Risk and Solvency of a Notional Defined Contribution public pension scheme

Jennifer Alonso García
(joint work with Pr. Pierre Devolder and Carmen Boado-Penas)

Université Catholique de Louvain (UCL)

jennifer.alonso@uclouvain.be

20/09/2013
Funding methodologies

- Pay as you go (PAYG): current contributions pay current pensions (inter-generational solidarity)
- Funding: contributions are put in a fund which earns a market interest rate (market risk has to be taken into account)

Benefit formula

- Defined Benefit (DB): the pensions are known but not their financing method.
  - People know exactly what they are going to receive.
  - Funding problem is up to the provider (state or the employer)
  - Benefits are rarely based in the amount of contributions and don’t consider longevity risk
- Defined Contributions (DC): the financing method is known and fixed
  - People don’t know exactly what they are going to receive
  - Pensions are based on the accumulated capital and life expectancy
A quick overview of the pension system in Belgium

- **1st pillar**: social security pension schemes.
  - Defined Benefits (DB) financed through Pay as you Go (PAYG)
  - Compulsory to all workers
  - 3 main systems in Belgium (employees, self-employed, public servants)

- **2nd pillar**: Occupational pension schemes
  - DB vs Defined Contributions (DC) financed through funding.

- **3rd pillar**: individual saving
Employees and Self-Employed 1\textsuperscript{st} pillar pension system

**Employees**
- Legal retirement age: 65
- Pension calculated through a defined benefit formula: 60\% or 75\% of the indexed average career wage
- Contributions for employees: 7,5\% worker and 8,86\% employer
- Salary ceiling in the benefit’s formula (50,000 €) but not in the contributions

**Self-employed**
- Same legal retirement age and benefit formula as for the employees
- The benefit is multiplied by a ratio based on the level of contributions between self-employed (one source of funding) and employees (two sources of funding) (around 50\%)
Public servants 1st pillar pension system

- Legal retirement age: 65
- Pension calculated through a defined benefit formula:
  75% of the last 10 years average earnings (before the reform it was 5 years)
- Contributions: no individual contributions. Pensions are taken from the state budget.
- No salary ceiling in the benefit’s formula.
- However the yearly pension cannot exceed 46.883 €
Why considering a pension reform?

- Rising longevity: people are living longer and longer but retire at the same age as 50 years ago.
  - Life expectancy in 1960: 70 years
  - Life expectancy in 2011: 80 years

- Drop in fertility
  - Gross fertility rate in 1960: 17 births per 1000 inhabitants
  - Gross fertility rate in 2011: 12 births per 1000 inhabitants

- Lack of actuarial fairness: No direct link between the contributions made and amount of pension received at retirement.

If nothing is done the expected spending in the public spending will increase much quicker than our neighbor countries
→ Our current pension system has to be revisited
→ Solvency or liquidity factors have to be created to increase transparency in the sustainability of the system.
Current pension systems

Funding methodologies

- Pay as you go (PAYG): current contributions pay current pensions (inter-generational solidarity)
- Funding: contributions are put in a fund which earns a market interest rate (market risk has to be taken into account)

Benefit formula

- DB: pensions are calculated through a specific formula
  - People know exactly what they are going to receive
  - Benefits are rarely based in the amount of contributions and don’t consider longevity risk
- DC: pension formula depend directly on the contributions
  - People don’t know exactly what they are going to receive
  - Pensions are based on the accumulated capital and life expectancy

→ Notional Defined Contributions mix both PAYG and DC
→ The notional rate doesn’t depend on the markets but on the evolution of contributions.
The Model

**Age:** $x = y, y+1, y+2, y+3$

<table>
<thead>
<tr>
<th>Active population</th>
<th>Retired population</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>$y+1$</td>
</tr>
<tr>
<td>$y+2$</td>
<td>$y+3$</td>
</tr>
</tbody>
</table>

