risk aversion and risk apportionment

Recently, Liu and Meyer (2012b) used the rate of substitution of one risk increase for another to provide a unifying generalization of $A_u(x)$ to the nth degree, leading to the nth/mth absolute risk aversion measure of the utility function u defined as

$$A_{u,n,m}(x) = \frac{(-1)^{n+1}u^{(n)}(x)}{(-1)^{m+1}u^{(m)}(x)} \text{ where } n > m \geq 1.$$

This concept unifies many definitions proposed in the literature. For $n = 2$ and $m = 1$, we recover the Arrow-Pratt absolute risk aversion measure for $u(\cdot)$, i.e. $A_{u,2,1} = A_u$ defined in Section 1. Kimball (1990, 1992) established a link between the intensity of the precautionary savings motive and what he termed the degree of absolute prudence measured by $A_{u,3,2}$. Chiu (2005) gave a choice theoretic interpretation to $A_{u,3,2}$ that paralleled that given by Arrow (1974) and Pratt (1964) for $A_{u,2,1} = A_u$. In a paper dealing essentially with an interpretation of the signs of successive derivatives of $u(\cdot)$, Caballe and Pomansky (1996) studied the properties of $A_{u,n,n-1}$. Denuit and Eeckhoudt (2010a) exploited a representation of the Friedman-Savage utility premium to give a general foundation for $A_{u,n,n-1}$. Also, $A_{u,n,1}$ has been successfully used in different economic problems; see Denuit and Eeckhoudt (2010b) and the references therein.

Eeckhoudt and Schlesinger (2006) introduced risk apportionment of order n, $n = 1, 2, \ldots$, by imposing preferences over simple lotteries[3]. These higher-order risk attitudes entail a

[3]These lotteries were characterized by Roger (2011) who established that they only differ by their moments

preference for combining relatively good outcomes with bad ones and can be interpreted as a desire to disaggregate the harms of unavoidable risks and losses. Eeckhoudt, Schlesinger and Tsetlin (2009) further established that a decision maker exhibiting risk apportionment prefers not to group the two relatively bad lotteries in the same state, where bad is defined via higher-order stochastic dominance. Eeckhoudt and Schlesinger (2006) proved that a differentiable utility function u satisfies risk apportionment of order n if, and only if, it fulfills the condition[4] $(-1)^{n+1}u^{(n)} > 0$.

Caballe and Pomansky (1996) indicated that if $A_{u,n,n-1}$ is decreasing for $n = 2, 3, \ldots$ then the utility function is mixed risk averse (see their Proposition 3.2). The next result extends this idea to $A_{u,n,m}$ for arbitrary integers $n > m \geq 1$.

Property 2.1. *If u exhibits risk apportionment of orders m, $m+1$ and n then*

$$A^{(1)}_{u,n,m} \leq 0 \Rightarrow u \text{ exhibits risk apportionment of order } n+1.$$

Proof. A straightforward calculation shows that

$$A^{(1)}_{u,n,m} = (-1)^{n-m} \frac{u^{(n+1)}u^{(m)} - u^{(n)}u^{(m+1)}}{(u^{(m)})^2}$$

which is well defined as u exhibits risk apportionment of order m so that the denominator is not zero. From this expression for $A^{(1)}_{u,n,m}$, we see that $A^{(1)}_{u,n,m} \leq 0$ implies

$$(-1)^{n-m}u^{(n+1)}u^{(m)} \leq (-1)^{n-m}u^{(n)}u^{(m+1)} < 0$$

where the last inequality follows from $(-1)^{n+1}u^{(n)} > 0$ and $(-1)^m u^{(m+1)} > 0$. As $(-1)^{m+1}u^{(m)} > 0$ we must have $(-1)^{n+1}u^{(n+1)} < 0$, that is, $(-1)^n u^{(n+1)} > 0$, as announced. □

3 Stochastic dominance rules

Let F_X be the distribution function for the random variable X. Starting from $F_X^{[1]} = F_X$, we define $F_X^{[2]}, F_X^{[3]}, \ldots$ recursively from repeated integrals:

$$F_X^{[k+1]}(x) = \int_a^x F_X^{[k]}(y)dy, \quad k = 1, 2, \ldots$$

Integration by parts shows that

$$F_X^{[k]}(x) = \frac{1}{(k-1)!}\int_a^x (x-t)^{k-1}dF_X(t) = \frac{E[(x-X)_+^{k-1}]}{(k-1)!}, \quad k = 2, 3, \ldots$$

so that

$$F_X^{[k]}(b) = \frac{E[(b-X)^{k-1}]}{(k-1)!}.$$

of order greater than or equal to n. See also Ebert (2013).

