Young Researchers' Day

6 February, 2015

9^{00}	**Anna Kiriliouk**	Nonparametric estimation of extremal dependence
and	**Johan Segers**	
9^{45}	**Samuel Gbari**	Stochastic approximations in mortality projection models
10^{00}	**Nicolas Asin**	Adaptive nonparametric estimation in the presence of dependence

Coffee Break

11^{00}	**Jennifer Alonso Garcia**	Guarantee valuation in Notional Defined Contribution pension systems
11^{15}	**Nathan Uyttendaele**	Nested Archimedean Copulas: A Short Research Story
11^{45}	**Gildas Mazo**	Construction and estimation of high-dimensional copulas

The seminar is followed by the annual lunch of ISBA at "Trattoria", Avenue Georges Lemaître 5.

Nonparametric estimation of extremal dependence

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Let $(X_1, Y_1), \ldots, (X_n, Y_n)$ denote independent and identically distributed bivariate random vectors with marginal distribution functions F_X and F_Y, representing the negative log-returns of two series of stock prices. By taking negative log-returns we force (extreme) losses to be in the upper tails of the distribution functions. It is such extreme values, and in particular their simultaneous occurrence, that are the focus of this talk. We are interested in quantifying the degree of tail dependence between the two stocks, i.e., the frequency of joint occurrences of large losses. To this end, we will introduce the *stable tail dependence function* $\ell : [0,\infty)^2 \to [0,\infty)$, defined by

$$\ell(x,y) = \lim_{t \to 0} t^{-1} \mathbb{P}[1 - xt \le F_X(X_1) \text{ or } 1 - yt \le F_Y(Y_1)], \qquad (x,y) \in [0,\infty)^2,$$

which represents the scenario where at least one of the variables is large, i.e., at least one of the stocks in our portfolio is crashing. We will link the stable tail dependence function to the *convergence of componentwise maxima*: if $M_{n,X} = \max(X_1, \ldots, X_n)$ and $M_{n,Y} = \max(Y_1, \ldots, Y_n)$, then for suitable sequences of normalizing constants $a_n > 0$, $b_n \in \mathbb{R}$, $c_n > 0$, $d_n \in \mathbb{R}$, we can characterize the limiting distribution of

$$\mathbb{P}\left[\frac{M_{n,X} - b_n}{a_n} \le x, \frac{M_{n,Y} - c_n}{d_n} \le y\right], \qquad \text{as } n \to \infty.$$

References

[1] Kiriliouk, A., Segers, J. and Warchoł, M. (2015), Nonparametric estimation of extremal dependence. To appear in the book *Extreme Value Modelling and Risk Analysis: Methods and Applications*, edited by Jun Yan.

Stochastic approximations in mortality projection models

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In portfolios of life annuity contracts, the payments made by an annuity provider are driven by the random numbers of survivors which have an intricate distribution function leading to complex simulation procedure. We first provide accurate stochastic approximations for the random numbers of survivors in a mortality model with a univariate time index. Hence, we derive approximations for various quantities of interest for practitioners. Then, we extend our results to a mortality model with a bivariate time index. These approximations which are valid whatever the portfolio size, account not only for systematic longevity risk but also for the diversifiable fluctuations around the unknown life table. They also provide practitioners with a useful tool avoiding simulations within simulations problem in Solvency 2 computations.

References

[1] M. Denuit and J. Dhaene (2007), Comonotonic bounds on the survival probabilities in the lee-carter model for mortality projection. *Computational and applied mathematics*, 203, 169–176.

[2] S. Gbari and M. Denuit (2014), Efficient approximations for numbers of survivors in the lee-carter model. *Insurance Mathematics and Economics*, 59, 71–77.

[3] S. Gbari and M. Denuit (2014), Stochastic approximations in CBD mortality projection models. *Discussion paper.*

Adaptive nonparametric estimation in the presence of dependence

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We consider nonparametric estimation problems in the presence of dependent data, notably nonparametric regression with random design and nonparametric density estimation. The proposed estimation procedure is based on a dimension reduction. The minimax optimal rate of convergence of the estimator is derived assuming a sufficiently weak dependence characterized by fast decreasing mixing coefficients. We illustrate these results by considering classical smoothness assumptions. However, the proposed estimator requires an optimal choice of a dimension parameter depending on certain characteristics of the function of interest, which are not known in practice. The main issue addressed in our work is an adaptive choice of this dimension parameter combining model selection and Lepski's method. It is inspired by the recent work of []. We show that this data-driven estimator can attain the lower risk bound up to a constant provided a fast decay of the mixing coefficients.