**Population at time $t$:**

\[
l(x, t) = l(y, t-x+y)p(x, t) = l(y, 0) \exp \left( \sum_{i=1}^{t-x+y} R_i \right) p(x, t)
\]

where:
- $l(y, t-x+y)$ = Entry population at time $t-x+y$
- $p(x, t)$ = Time-dependent survival probability to attain age $x$ at time $t$

**Wages at time $t$:**

\[
S(x, t) = S(x, 0) \exp \sum_{i=1}^{t} \gamma_i
\]

The following stochastic processes are defined in the probability space $(\Omega, \mathcal{F}, P)$:
- $R_i$ = Increase rate in the entrant population during period $i-1$ to $i$
- $\gamma_i$ = Increase rate of the salaries during period $i-1$ to $i$

Further assumption: no mortality risk until retirement. Thus:
- $p(x, t) = 1$ for $x = y, y+1, y+2$
- $p(y+3, t) = p_t$ for simplicity.
Contributions and notional rate

At time $t$, all members of the active population contribute a rate $\pi$ of their salaries to the pension system:

$$C(t) = \pi S(y, t)l(y, t) + \pi S(y + 1, t)l(y + 1, t)$$

$$= \pi l(y, 0) \exp \left( \sum_{i=1}^{t} \gamma_i + \sum_{i=1}^{t-1} R_i \right) K_C(t)$$

where: $K_C(t) = S(y, 0)e^{R_t} + S(y + 1, 0)$

The notional rate index $I(t)$ is taken as the evolution of the global contribution base

$$I(t) = \frac{C(t)}{C(t - 1)} = e^{\gamma_t + R_{t-1}} \frac{K_C(t)}{K_C(t - 1)}$$

This notional rate is affected by both salary and demographic risks.
Pension calculation and expenditure

The individual contributions are compounded at the notional rate during the contribution period and become the Notional capital $NDC_{CO}(y + 2, t)$. The pension for the new retirees is based on this notional capital and the annuity $a_t$ at time of retirement $t$:

$$P(y + 2, t) = \frac{NDC_{CO}(y + 2, t)}{a_t l(y + 2, t)}$$

with:

$$a_t = E_t \left[ 1 + p_{t+1} \frac{RP(t + 1)}{l(t + 1)} \right]$$

$$RP(t + 1) = \text{indexation rate at time } t + 1$$

The expenditure in pension becomes:

$$O(t) = P(y + 2, t) l(y + 2, t) + P(y + 2, t - 1) R(t) l(y + 3, t)$$

$$= C(t) K_0(t)$$

The expenditure in pension $O(t)$ at time $t$ is thus proportional to the contributions made at the same period.
Most natural way to study the sustainability is to compare income and expenses, i.e.,
\[ LR_t = \frac{C(t) + F^-(t)}{O(t)} \]; where \( F^-(t) \) is a buffer fund.

As seen previously expenses are proportional to the income in this framework.

Even if longitudinal equilibrium may be attained, cross-sectional equilibrium is not guaranteed.

Result 1

Contributions are in general not equal to the expenditure on pensions in the presented 4-period OLG unfunded dynamic model, i.e., \( C(t) \neq O(t) \ \forall t \).

→ Equality is only found when the population is in steady state
→ The population in Europe isn’t in steady state but it’s rather dynamic.
Another way of assessing the health of the pension system is through the analysis of the current liabilities, \( V(t) \) toward all participants in the pension system and compare them to the current assets, \( SR_t = \frac{\text{Assets} + F^-(t)}{V(t)} \); where \( F^-(t) \) is a buffer fund.

\[
V(t) = \sum_{x=y}^{y+3} NDC(x, t) = C(t)K_V(t)
\]

where:

\( NDC(x, t) \) is the accumulated notional capital for all ages.