[4]Notice that we impose here a strict inequality to define risk apportionment, to avoid dividing by zero when defining $A_{u,n,m}$ as ratios of derivatives of the utility function u.

The nth-degree stochastic dominance rule $\preceq_n$ can then be defined as follows: given two random variables X and Y, $X \preceq_n Y \Leftrightarrow F_X^{[n]}(x) \geq F_Y^{[n]}(x)$ for all $x \in [a, b]$ and $F_X^{[k]}(b) \geq F_Y^{[k]}(b)$ for $k = 1, 2, \ldots, n$. The particular case studied in Ekern (1980) imposes the equality of the first $n-1$ moments of the random variables X and Y to be compared through $\preceq_n$, i.e.

$$E[X^k] = E[Y^k] \text{ for } k = 1, 2, \ldots, n-1 \Leftrightarrow F_X^{[k]}(b) = F_Y^{[k]}(b) \text{ for } k = 1, 2, \ldots, n.$$

These rules are known to express the common preference by all the decision makers exhibiting appropriate risk apportionment. To see this precisely, let us recall the following standard expansion formula, easily obtained using integration by parts: if the utility function u is s times continuously differentiable then we have for any positive integer s that

$$E[u(X)] = \sum_{j=0}^{s-1} (-1)^j u^{(j)}(b) F_X^{[j+1]}(b) + (-1)^s \int_a^b u^{(s)}(x) F_X^{[s]}(x) dx. \tag{3.1}$$

From (3.1) with $s = n$, we see that the the common preference by all the decision makers satisfying risk apportionment of orders 1 to n correspond to $\preceq_n$. Moreover, for random variables X and Y such that $E[X^k] = E[Y^k]$ for $k = 1, 2, \ldots, n-1$, $X \preceq_n Y$ holds if Y is preferred over X by all the expected utility maximizers whose preferences exhibit risk apportionment of order n.

For $n = 1$, $\preceq_1$ is the well-known first-order stochastic dominance which expresses the common preference by all the profit-seeking decision makers. For $n = 2$, the second-order stochastic dominance $\preceq_2$ reflects the common preference by all the profit-seeking risk-averse decision makers, i.e. those preferring the certain outcome $E[X]$ over the random variable X, whatever X. For $n = 3$, $\preceq_3$ expresses the common preference by all the profit-seeking, risk-averse and prudent decision makers, i.e. those having a precautionary savings motive. For $n = 4$, $\preceq_4$ expresses the common preference by all the profit-seeking, risk-averse, prudent and temperant decision makers, i.e. those who dislike mutually aggravating risks. For $n = 5$, $\preceq_5$ expresses the common preference by all the profit-seeking, risk-averse, prudent, temperant decision makers exhibiting edginess. More generally, $\preceq_n$ represents the common preference by all the decision makers whose preferences satisfy risk apportionment of orders 1 to n in that they prefer to disaggregate risk across states of nature. Letting n tend to $+\infty$ gives utilities with all odd derivatives positive and all even derivatives negative. In this case, utility functions are completely monotone and express mixed risk aversion. See Brockett and Golden (1987) and Caballe and Pomansky (1996). A precise economic meaning for the sign of the nth derivative of the utility function u has been given in Eeckhoudt and Schlesinger (2006) and Denuit and Eeckhoudt (2010b) based on sequences of lotteries. See also Denuit, Lefevre and Scarsini (2001) and Eeckhoudt, Schlesinger and Tsetlin (2009).

Ekern (1980)'s definition includes well-known special cases. Famous examples are the mean preserving increase in risk of Rothschild and Stiglitz (1970) corresponding to $n = 2$ or the increase in downside risk defined by Menezes, Geiss and Tressler (1980) in which mean and variance are kept constant while there is a dispersion transfer from high to low wealth levels (i.e. $n = 3$). More recently, Menezes and Wang (2005) defined an increase in outer risk corresponding to $n = 4$.