References

[1] Goldenshluger, A., & Lepski, O. (2011). Bandwidth selection in kernel density estimation: oracle inequalities and adaptive minimax optimality. *The Annals of Statistics*, 39(3), 1608-1632.

Guarantee valuation in Notional Defined Contribution pension systems

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The notional defined contribution pension scheme combines pay-as-you-go financing and a defined contribution pension formula. The returns on contributions are calculated utilizing a notional rate that reflects the financial health of the system, which is linked to an external index set by law, such as the growth rate of GDP, average wages, or contribution payments [4]. However, the volatility of this rate may introduce a pension adequacy problem in the system and therefore guarantees may be needed. Here we focus on the minimum return on the contributions made to the pension scheme and we calculate its price by means of put option pricing [5]. In pay-as-you-go financed pension system we face the problem that we guarantee returns based on assets which cannot be traded, due to their unfunded nature. This problems leads to contingent claim valuation in incomplete markets. In this context uniqueness of a risk-neutral measure is not assured and individual risk preference has to be introduced [1]. In this paper we use the theory of utility indifference pricing ([2]) to obtain a closed-form solution of the guarantee under exponential utility and with presence of stochastic interest rates [3]. The idea is to price the option on the untraded asset by using a traded asset which is correlated to it as a proxy. The obtained formulae is used to value different interest rate guarantees on a notional defined contribution pension scheme.

References

[1] Henderson, V. (2002). Valuation of claims on nontraded assets using utility maximization. *Mathematical Finance*, 12(4), 351-373.

[2] Hodges, S. D., & Neuberger, A. (1989). Optimal replication of contingent claims under transaction costs. *Review of futures markets*, 8(2), 222-239.

[3] Korn, R., & Kraft, H. (2002). A stochastic control approach to portfolio problems with stochastic interest rates. *SIAM Journal on Control and Optimization*, 40(4), 1250-1269.

[4] Palmer, E. (2006). What is NDC?, in: R. Holzmann and E. Palmer, eds., *Pension Reform: Issues and Prospects for Notional Defined Contribution (NDC) Schemes*, (Washington, D.C.: The World Bank), chapter 2. ISBN 0-8213-6038-8.

[5] Pennacchi, G. G. (1999). The value of guarantees on pension fund returns. *Journal of Risk and Insurance*, 219-237.

Nested Archimedean Copulas: A Short Research Story

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Copulas are wonderful functions, allowing to study how random variables interact with one another or to build new cumulative distribution functions with arbitrary margins. In this talk, the three main properties any copula must have in order to be a proper copula will be reviewed. It will then be shown how checking for these properties allowed a researcher and his thesis advisor to settle an argument they had about what a nested Archimedean copula can and cannot be, nested Archimedean copulas being formally introduced along with the general notion of copulas.

Construction and estimation of high-dimensional copulas

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In the last decades, copulas have been more and more used in statistical modeling. Their popularity owes much to the fact that they allow to separate the analysis of the margins from the analysis of the dependence structure induced by the underlying distribution. This renders easier the modeling of non Gaussian distributions, and, moreover, it allows to take into account non linear dependencies between random variables. Finance and hydrology are two examples of scientific fields where the use of copulas is nowadays standard. Since there exists many families of bivariate copulas, it is always possible for the user to choose one that suits his/her needs. Unfortunately, the multivariate case is not that simple. The range of these models is still not rich enough for the user to choose one that satisfies all the desired properties. This talks – strongly based on our PhD defense, if not identical – addresses this issue. We propose two classes of multivariate copulas with novel properties, resulting in an enlargement of the range of the existing models. The first model writes as a product of bivariate copulas and is underlain by a tree structure where each edge represents a bivariate copula. Hence, we are able to model different pairs with different dependence properties. The second one is a factor model, with a singular component, built on a nonparametric class of bivariate copulas. It exhibits a good balance between tractability and flexibility. Since the copulas belonging to the second proposed class have a singular component, the standard methods of inference do not permit to estimate their parameters. For this reason, we also deal with the estimation of copulas in general, and establish the asymptotic properties of a least-squares estimator based on dependence coefficients without imposing regularity conditions on the copulas. The models and methods have been applied to hydrological data (flow rates and rain falls).