→ **Problem:** 1\(^{st}\) pillar pensions are mostly unfunded
→ How can we estimate this non-existent asset?
⇒ The assets are estimated according to some accounting measure called the Contribution Asset.
The Contribution Asset is thus:

\[ CA(t) = C(t)TD(t) = C(t)(A_t^R - A_t^C) \]

\[ A_t^R = \frac{\sum_{x=y+2}^{y+3} xP(x, t)l(x, t)}{\sum_{x=y+2}^{y+3} P(x, t)l(x, t)} = \text{Pay-out duration} \]

\[ A_t^C = \frac{\sum_{x=y}^{y+1} xC(x, t)l(x, t)}{\sum_{x=y}^{y+1} C(x, t)l(x, t)} = \text{Pay-in duration} \]

Same problem as before, this accounting measure only gives equilibrium if the population is in steady state:

**Result 2**

Contribution asset is in general not equal to the liabilities in the presented 4-period OLG unfunded dynamic model, i.e., \( CA(t) \neq V(t) \ \forall t \)
As seen in the previous sections, both liquidity and solvency are not assured by the NDC framework;

An Automatic Balance Mechanism (ABM) $B_x(t)$ is thus introduced through the notional rate: $l_x(t) = l(t)B_x(t)$ for $x=LR,SR$.

For the liquidity case is: $B_{LR}(s) = \frac{C(s)+F^-(t)}{C(s)K^{LR}_O(s)}$

For the solvency case is: $B_{SR}(s) = \frac{CA(s)+F^-(s)}{V(s)}$

→ **Issue?**: How can we choose between these two ABM? We aim to choose the ABM which has a lower variance.
Definition of the processes

The demographic and salary processes follow a geometric Brownian motion:

\[
D = D_s = \frac{l(y, s)}{l(y, s - 1)} = e^{R_s} = e^{R - \frac{\sigma_R^2}{2} + \sigma_R Z_R}
\]

\[
S = S_s = \frac{S(x, s)}{S(x, s - 1)} = e^{\gamma_s} = e^{\gamma - \frac{\sigma_\gamma^2}{2} + \sigma_\gamma Z_\gamma}
\]

where the random variables \( Z_R \) and \( Z_\gamma \) are correlated, i.e., \( E[Z_R Z_\gamma] = \rho \)

The notional rate becomes:

\[
I(s) = \frac{C(s)}{C(s - 1)} = S_s D_{s-1} \frac{S(y, 0) D_s + S(y + 1, 0)}{S(y, 0) D_{s-1} + S(y + 1, 0)}
\]

\[
= e^{R + \gamma - \frac{\sigma_R^2 + \sigma_\gamma^2}{2} + \sigma_{R, \gamma} Z}
\]

Where:

- \( \sigma_{R, \gamma}^2 = \sigma_R^2 + \sigma_\gamma^2 + 2 \rho \sigma_R \sigma_\gamma \)
- \( Z \sim N(0, 1) \)
The replacement ratio denotes the relationship between the first pension received and salary just prior retirement.

In this framework it corresponds to:

\[ RR(s) = \frac{P(y+2,s)}{S(y+1,s-1)} = \frac{l_x(s)}{a_s} \left\{ \frac{S(y,0)}{S(y+1,0)} \frac{1}{S_{s-1}} l_x(s - 1) + 1 \right\} \]

The distribution of \( l_x(s) \) is unknown due to complicated form of the ABM.

The sometimes known as the law of the unconscious statistician will have to be used:

**Law of the unconscious statistician**

Let \( g \) be an arbitrary function of function on the real line, that is all sets \( x : g(x) = y \) belong to the \( \sigma \)-algebra generated by intervals. Define \( Y = g(X) \). Then

\[ E[Y] = \int g(x)dF_X(x) \]