The following intermediate cases have been considered by Denuit and Eeckhoudt (2013). Let m be a positive integer and let n be an integer such that $n \geq m+1$. Consider two random variables X and Y such that $E[X^k] = E[Y^k]$ holds for $k = 1, \ldots, m$. Then $X \preceq_n Y$ holds if $E[u(X)] \leq E[u(Y)]$ for every utility function u exhibiting risk apportionment of orders $m+1$ to n. The equality of the first m moments of X and Y allows one to drop the conditions on the first m derivatives $u^{(1)}, \ldots, u^{(m)}$, the particular case $m = n-1$ corresponding to Ekern (1980). This illustrates the trade-off between conditions imposed on derivatives and the equality of the first moments. Among the intermediate cases imposing the equality of the first $m \leq n-1$ moments, mean-preserving stochastic dominance has been considered by Denuit and Eeckhoudt (2012, 2013). This rule restricts $\preceq_n$ to pairs of random variables with the same expected value.

4 Stochastic dominance rules and decreasing absolute risk aversion

In this section, we aim to provide an alternative characterization of stochastic dominance rules where decreasing absolute risk aversion replaces risk apportionment. Let us start with the following general result.

Proposition 4.1. *Let m be a positive integer and let n be an integer such that $n \geq m+1$. Consider two random variables X and Y such that $E[X^j] = E[Y^j]$ for $j = 1, \ldots, m$. Then, $E[u(X)] \leq E[u(Y)]$ for every utility function u exhibiting risk apportionment of orders m, $m+1$, n, and $n+1$ if, and only if, $E[u(X)] \leq E[u(Y)]$ for every utility function u exhibiting risk apportionment of orders m, $m+1$, and n such that $A^{(1)}_{u,n,m} \leq 0$.*

Proof. The "⇒"-part is a direct consequence of Property 2.1. In order to establish the validity of the "⇐"-part, let us consider the expansion in formula (3.1) with $s = m$. Since $E[X^j] = E[Y^j]$ for $j = 1, \ldots, m-1 \Rightarrow F_X^{[j+1]}(b) = F_Y^{[j+1]}(b)$ for $j = 1, \ldots, m-1$, expansion (3.1) allows us to write the utility premium as

$$E[u(Y)] - E[u(X)] = (-1)^m \int_a^b u^{(m)}(x)\Big(F_Y^{[m]}(x) - F_X^{[m]}(x)\Big)dx.$$

Now, assume that $E[u(X)] \leq E[u(Y)]$ holds for every utility function u exhibiting risk apportionment of orders m, $m+1$ and n such that $A^{(1)}_{u,n,m} \leq 0$. Let us show that to each utility function u exhibiting risk apportionment of orders m, $m+1$, n, and $n+1$, we can associate a utility function v exhibiting risk apportionment of orders m, $m+1$, and n such that $A^{(1)}_{v,n,m} \leq 0$ and $E[u(X)] - E[u(Y)] = E[v(X)] - E[v(Y)]$, i.e. the utility premiums for u and v coincide. The utility function v is such that

$$(-1)^{m+1}v^{(m)} = (-1)^{m+1}u^{(m)} + k$$

for some constant $k \geq 0$. As

$$\int_a^b \Big(F_Y^{[m]}(x) - F_X^{[m]}(x)\Big)dx = 0$$

since $E[X^j] = E[Y^j]$ for $j = 1, \dots, m$, we have

$$\begin{aligned} E[v(Y)] - E[v(X)] &= (-1)^m \int_a^b v^{(m)}(x)\Big(F_Y^{[m]}(x) - F_X^{[m]}(x)\Big)dx \\ &= (-1)^m \int_a^b \Big(u^{(m)}(x) - k\Big)\Big(F_Y^{[m]}(x) - F_X^{[m]}(x)\Big)dx \\ &= (-1)^m \int_a^b u^{(m)}(x)\Big(F_Y^{[m]}(x) - F_X^{[m]}(x)\Big)dx \\ &= E[u(Y)] - E[u(X)]. \end{aligned}$$

Now, let us show that, for k large enough, $A^{(1)}_{v,n,m} \le 0$ when the utility function u exhibits risk apportionment of orders m, $m+1$, n and $n+1$. As

$$\begin{aligned} A^{(1)}_{v,n,m} &= (-1)^{n-m}\left(\frac{v^{(n+1)}}{v^{(m)}} - \frac{v^{(n)}v^{(m+1)}}{(v^{(m)})^2}\right) \\ &= (-1)^{n-m}\left(\frac{u^{(n+1)}}{u^{(m)} + (-1)^{m+1}k} - \frac{u^{(n)}u^{(m+1)}}{(u^{(m)} + (-1)^{m+1}k)^2}\right) \end{aligned}$$