Further **assumption**: the indexation process is equal to the adapted notional rate process, i.e., \( RP(t) = l_x(t) \)
The expected value of the $k^{th}$ power of the liquidity-ratio based ABM is thus:

$$E[B_{LR}(s)^k] = E[g_{LR}(s,X)^k] = \int_0^\infty g_{LR}(s,x)^k f_X(x) dx$$

where:

$$g_{LR}(s,X) = \left\{ \frac{K_{NC}^{LR}(s)}{a_s} + K_{NC}^{LR}(s-1)\frac{g_{LR}(s-1,X)(a_{s-1} - 1)}{a_{s-1}} \right\}^{-1}$$

where $X \sim \text{logN}(R - \frac{\sigma_R^2}{2}, \sigma_R^2)$ with density function $f_X(x)$

$$K_{NC}(s) = f_{NC}(D_{s-1}, D_{s-2}, g_{LR}(s-1,X))$$

The $ABM_{LR}$ only depends on the random variable $X$ which corresponds to the demographic risk.
The expected value of the $k^{th}$ power of the solvency-ratio based ABM is thus:

$$E[B_{BR}(s)^k] = E[g_{BR}(s, X)^k] = \int_0^\infty \int_0^\infty g_{BR}(s, x, y)^k f_X(x)f_Y(y)dx\,dy$$

where:

$$g_{BR}(s, X, Y) = \frac{TD(s, X, Y) + f(r, s, X, Y)}{K_V(s, X, Y)}$$

where $X \sim \logN(R - \frac{\sigma_R^2}{2}, \sigma_R^2)$ with density function $f_X(x)$ and

where $Y \sim \logN(R + g - \frac{\sigma_R^2 + \sigma_\gamma^2}{2}, \sigma_{R,\gamma}^2)$ with density function $f_Y(y)$

The ABM$_{SR}$ depends on:

- The random variable $X$ corresponding to the demographic risk;
- The random variable $Y$ corresponding to the notional rate. This dependence comes through the creation of the fund.
The variances and expected values will be studied in 3 different scenarios for both ABM:

1. Base: No longevity trend, $p_t = p \ \forall t$;
2. Up: Upward longevity trend, $p_t > p_{t-1} \ \forall t$;
3. Down: Downward longevity trend, $p_t < p_{t-1} \ \forall t$.

Furthermore, the impact of the exogenous shock $\delta$ will be studied for the three scenarios for both ABM by setting $D_t = D e^{\delta}$.

The following hypotheses are taken:

- $R = 0, 25\%$
- $\sigma_R = 5\%$
- $\gamma = 1, 5\%$
- $\sigma_\gamma = 10\%$
- $S(y, 0) = 30000$
- $S(y + 1, 0) = 45000$
- $\rho = -0.25$
- $p_0 = 0.5$
- $i = 2\%$
- $\delta = 5\%$

The graphics will show the ratio between the variance with ABM over the variance without ABM.
Numerical illustration-No baby boom

Figure: Replacement rate’s variances - No baby boom

(a) Base scenario
(b) Up scenario
(c) Down scenario

- Base scenario: variance is the same for the three cases. As $p_t = p$ the $E[B_x(s)] = 1$ for $x=LR,SR$
- Up (down) scenario: variance is always higher (lower respectively) for both $ABM_{LR}$ and $ABM_{SR}$ than the scenario without ABM
- After two periods the $ABM_{LR}$ has a lower variance than $ABM_{SR}$.
- On the long run both variances converge to the same level.
Base scenario: variances are very volatile but converge to the variance when no ABM is used once that the baby boom effect has disappeared.

Up and down scenario: both ABM variances are really volatile during the first five periods. It seems difficult to choose between the better one in terms of variance.

On the long run both variances converge to the same level.
Further research

- **Annuity design:**
  - Choice between different levels of indexation;
  - Choice between projected or observed mortality values;
  - Choice of adjustments, if any, according to the real mortality experience.

- Influence of the decisions if a **mixed** plan: optimal choice between funding and PAYG.

- Pension reform **transition**: Cost of this transition and ways of optimizing it.

- NDC plans with **minimum pensions**: Calculation of the cost of the guarantee through option pricing.


Thank you