we have

$$\begin{aligned} A^{(1)}_{v,n,m} \le 0 &\Leftrightarrow (-1)^{n-m}\frac{u^{(n+1)}}{u^{(m)} + (-1)^{m+1}k} \le (-1)^{n-m}\frac{u^{(n)}u^{(m+1)}}{(u^{(m)} + (-1)^{m+1}k)^2} \\ &\Leftrightarrow (-1)^{n-m}u^{(n+1)}\big(u^{(m)} + (-1)^{m+1}k\big) \le (-1)^{n-m}u^{(n)}u^{(m+1)} \\ &\Leftrightarrow (-1)^{n+1}ku^{(n+1)} \le (-1)^{n-m}\big(u^{(n)}u^{(m+1)} - u^{(n+1)}u^{(m)}\big) \end{aligned}$$

so that it suffices to take

$$k \ge (-1)^{m+1}\frac{u^{(n)}(x)u^{(m+1)}(x) - u^{(n+1)}(x)u^{(m)}(x)}{u^{(n+1)}(x)} \text{ for all } x \in [a,b],$$

which ends the proof. □

For $m = 1$ and $n = 2$, we recover the Fishburn and Vickson result stated as Theorem 3 in Liu and Meyer (2012a), which is used there to shed light on the measure of downside risk aversion. Notice that the equality of the first $m-1$ moments of X and Y is needed to avoid putting restrictions on the sign of derivatives $u^{(1)}, \dots, u^{(m-1)}$ of the utility function u, i.e. risk apportionment of orders 1 to $m-1$ is not needed. This is important because the auxiliary function v appearing in the proof is obtained by adding a polynomial to u so hat even if u exhibits risk apportionment of orders 1 to $m-1$, this is not necessarily the case for v. The equality of the mth moments is needed to ensure that the utility premiums for u and v coincide.

Let us now apply Proposition 4.1 to characterize higher-degree stochastic dominance by means of decreasing absolute risk aversion.

Proposition 4.2. *Let m be a positive integer and let n be an integer such that $n \ge m+1$. Consider two random variables X and Y such that $E[X^j] = E[Y^j]$ for $j = 1, \dots, m$. Then, $X \preceq_{n+1} Y$, i.e. $E[u(X)] \le E[u(Y)]$ for every utility function u exhibiting risk apportionment of orders $m, m+1, \dots, n+1$ if, and only if, $E[u(X)] \le E[u(Y)]$ for every utility function u exhibiting risk apportionment of orders m and $m+1$ such that $A^{(1)}_{u,k,m} \le 0$ for $k = m+1, \dots, n$.*

Proposition 4.2 is a direct consequence of Proposition 4.1 and its proof is thus omitted. Let us now consider some special cases of particular interest. For $m = 1$, we obtain the following corollary for the mean-preserving stochastic dominance considered in Denuit and Eeckhoudt (2012).

Corollary 4.3. *Consider two random variables X and Y such that $E[X] = E[Y]$ and an integer $n \geq 2$. Then, $X \preceq_{n+1} Y$ if, and only if, $E[u(X)] \leq E[u(Y)]$ for every utility function u such that $u' > 0$, $u'' < 0$ and $A^{(1)}_{u,k,1} \leq 0$ for $k = 2, \ldots, n$.*

For $n = 2$, we recover the result of Fishburn and Vickson (1978). Given two random variables X and Y with equal means, Corollary 4.3 shows, for example, that $X \preceq_4 Y$ holds if, and only if, Y is preferred to X by all decision makers with utility function $u(\cdot)$ such that $u' > 0$, $u'' > 0$, $A^{(1)}_{u,2,1} \leq 0$ and $A^{(1)}_{u,3,1} \leq 0$.

Now, if higher moments of the random variables X and Y to be compared also coincide, it is possible to reduce the set of conditions imposed on the utility function u. This is done hereafter for $\preceq_{n+1}$ when the first n moments of X and Y coincide, as considered in Ekern (1980). This covers the spacial cases $n = 2$ defined by Menezes, Geiss and Tressler (1980) and $n = 3$ considered in Menezes and Wang (2005).

Corollary 4.4. *Consider two random variables X and Y such that $E[X^j] = E[Y^j]$ for $j = 1, \ldots, n$ for some integer $n \geq 2$. Then, $X \preceq_{n+1} Y$, if, and only if, $E[u(X)] \leq E[u(Y)]$ for every utility function u exhibiting risk apportionment of orders $n-1$ and n such that $A^{(1)}_{u,n,n-1} \leq 0$.*

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